# CONTEMPORARY TRENDS IN NUCLEAR SYMMETRY ENERGY INVESTIGATION

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***Abstract:*** *The nuclear symmetry energy is one of the most relevant physical parameters that affect many nuclear processes and phenomena. This paper presents some of the recently obtained results related to different aspects of nuclear symmetry energy investigation.*
***Keywords:*** *Nuclear symmetry energy, Binding energy.*

The nuclear symmetry energy characterizes the variation of the binding energy as the neutron to proton ratio of a nuclear system is varied at a fixed value of the total number of particles. It is a quantity of crucial importance in different areas of nuclear physics, including structure of ground-state nuclei, dynamics of heavy-ion reactions, physics of giant collective excitations and physics of neutron stars.

In nuclear matter one considers the energy per particle ***E/A***, which is a function of the total density $\rho$ and the asymmetry $\beta=(N-Z)/A$, where ***N, Z, A*** are the neutron and proton numbers and the total particle number $A=Z+N$, respectively. One then expands the energy per particle as a function of $\beta$ around $\beta=0$ at a given density $\rho$ [4]:

$$\frac{E}{A}(\rho,\beta)=\frac{E}{A}(\rho,0)+S_N(\rho)\beta^2+\cdots.$$

The coefficient $S_N(\rho)$ is the nuclear symmetry energy. For a free Fermi gas of protons and neutrons

$$\frac{E}{A}=\frac{E}{A}(\beta=0)+S_N\beta^2+\cdots,$$

and the symmetry energy is $S_N=\frac{1}{3}E_F$ , where $E_F$ is the Fermi energy of symmetric nuclear matter. At saturation density $\rho_0$ $S_N\approx 12\ MeV$ which means a steep increase of the energy with asymmetry. For interacting nuclear matter this value is more than twice larger and the energy surface is steeper with respect to rising asymmetry. This property of the symmetry energy is of fundamental significance for many nuclear phenomena.

The above expansion of the energy per particle can be used also for finite nuclei to define $S_N$ as a function of the particle number $A$. Different approaches have been used to extract the values of $S(\rho)$ for nuclear matter from the experimental data - **LDM**, **EDF** generated by Skyrme forces, etc. $S(\rho)$ is usually expanded around the saturation density $\rho_0$ :

$$S(\rho)=S(\rho_0)+\left(\frac{dS}{d\rho}\right)_{\rho_0}(\rho-\rho_0)+\frac{1}{2}\left(\frac{d^2S}{d\rho^2}\right)_{\rho_0}(\rho-\rho_0)^2+\frac{1}{6}\left(\frac{d^3S}{d\rho^3}\right)_{\rho_0}(\rho-\rho_0)^3+\cdots.$$

We denote $S_o=S(\rho_0)$ and initiate parameters, named curvature ***K*** and slope ***L***, as follows:

$$K=3\rho_0\left(\frac{dS}{d\rho}\right)_{\rho_0},\ L=9{\rho_0}^2\left(\frac{d^2S}{d\rho^2}\right)_{\rho_0}.$$

The predictions for the symmetry energy vary quite substantially, e.g. $S_o = 28 \div 38\, MeV$. An empirical value of about ***29 MeV*** has been extracted from finite nuclei by fitting the ground-state energies using the generalized Weizsäcker mass formula:

$$E(N,Z) = E_V + E_S + E_a + E_C + E_{mic} = -a_V A + a_S A^{\frac{2}{3}} + a_a \frac{(N-Z)^2}{A} + a_C \frac{Z^2}{A^{1/3}} + E_{mic}\,,$$

where $a_V, a_S, a_a$ and $a_C$ are the volume, surface, symmetry, and Coulomb coefficients, respectively. The microscopic energy $E_{mic}$ contains shell- and pairing-energy corrections [7].

