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SERIES 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 7

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An application of the extended mirror images method in constructing equivalent models for calculating the vector potential of the magnetic field produced from current fibre in inhomogeneous dielectric medium

AN APPLICATION OF THE EXTENDED MIRROR IMAGES METHOD IN CONSTRUCTING EQUIVALENT MODELS FOR CALCULATING THE VECTOR POTENTIAL OF THE MAGNETIC FIELD PRODUCED FROM CURRENT FIBRE IN INHOMOGENEOUS DIELECTRIC MEDIUM

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Abstract: *In this paper the foundation of the extended mirror images method is discussed. The application of the method in determining of the vector potential of the magnetic field produced by a current fibre is described in details. In a case of different dielectric media or ground planes parallel to the fibre the real case is replaced by a suitable equivalent model constructed by one, two or more current fibres located in a homogeneous medium. Equivalent models for calculating the vector potential concerning microwave transmission lines like microstrip, reversed microstrip and embedded strip lines are proposed.*
Keywords: *microwaves, strip transmission lines, microstrip, reversed microstrip, embedded strip line*

INTRODUCTION

Strip structures, like single microstrip, reversed microstrip and embedded microstrip transmission lines (Fig. 1), as well as similar lines containing more then one conductor are used to construct MICs, antennas and many other microwave devises. The interest in this field rises nearly 50 years ago and a lot of different approaches like conformal mapping [5], the method of grids [2], different methods based on a Green function [1] and others were proposed Different empirical formulae for calculating the electrical parameters of some lines are also available in the literature. In spite of the microstrip transmission line is well studied the other two lines in Fig.1 are not so well investigated until now. If transverse electromagnetic wave is transmitted by the strip line then the universal method for calculating the characteristic impedance ***Z*** can be used [3]. The main advantage of this method is that it can be applied directly in a case when a strip line has arbitrary number of parallel conductors no matter of the conductors' cross sections shapes.

The disadvantage of this method is that all conductors of the strip line must be disposed in a homogeneous dielectric medium.

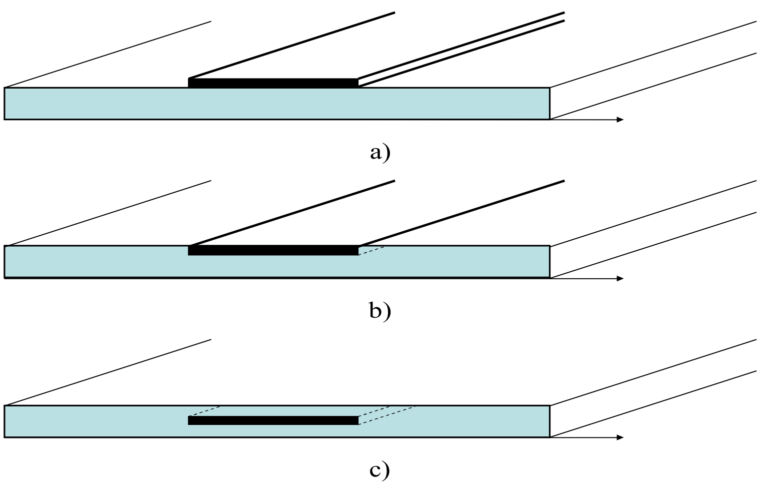


Fig. 1. a) Microstrip; b) reversed microstrip; c) embedded strip line

So this method can't be used directly for calculating the characteristic impedance of microwave transmission lines usually used in practice because they have not homogeneous dielectric filling. The universal method together with the extended mirror images method is used in [4] for calculating the characteristic impedance of microstrip and coupled strip transmission lines. The role of the extended mirror images method is to exchange the real configuration – ground plane, dielectric slab, strip conductor and ear (Fig.1a) to equivalent model containing strip conductors located in homogeneous medium (ear). The results received in [4] give a reason for the idea that the extended mirror images method could be applied for calculation the characteristic impedance of reversed and embedded microstrip and relevant coupled transmission lines.

The aim in this study is a systematic description on how to construct the equivalent models with the help of the extended mirror images theory and how well they apply to different microwave strip transmission lines. For completeness some results shown in [4] are repeated. The idea is developed for the vector potential ***A(x,y)*** of the magnetic field created by an indefinitely thin and long fibre driving DC whit magnitude I_0 if there are ground planes and dielectric slabs parallel to the fibre.

