PROCEEDINGS

of the Union of Scientists - Ruse

Book 5

Mathematics, Informatics and Physics

Volume 8, 2011

RUSE

ISSN 1311-9974

The Ruse Branch of the Union of Scientists in Bulgaria was founded in 1956. Its first Chairman was Prof. Stoyan Petrov. He was followed by Prof. Trifon Georgiev, Prof. Kolyo Vasilev, Prof. Georgi Popov, Prof. Mityo Kanev, Assoc. Prof. Boris Borisov, Prof. Emil Marinov. The individual members number nearly 300 recognized scientists from Ruse, organized in 13 scientific sections. There are several collective members too – organizations and companies from Ruse, known for their success in the field of science and higher education, or their applied research activities. The activities of the Union of Scientists – Ruse are numerous: scientific, educational and other humanitarian events directly related to hot issues in the development of Ruse region, including its infrastructure, environment, history and future development; commitment to the development of the scientific organizations in Ruse, the professional development and growth of the scientists and the protection of their individual rights.

The Union of Scientists – Ruse (US – Ruse) organizes publishing of scientific and popular informative literature, and since 1998 – the “Proceedings of the Union of Scientists- Ruse".

BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 8

CONTENTS

BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 8

Education

Participation in International Projects

DETERMINING THE LATERAL OSCILATIONS NATURAL FREQUENCY OF A BEAM FIXED AT ONE END

Tsanko Karadzhov, Nikolay Angelov

Technical University of Gabrovo

Abstract: *A method for determining the transverse vibrations of a beam fixed at one end has been discussed. During the laboratory exercise, students explore the relationships between quantities and compare experimental and theoretical results. A relationship between the disciplines of Physics and Noise and Vibration, which further motivates the students after they find out that each subject is important for the learning process, has been shown.*
Keywords: *beam, transverse vibrations, modulus of elasticity.*

INTRODUCTION

Oscillation processes play an important role in engineering. According to their physical nature oscillations can be divided into mechanical, thermal and electrical. As it is well known, the following types of oscillations can be further distinguished - free continuous oscillations, freely dying oscillations, induced oscillations, parametric oscillations, self-excited oscillations, etc. Regardless of their type they are defined by the same physical laws and properties. Quite frequently oscillation processes in engineering are a sum of free oscillations, which makes the study of the latter extremely important in practice. Such oscillations are the lateral oscillations of a beam, a bar or a strip fixed at one end. These oscillations are used not only in engineering but also in some musical instruments like harmonica, xylophone and tuning fork [1].

EXPOSITION

The goal of the laboratory exercise for the students is to extend their knowledge of oscillation processes by learning a method of determining the natural frequencies of free, lateral oscillations of a beam fixed at one end; to create graphs to demonstrate the relationship between the natural oscillations and the length and thickness of the beam; to compare the theoretical and experimental curves.

The lateral oscillations of a beam are normally caused by the elastic bending strains. They are defined by a partial-differential equation. For the solution of the equation to be one-valued the respective initial and boundary conditions are used. The differential equation of free lateral oscillations of straight prismatic beams with distributed mass is worked out by using dynamic force analysis [2, 3, 4]. It is known from theory that the differential equation of the elastic line is as follows:

$$EI\frac{\partial^2 y}{\partial x^2} = M_b \qquad (1)$$

where *I* is the geometric inertia moment of the beam; *E* – modulus of elasticity; *y=y(x,t)* - deflection, M_b – bending moment.

The dependency

$$q = \frac{\partial^2 M_b}{\partial x^2} \qquad (2)$$

is used when spread load with intensity *q(x,t)* is present.

With free lateral oscillations the intensity of the lateral oscillations is determined by the apparent forces

$$q = S\rho \frac{\partial^2 y}{\partial t^2} \tag{3}$$

where S is the cross sectional area, ρ – the density of the material.

The condition of a prismatic beam is S=const and I=const. Double differentiating of (1) in relation to x and comparing (1), (2) and (3) results in

$$\frac{EI}{S\rho}\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} = 0 \tag{4}$$

Equation (4) defines free lateral oscillations of a straight prismatic beam. It is a linear, homogenous, partial-differential equation with constant coefficients. Its solution is of the type:

$$y(x,t) = X(x).T(t) \tag{5}$$

where *X(x)* is a function, depending on x; *T(t)* is a function, depending on t.