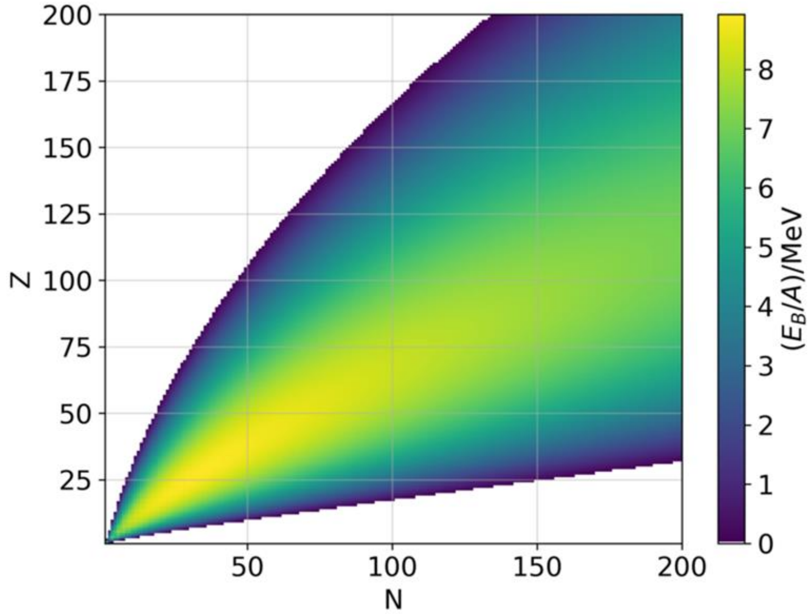


**Fig. 1:** A graphical representation of the semi-empirical binding energy formula. The binding energy per nucleon in ***MeV*** (highest numbers in yellow, in excess of ***8.5 MeV*** per nucleon) is plotted for various nuclides as a function of ***Z,*** the atomic number (y-axis), vs. ***N***, the number of neutrons (x-axis). The highest numbers are seen for ***Z=26 (Fe)***.
https://en.wikipedia.org/wiki/Semi-empirical_mass_formula

In [7] the authors study the correlation between the thickness of the neutron skin in finite nuclei and the nuclear symmetry energy for isotopic chains of even-even ***Ni***, ***Sn***, and ***Pb*** nuclei in the framework of the deformed self-consistent mean-field **Skyrme HF+BCS** method. By means of the coherent density fluctuation model (**CDFM**) the symmetry energy, the neutron pressure and the asymmetric compressibility in finite nuclei are calculated using the symmetry energy as a function of density within the Brueckner energy-density functional. The mass dependence of the nuclear symmetry energy and the neutron skin thickness are also studied together with the role of the neutron-proton asymmetry. A correlation between the parameters of the equation of state (**EOS**)-the symmetry energy and its density slope, and the neutron skin is suggested in the isotopic chains of even-even ***Ni***, ***Sn***, and ***Pb*** nuclei. The results for the symmetry energy obtained using four different Skyrme parametrizations are compared in **Fig. 2**. The magnitude of the symmetry energy values slowly changes and turns out to be approximately in the range of ***27÷29 MeV***. The nuclear matter calculations relying on realistic nucleon-nucleon interactions appear to yield volume symmetry energy values within the same range depending on the interaction.

The results reveal the existence of an approximate linear correlation between the skin thickness and the nuclear symmetry energies for isotopic chains of even-even ***Ni***, ***Sn***, and ***Pb*** nuclei. They confirm that the neutron skins in nuclei with a large neutron excess near and beyond the drip line are very clearly related to the asymmetry parameter. The subsequently obtained results for ***Kr*** and ***Sm*** isotopes [8] reveal similar trends.

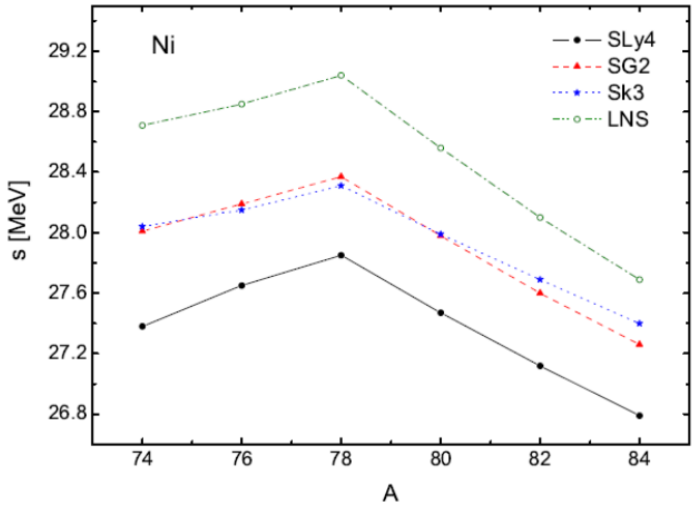


**Fig. 2:** The symmetry energies ***s*** within the **CDFM** framework for ***Ni*** isotopes calculated with **SLy4**, **SG2**, **Sk3**, and **LNS** Skyrme forces.