CONSTRUCTING OF EQUIVALENT MODELS BY EXTENDED MIRROR IMAGES METHOD

1. Vector potential created by a current fibre in a homogeneous medium

In constructing of all further models it is assumed that the magnetic field is created by one, two or more current fibres laying in homogeneous dielectric medium.
So the vector potential of the magnetic field at a definite point can be considered as a superposition of the individual vector potentials created by each fibre at this point.

Thus, the base of all models is the vector potential created by one current fibre located in an infinite homogeneous dielectric medium.

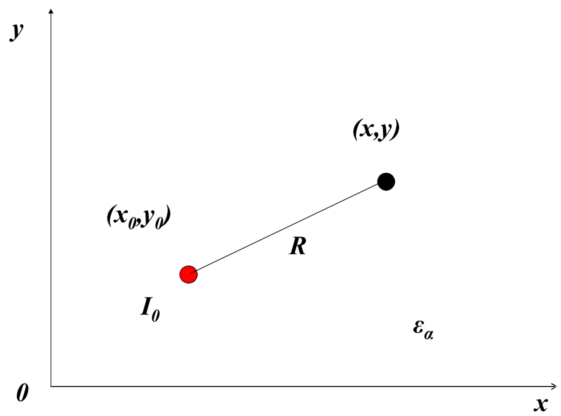


Fig. 2. Current fibre, located in homogeneous medium with electrical material constant ε_α

Due to the axis symmetry, it is more comfortable the coordinate system to be taken such that the current fibre is parallel to the axis ***z*** and all the others quantities to be represented in the plane ***z=0***. If the current fibre crosses the plane at the point (x_0,y_0) (Fig. 2), then an magnetic field is created at the point ***(x,y)*** with vector potential $\vec{A}$ having one component along the ***z*** axis, and could be presented with the equality as shown in [6]:

$$A(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_\alpha}\ln\left[(x-x_0)^2+(y-y_0)^2\right] = -\frac{\mu_0 I_0}{4\pi\varepsilon_\alpha}\ln R^2 , \qquad (1)$$

where, I_0 is the current flowing along the fibre in z direction, μ_0 is the magnetic constant in vacuum, ε_α is the relative dielectric permittivity (electric material constant) of the medium, and $\boldsymbol{R}$ is the distance between the current fibre and the point, in which the potential is defined.

2. Current fibre in an inhomogeneous dielectric medium

The special feature of the strip transmission lines is that the different dielectric media are separated from one another with surfaces parallel to the strip conductors or to the current fibre for the investigated case. That is why, firstly the case when the space is separated into two media with different dielectric fillings from the plane $\boldsymbol{y=d}$ is inspected. Conditionally, the area $\boldsymbol{y \geq d}$ is marked with $\boldsymbol{\alpha}$ and all the quantities, concerning this medium have the indices $\boldsymbol{\alpha}$ and the area $\boldsymbol{y \leq d}$ is marked with $\boldsymbol{\beta}$. Current fibre is located in the area $\boldsymbol{\alpha}$ when it crosses the plane $\boldsymbol{z=0}$ in the point $\boldsymbol{(x_0, d+a)}$ (see Fig. 3a).

The current fibre creates magnetic field in the both areas. So for the calculation of the vector potential for each of them it is necessary to construct two different equivalent models (Fig.3b,c)