The displacement and the angle of inclination at the built-in end are equal to zero. The following boundary conditions are derived:

$$X(0) = 0 \quad \text{and} \quad \frac{\partial X}{\partial x}(0) = 0 \tag{6}$$

for the built-in end

$$X(\ell) = 0 \quad \text{and} \quad \frac{\partial X}{\partial x}(\ell) = 0 \tag{7}$$

for the free end.

For the natural frequencies of the lateral oscillations of a beam fixed at one end the following expression is derived:

$$f_i = 0{,}046(2i-1)^2 \frac{\pi^2 h}{4\ell^2}\sqrt{\frac{E}{\rho}}, \qquad i = 1, 2, 3, 4, \tag{8}$$

where h is the thickness of the beam and ℓ is the length of the beam.

For the first three natural frequencies of the lateral oscillations of a beam fixed at one end the following expression is derived

$$f_1 = 0{,}046 \frac{\pi^2 h}{4\ell^2}\sqrt{\frac{E}{\rho}}, \tag{9}$$

$$f_2 = 0{,}046 \frac{9\pi^2 h}{4\ell^2}\sqrt{\frac{E}{\rho}}, \tag{10}$$

$$f_3 = 0{,}046 \frac{25\pi^2 h}{4\ell^2}\sqrt{\frac{E}{\rho}}. \tag{11}$$

Scheme of the experiment and tasks

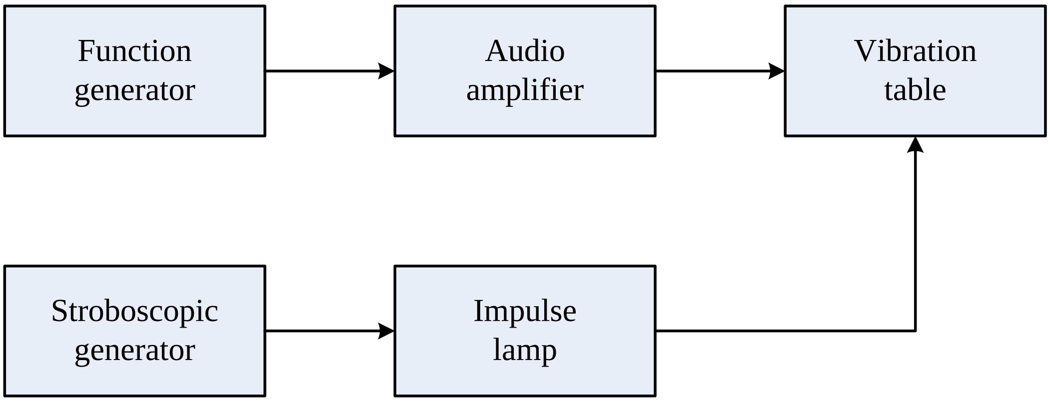

Fig.1. Scheme of the experiment

The sample is fixed on the vibrating table and the frequency of oscillations is changed by function generator while the beam begins to vibrate with maximum amplitude – until a resonance is achieved. The frequency of the blink of the pulse lamp of the stroboscopic generator changes until the image of the beam becomes stationary. Then the frequency is given from the stroboscopic generator.

The following tasks are to be accomplished:

1. To determine the relationship between the natural oscillations of a beam fixed at one end and its length – $f_{exp} = f(\ell)$ and to check it with the theoretical relationship $f_{theor} = f(\ell)$.

The students are to determine the relationship either for the first, or the second or the natural frequency following the instructions of the teacher.

Example: Determine the relationship $f_1 = f_1(\ell)$ for the first natural frequency of a steel beam with the following characteristics:

modulus of elasticity $E = 3{,}12.10^{11}$ N/m^2;
density $\rho = 7{,}74.10^3$ kg/m^3;
thickness $h = 2{,}80$ mm.