The kinks displayed in **Fig. 2** by the ***Ni*** isotopes and similar kinks in the case of ***Sn*** isotopes can be attributed to the shell structure of those exotic nuclei. The isotopic chains of ***Ni*** and ***Sn*** are of certain interest for nuclear structure calculations because of their proton shell closures at ***Z=28*** and ***Z=50***, respectively.

The used microscopic theoretical approach is capable also to predict important nuclear matter quantities in deformed nucleon-rich exotic nuclei and their relation to surface properties of these nuclei. This is confirmed by the good agreement with other theoretical predictions and some experimentally extracted ground-state properties.

A comprehensive study of various ground-state properties of neutron-rich and neutron-deficient ***Mg*** isotopes with ***A=20÷36*** is performed in [9]. The results of the calculations show that the behaviour of the nuclear charge radii and nuclear symmetry energy in the ***Mg*** isotopic chain is closely related to nuclear deformation.

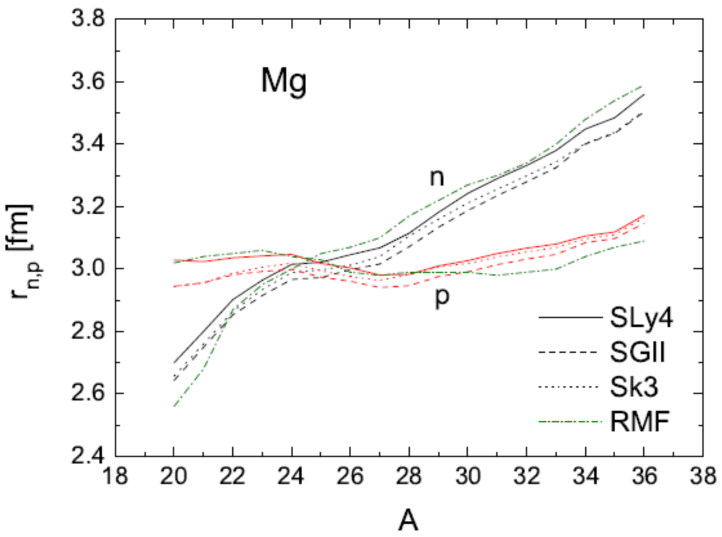


**Fig. 3:** Proton $r_p$ (red curves) and neutron $r_n$ (black curves) ***rms*** radii of ***Mg*** isotopes calculated by different Skyrme forces compared to the **RMF** theory predictions (green curves).

In **Fig. 3** are displayed the obtained results for the neutron and proton mean square radii in the ***Mg*** isotopic chain for different Skyrme forces compared to the relativistic mean field (**RMF**) predictions.

It is well known that for a proper description of the symmetry energy volume and surface contributions should be taken into consideration [5, 6]. Such contributions are included in [2] within the **CDFM**. The volume and surface components of the nuclear symmetry energy and their ratio are calculated using the results of the model for finite nuclei based on the **Brueckner EDF** for nuclear matter. In addition, results obtained by using the **Skyrme EDF** are presented. The **CDFM** weight function is obtained using the proton and neutron densities from the self-consistent **HF+BCS** method with Skyrme interaction. The isotopic sensitivity of the volume and surface contributions to the symmetry energy and their ratio are studied for ***Ni***, ***Sn***, and ***Pb*** isotopic chains (**Fig. 4**). The results are compared with estimations of other approaches which have used available experimental data on binding energies, skin thickness, excitation energies as well as with results of other theoretical methods.