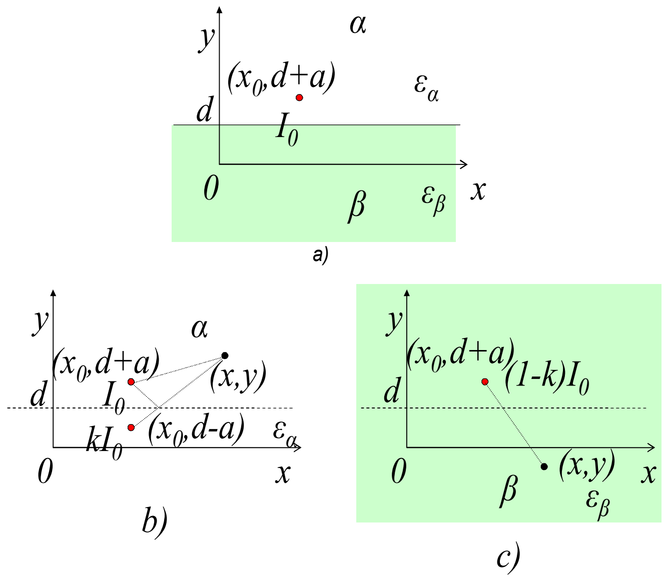


Fig. 3. a) Current fibre, located in inhomogeneous dielectric media; b) the model equivalent for region α ; c) the equivalent models for region β .

For the area **α** equivalent model is constructed from two current fibres, situated in homogeneous medium with material constant $\boldsymbol{\varepsilon_\alpha}$. The first is the original one with coordinates $\boldsymbol{(x_0, d+a)}$ with current of $\boldsymbol{I_0}$, and the second is the mirror image relative the plane $\boldsymbol{y=d}$ with coordinates $\boldsymbol{(x_0, d-a)}$ with a current of $\boldsymbol{kid_0}$ (Fig. 3.b). The quantity $\boldsymbol{\kappa}$ could be accepted as a coefficient of reflection from the boundary surface between the two dielectrics. It depends on the electrical material constants $\boldsymbol{\varepsilon_\alpha}$ and $\boldsymbol{\varepsilon_\beta}$ respectively of the both areas and is defined through the formula:

$$k = \frac{\varepsilon_\alpha - \varepsilon_\beta}{\varepsilon_\alpha + \varepsilon_\beta} \qquad (2)$$

The vector potential $\boldsymbol{A(x,y)}$ in the area $\boldsymbol{y \geq d}$ is calculated according the principle of the superposition – the sum of the vector potentials of the two 'sources' – the real fibre and its image, and it follows:

$$A_\alpha(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_\alpha}\{\ln[(x-x_0)^2+(y-(d+a))^2]+k\ln[(x-x_0)^2+(y-(d-a))^2]\}. \qquad (3)$$

The vector potential depends on ε_β through the coefficient ***k***. When ε_β increases infinitely (e.g. for ideal conductor $\varepsilon_\beta = \infty$), the coefficient $k = -1$ and the equation (3) expresses the application of the classical method of mirror images.

The equivalent model for calculating the vector potential in the area $y \le d$ shows that the source of the field is the same. Current fibre is located at point $(x_0, d+a)$ in homogeneous dielectric medium with material constant ε_β, but the current is $(1-k)I_0$. For this area, the vector potential is calculated following the formula: (6)

$$A_\beta(x,y) = -\frac{\mu_0(1-k)I_0}{4\pi\varepsilon_\beta}\left\{\ln\left[(x-x_0)^2 + (y-(d+a))^2\right]\right\} \tag{4}$$

where the constant ***k*** has the same meaning as previously discussed. It is obvious, that when $\varepsilon_\beta \to \infty$, $A_\beta(x,y) \to 0$.

At the points of the boundary surface (x,d) the expressions for the vector potential (3) and (4) are equivalent.

$$A_\alpha(x,d) = A_\beta(x,d) = -\frac{2\mu_0 I_0}{4\pi(\varepsilon_\alpha + \varepsilon_\beta)}\ln\left[(x-x_0)^2 + a^2\right]. \tag{5}$$

In the case, when the current fibre is located in the area β (Fig.4), the equivalent models are constructed analogously to the previous one.

$$A'_\alpha(x,y) = -\frac{\mu_0(1-k')I_0}{4\pi\varepsilon_\alpha}\left\{\ln\left[(x-x_0)^2 + (y-(d-a))^2\right]\right\}; \tag{6.1}$$

$$A'_\beta(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_\beta}\{\ln[(x-x_0)^2 + (y-(d-a))^2] + k'\ln[(x-x_0)^2 + (y-(d+a))^2]\} \tag{6.2}$$