Experimental results:

Table 1

ℓ, mm	265	250	235	220	205	190	175	160
f_{1e}, Hz	29,1	32,7	37,0	42,2	48,6	56,5	66,6	79,5
f_{1t}, Hz	28,7	32,3	36,5	41,7	48,0	55,9	65,9	78,8

Example: Determine the relationship $f_1 = f_1(\ell)$ for the first natural frequency of a synthetic resin- bonded paper beam with the following characteristics:

modulus of elasticity $E = 2{,}19.10^{10}$ N/m^2;
density $\rho = 1{,}294.10^3$ kg/m^3;
thickness $h = 3{,}00$ mm.

Experimental results:

Table 2

ℓ, mm	265	250	235	220	205	190	175	160
f_{1e}, Hz	20,3	22,8	25,8	28,4	33,8	39,4	46,3	55,4
f_{1t}, Hz	19,9	22,4	25,4	28,9	33,3	38,8	45,7	54,7

The results (Table 1 and Table 2) show that the experimental dependence $f_1 = f_1(\ell)$ for a beam of steel and synthetic resin- bonded paper hardly differs from the theoretical.

Fig. 2 presents graphs of the dependence $f_1 = f_1(\ell)$ for steel and synthetic resin-bonded paper.

According to the formulae (9) for determination of the frequency one gets that it is higher for steel in comparison with that for synthetic resin-bonded paper - $(\frac{E}{\rho})_{st} > (\frac{E}{\rho})_{sr}$

That dependence is confirmed from the experimental results, too.

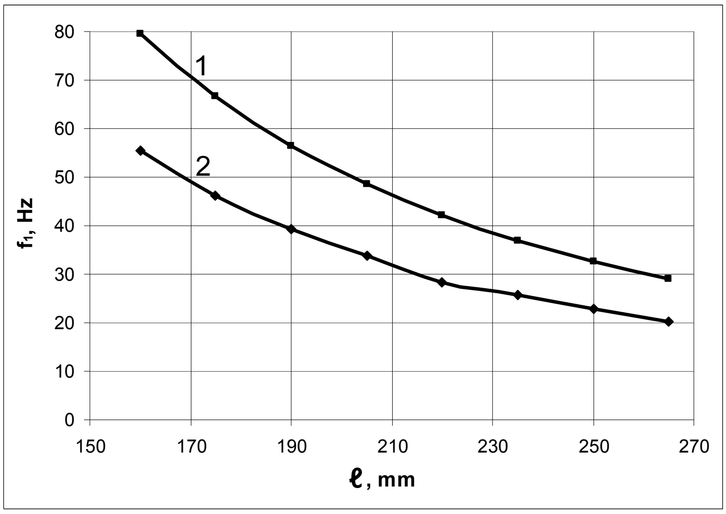

Fig.2. Graphs of the experimental dependence $f_1 = f_1(\ell)$: 1 – for steel; 2 – for synthetic resin- bonded paper.

2. To determine the relationship between the natural oscillations of a beam fixed at one end and its thickness – $f_{exp} = f(h)$ and to check it with the theoretical relationship $f_{theor} = f(h)$.

The students are to determine the relationship for the first, or the second or the natural frequency either following the instructions of the teacher.

Example: Determine the relationship $f_1 = f_1(h)$ for the first natural frequency of a steel beam with the following characteristics:

modulus of elasticity $E = 3{,}12.10^{11}$ N/m^2;
density $\rho = 7{,}74.10^3$ kg/m^3;
length $\ell = 250$ mm.

Experimental results:

Table 3

h, mm	2,20	2,40	2,60	2,80	3,00	3,20	3,40	3,60
f_{1e}, Hz	25,0	27,2	29,4	31,7	34,1	36,4	38,6	40,9
f_{1t}, Hz	25,4	27,7	30,0	32,3	34,6	36,9	39,2	41,5

Example: Determine the relationship $f_1 = f_1(h)$ for the first natural frequency of a synthetic resin- bonded paper beam with the following characteristics:

modulus of elasticity $E = 2{,}19.10^{10}$ N/m^2;
density $\rho = 1{,}294.10^3$ kg/m^3;
length $\ell = 250$ mm.

Experimental results:

Table 4

h, mm	2,20	2,40	2,60	2,80	3,00	3,20	3,40	3,60
f_{1e}, Hz	16,1	17,5	19,0	20,6	22,1	23,5	25,1	26,5
f_{1t}, Hz	16,4	17,9	19,4	20,9	22,4	23,9	25,4	26,9

The results (Table 3 and Table 4) show that there is good correlation between experimental and theoretical results of dependence $f_1 = f_1(h)$ for steel and synthetic resin-bonded paper beams.