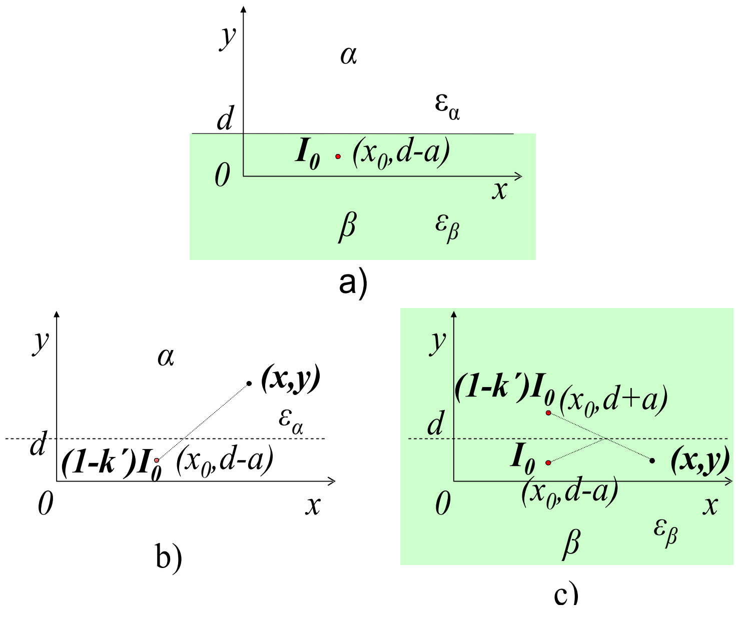


Fig. 4. a) Current fibre, located in an inhomogeneous dielectric medium; b) the equivalent model for region α; c) equivalent model for region β.

k´ is: $k' = \dfrac{\varepsilon_\beta - \varepsilon_\alpha}{\varepsilon_\alpha + \varepsilon_\beta} = -k.$

When the dependence of ***A*** on ε_α is investigated, the same conclusions follow: If $\varepsilon_\alpha \to \infty$ then the vector potential of the magnetic field is different from zero only in the area $y \le d$ and (6.2) corresponds to applying the classical method of the mirror images.

In this case it is also fulfilled: $A'_\alpha(x,d) = A'_\beta(x,d)$

3. Current fibre located above a dielectric slab

The next task is to construct an equivalent model for the case, when the current fibre is located above a dielectric slab (Fig.5). The space is separated into three regions, marked respectively with ***α, β*** and ***γ***. The existence of two parallel boundaries between the dielectrics leads to equivalent models, consisting infinite number of images. The construction of the models for the different areas in Fig. 5 is made in a different way and gives different expressions for calculation of the ***A*** for each of the three areas.

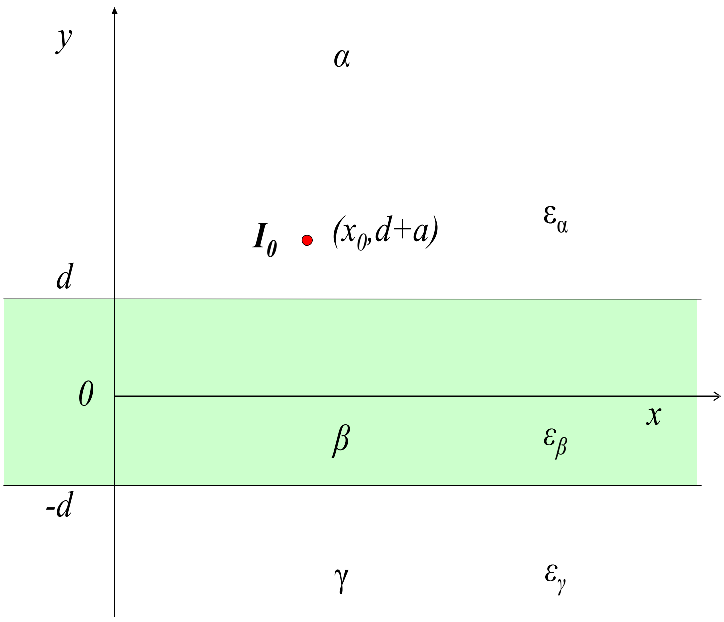


Fig. 5. Current fibre located above a dielectric slab

The three areas ***α***, ***β*** and ***γ*** have different homogeneous (for the area) dielectric filling, but for the analysis of strip transmission lines in Fig. 1 construction of the equivalent models for the case, when the dielectric environment is the same above and below the dielectric slab ($\varepsilon_\alpha=\varepsilon_\gamma$), is sufficient. This does not restrict the application of the examined method and it is applied in the same way for the cases when ($\varepsilon_\alpha \neq \varepsilon_\gamma$).

The equivalent model for the area ***α*** is an infinite number of current fibres, located in homogeneous medium with material constant ε_α with the corresponding coordinates and currents, as it is shown in Fig. 6. The vector potential of the magnetic field ***A(x,y)*** for $y \geq d$ is a superposition of the vector potentials in the point, created from all current fibre. ***k*** and ***k´*** are accepted as reflection coefficients from the first and the second dielectric boundary. In a similar way the equivalent models are constructed for the rest two areas of the space, shown in Fig. 6.

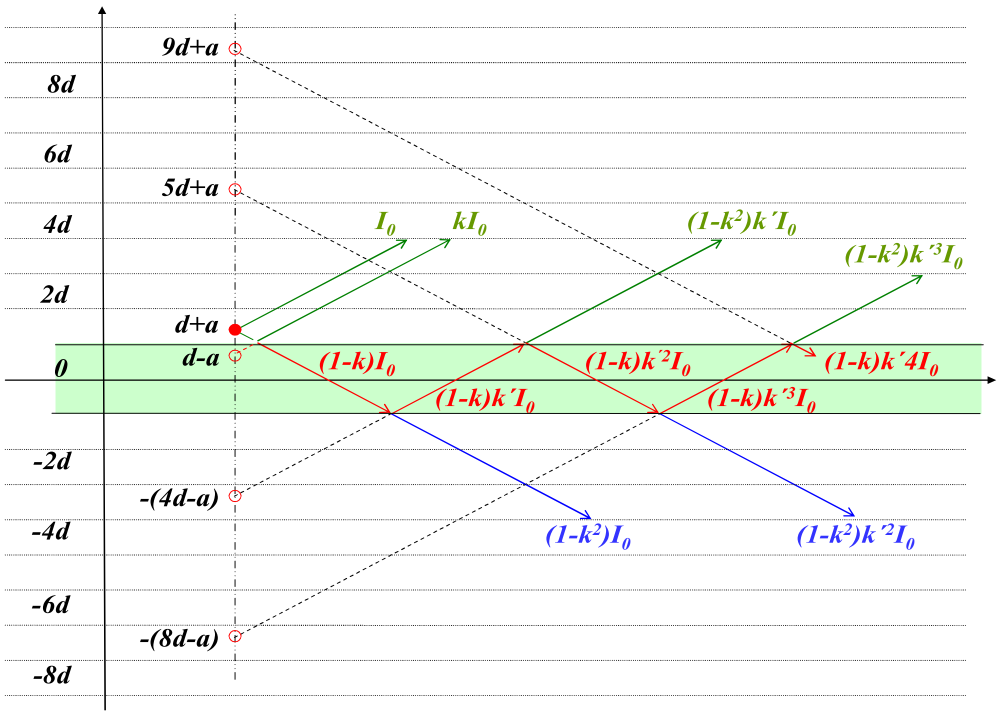


Fig. 6. Chart for construction of the equivalent models of a current fibre, located over infinite dielectric slab.

Analytically, the vector potentials for every of the areas are represented with the following formulas: (8.1; 8.2; 8.3).

$$A_\alpha(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_a}\{\ln[(x-x_0)^2+(y-(d-a))^2]+k\ln[(x-x_0)^2+(y-(d-a))^2]- \\ -(1-k^2)\sum_1^\infty k'^{(2n-1)}\ln[(x-x_0)^2+(y+((4n-1)d+a))^2] \quad ; \qquad (8.1)$$

$$A_\beta(x,y) = -\frac{\mu_0(1-k)I_0}{4\pi\varepsilon_\beta}\{\sum_{n=0}^\infty k'^{2n}\ln[(x-x_0)^2+(y-((4n+1)d+a))^2 - \\ -\sum_{n=1}^\infty k'^{(2n-1)}\ln[(x-x_0)^2+(y+((4n-1)d+a))^2]\} \quad ; \qquad (8.2)$$

$$A_\gamma(x.y) = -\frac{\mu_0(1-k)^2}{4\pi\varepsilon_\alpha}\sum_0^\infty k'^{2n}\ln[(x-x_0)^2+(y-((4n+1)d+a))^2]. \qquad (8.3)$$

Here again, if ***y = d*** then the equation ***A_α(x,d) = A_β(x,d)*** is fulfilled and if ***y = -d***, then the equation accepts the form of ***A_β(x,-d) = A_γ(x,-d)***.