Fig. 3 presents graphs of the dependence $f_1 = f_1(h)$ for steel synthetic resin- bonded paper.

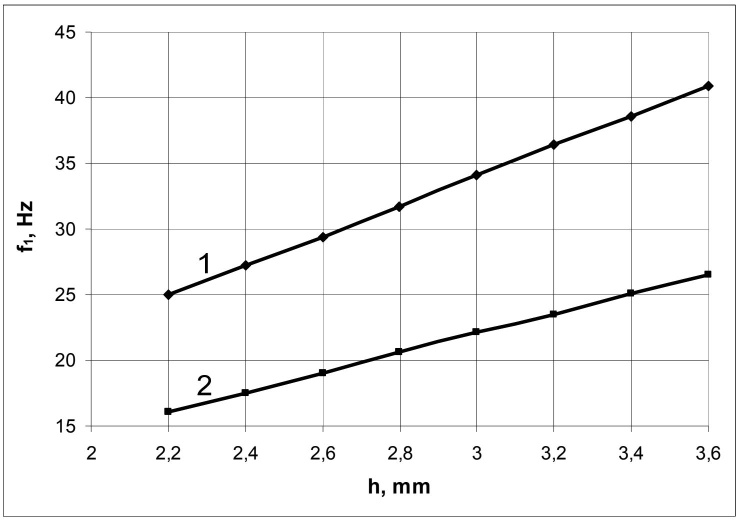

Fig.3. Graphs of the experimental dependence $f_1 = f_1(h)$:
1 – for steel; 2 – for synthetic resin- bonded paper.

Students use beams made of the following materials: steel, copper, aluminum, acrylic resin, synthetic resin- bonded paper and laminated fabric.

Combining a particular natural frequency with a particular material will make it possible for the teacher to assign individual tasks to the students.

CONCLUSION

This exercise demonstrates the relationship between engineering disciplines as well as the important role of continuity in the training of postgraduate engineers. For example in their lectures and seminars in Physics they have studied oscillation processes in spring pendulum, mathematical pendulum and liquid oscillations in a U-shaped tube. In the discipline Noise and Oscillations they study more complex oscillation processes, such as free lateral oscillations of a beam fixed at one end.

REFERENCES

[1] Borisov M. Physics, part I: Mechanics, „Science and Art ", Sofia, 1965
[2] Cheschanov B. Theory of oscillation, Technical University of Sofia, 1992, ISBN 954-438-001-9
[3] Lilkova-Markova S., D. Lolov. Numerical-analytical method to determine their own forms of rights beam with concentrated masses, Jubilee Conference of VSU, Suhodol, 2003
[4] Pisarev A. Mechanical vibrations, „Technika", Sofia,1985

CONTACT ADDRESSES
Assist. Prof. Tsanko Karadzhob, PhD
Technical University of Gabrovo
Department of Mechanical and Precision Engineering
4, H. Dimitar Str.
5300 Gabrovo, BULGARIA
Phone: +35966827532
E-mail: karadjov_st@abv.bg

Assist. Prof. Nikolay Angelov
Technical University of Gabrovo
Department of Physics
4, H. Dimitar Str.
5300 Gabrovo, BULGARIA
Phone: +35966827318
E-mail: bo232001@yahoo.com

ОПРЕДЕЛЯНЕ НА СОБСТВЕНИТЕ ЧЕСТОТИ НА НАПРЕЧНИ ТРЕПТЕНИЯ НА ГРЕДА, ЗАПЪНАТА В ЕДИНИЯ КРАЙ

Цанко Караджов, Николай Ангелов

Технически университет - Габрово

Резюме: *Разгледан е метод за определяне на собствените напречни трептения на греда, запъната в единия край. По време на лабораторното упражнение студентите изследват връзки между величините и сравняват експерименталните и теоретичните резултати. Показана е взаимовръзка между дисциплините „Физика” и „Шум и вибрации” , което допълнително мотивира студентите след като разберат, че всеки отделен предмет е важен за цялостния учебен процес.*

Ключови думи: *греда, напречни трептения, модул на еластичност*

ISSN 1311-9974