4. Current fibre located in a dielectric slab.

Equivalent models for the case, shown in Fig.7 are represented in Fig. 8 and the vector potential ***A'*** for the different areas is presented analytically with the following expressions: (9.1; 9.2; 9.3).

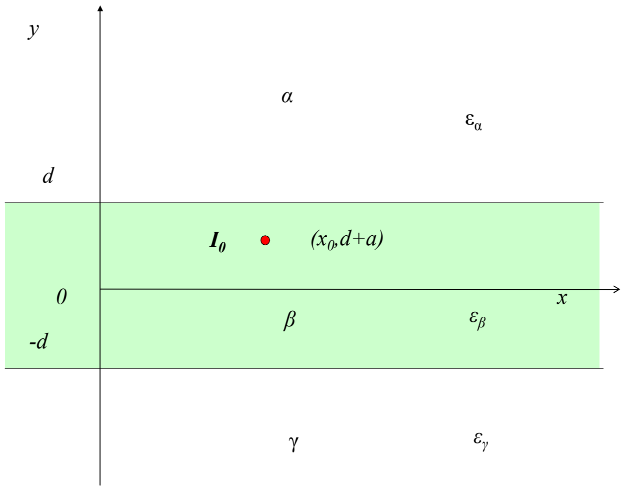


Fig. 7. Current fibre located in a dielectric slab

$$A'_{\alpha}(x,y) = -\frac{\mu_0(1-k')I_0}{4\pi\varepsilon_{\alpha}}\{\sum_{n=0}^{\infty} k^{2n}\ln[(x-x_0)^2+(y+((4n-1)d+a))^2]+ \\ +\sum_{n=1}^{\infty} k^{2n-1}\ln[(x-x_0)^2+(y+((4n-1)d-a))^2]\} \quad ; \qquad (9.1)$$

$$A'_{\beta}(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_{\beta}}\sum_{n=0}^{\infty} k'^{2n}\{\ln[(x-x_0)^2+(y-((4n+1)d-a))^2]+k'\ln[(x-x_0)^2+(y-((4n+1)d+a))^2]+ \\ +k'\ln[(x-x_0)^2+(y+((4n+3)d-a))^2]+k'^2\ln[(x-x_0)^2+(y+((4n+3)d+a))^2]\} \quad ; \qquad (9.2)$$

$$A_{\gamma}(x,y) = -\frac{\mu_0(1-k')I_0}{4\pi\varepsilon_{\alpha}}\{\sum_{n=0}^{\infty} k'^{2n}\ln[(x-x_0)^2+(y-((4n+1)d-a))^2]+ \\ +\sum_{n=1}^{\infty} k'^{(2n-1)}\ln[(x-x_0)^2+(y-((4n+1)d+a))^2]\} \quad . \qquad (9.3)$$

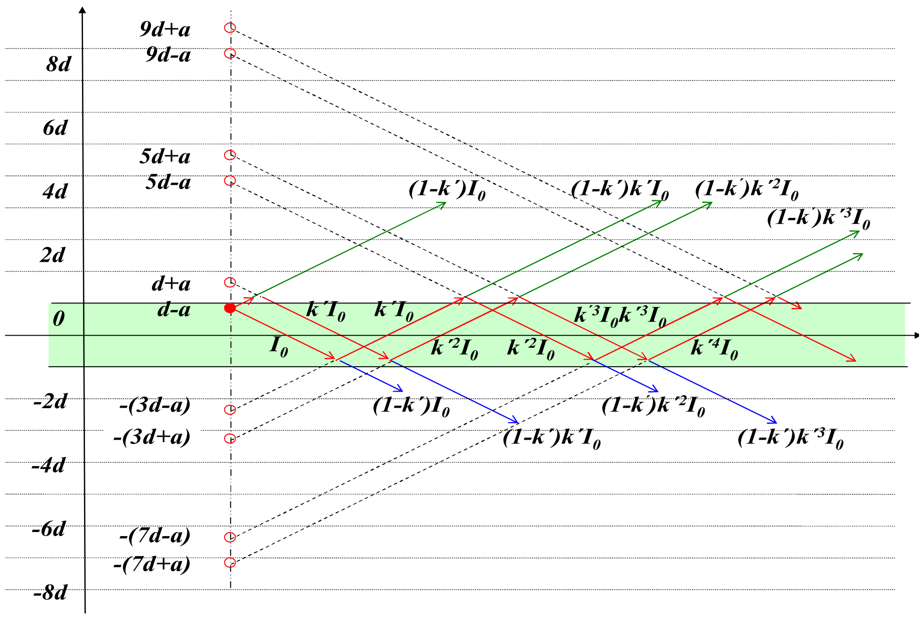


Fig. 8. Equivalent model for the case, when the fibre current is located in dielectric slab

If $\varepsilon_\alpha = \varepsilon_\gamma = \infty$ is accepted than the equations above show that only $A_\beta \neq 0$. In this case equation (9.2) presents the application of the classical mirror images method when the current fibre is located between two parallel conducting ground planes.

The extended method of mirror images can be used following the same procedure if there are two, three or more different dielectric slabs. This is giving the possibility more complicated microwave strip transmission lines to be investigated.

5. Current fibre located above a dielectric slab put on a conductive ground plane

The construction of the equivalent model for the case corresponding to microstrip is possible if the classical (Fig. 9), and the expanded mirror images methods are combined.

For microstrip, a certain interest is given to the vector potential of the field above the dielectric slab ***(y ≥ d)***. For the upper fibre this area corresponds to region α and for the lower fibre the same area corresponds to region γ according to Fig. 5

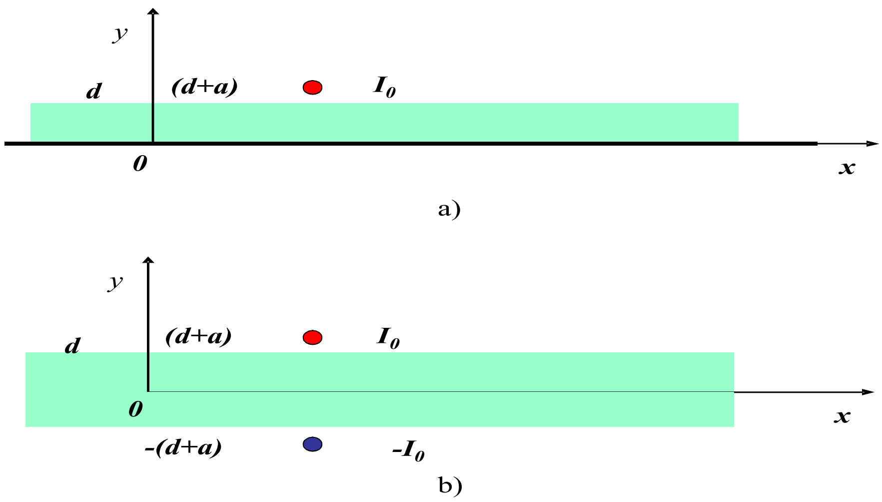


Fig. 9. a) The case relevant to microstrip transmission line; b) The model after applying the classical mirror images method.

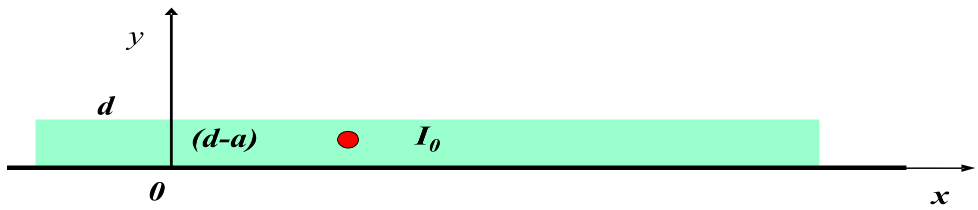
The magnetic field and its vector potential are a superposition of magnetic fields and vector potentials created by both current fibres presented by their equivalent models. The final result for the vector potential in this case is as follows:

$$A_\alpha(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_\alpha}\{\ln[(x-x_0)^2 + (y--d-a)^2] + k\ln[(x-x_0)^2 + (y-d+a)^2] - \\ -(1-k^2)\sum_0^\infty k^n \ln[(x-x_0)^2 + (y+(2n+1)d+a))^2]\} \quad . \qquad (10)$$

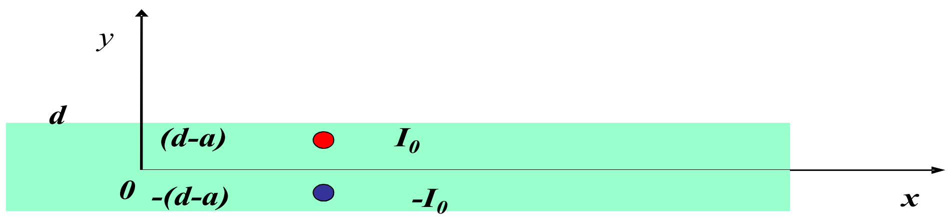
6. Current fibre located in a dielectric slab put on a conductive ground plane.

The equivalent model for this real situation reveals the possibility the reversed microstrip and embedded microstrip lines (Fig. 1b, 1c) to be analyzed. Like in the previous case the classical (Fig.10) and the extended mirror images methods are used together.

The magnetic field at any point in β region is a result of superposition of the magnetic fields of both fibres – the real one with coordinates ***(x_0,d-a)*** driving current I_0 and its classical image with coordinates ***(x_0,-(d-a))*** and current $-I_0$. Applying the extended mirror images method for both magnetic fields' sources for the vector potential of the magnetic field in ***β*** region the following expression is received:



a)



b)

Fig. 10. a) The case relevant to with reversed and embedded microstrip transmission lines; b) The model after applying the classical mirror images method.

$$A_\beta(x,y) = -\frac{\mu_0 I_0}{4\pi\varepsilon_\alpha}\sum_{n=0}^{\infty}(-k')^n\{\ln[(x-x_0)^2+(y-(2n+1)d+a)^2]-\ln[(x-x_0)^2+(y+(2n+1)d-a)^2]+ k'\ln[(x-x_0)^2+(y-(2n+1)d-a)^2]-k'\ln[(x-x_0)^2+(y+(2n+1)d+a)^2]\} \quad (11)$$

This result can be used for calculation of the characteristic impedance of reversed and embedded microstrip lines as well as it is done for microstrip in [5].

CONCLUSION

The extended method of mirror images includes in itself the classical mirror images method and finds more wide application. In this paper only horizontal slabs and ground planes are considered but it can be applied if there are vertical dielectric slabs and ground planes too. This is giving the possibility not only open but also partially or fully shielded microwave strip transmission lines constructed by one two or more strip conductors to be investigated.

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ПРИЛОЖЕНИЕ НА РАЗШИРЕНИЯ МЕТОД НА ОГЛЕДАЛНИТЕ ОБРАЗИ ЗА СЪЗДАВАНЕ НА ЕКВИВАЛЕНТНИ МОДЕЛИ ЗА ПРЕСМЯТАНЕ НА ВЕКТОРНИЯ ПОТЕНЦИАЛ НА МАГНИТНОТО ПОЛЕ СЪЗДАДЕНО ОТ ТОКОВА НИШКА В НЕХОМОГЕННА ДИЕЛЕКТРИЧНА СРЕДА

Владимир Войнов, Роза Войнова, Златка Матева

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Резюме: *В статията са разгледани основите на разширения метод на огледалните образи. Подробно е обсъдено прилагането на метода при определяне на векторния потенциал на магнитното поле създадено от токова нишка. В случая, когато има диелектрични пластини и заземени повърхнини, разположени успоредно на токовата нишка, реалната структура се заменя с еквивалентен модел съдържаш една, две или повече токови нишки намиращи се в хомогенна среда. Предложени са еквивалентни модели за пресмятане на векторния потенциал за случаите, съответстващи на микролентова, обърната микролентова и потопена лентови линии.*

Ключови думи*: микровълни, лентови предаващи линии, микролентова, обърната микролентова потопена лентова линия.*

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