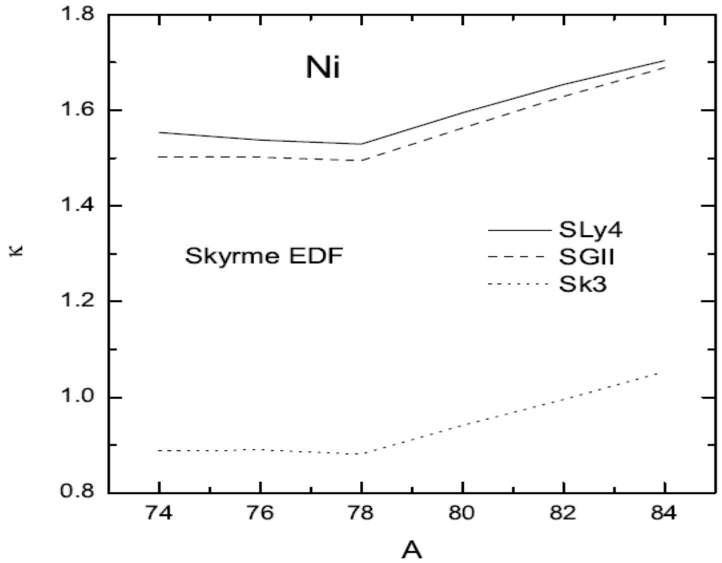


**Fig. 4:** The ratio of the volume and surface contributions to the nuclear symmetry energy $\boldsymbol{k = a_A^V / a_A^S}$ as a function of ***A*** for the isotopic chain of ***Ni*** in the case of **Skyrme EDF** with use of different modifications of Skyrme forces [2].

The temperature dependence of symmetry energy for isotopic chains of even-even ***Ni***, ***Sn*** and ***Pb*** nuclei has been investigated in the framework of the local density approximation (**LDA**) [3]. The Skyrme energy density functional with two Skyrme-class effective interactions, **SkM*** and **SLy4**, is used in the calculations. The temperature-dependent proton and neutron densities are calculated through the **HFBTHO** code that solves the nuclear Skyrme-Hartree-Fock-Bogoliubov (**Skyrme HFB**) problem by using the cylindrical transformed deformed harmonic-oscillator basis. The results for the thermal evolution of the symmetry energy coefficient in the interval ***T=0÷4 MeV*** show that its values decrease with temperature. The temperature dependence of the neutron and proton **rms** radii and corresponding nuclear skin thickness is also investigated, showing that the effect of temperature leads mainly to a substantial increase of the neutron radii and skins, especially in the more neutron-rich nuclei (**Fig. 5**). This feature may have consequences on astrophysical processes and neutron stars. The kink appearance in ***s(A)*** dependence confirms also the well-known fact that the shell effects can be expected up to ***T ≤ 2 MeV***.

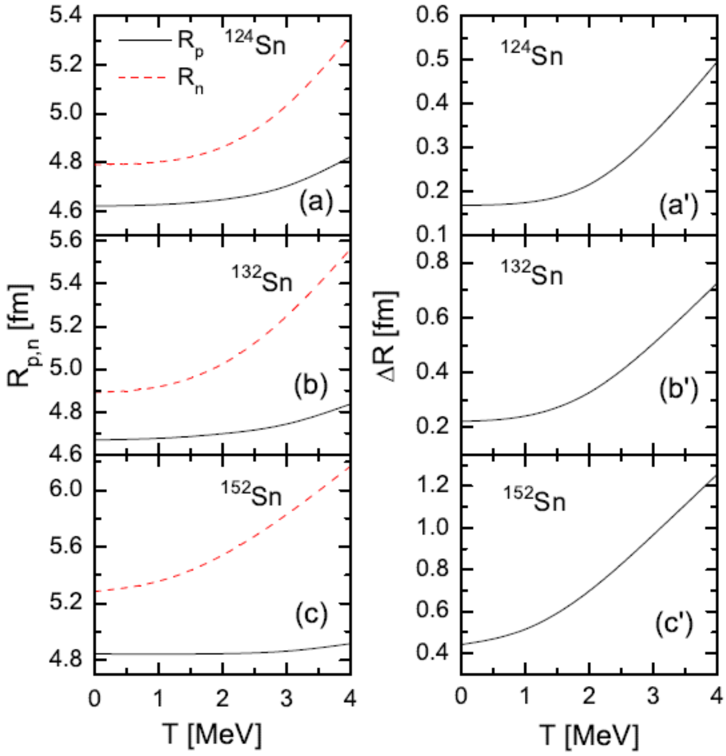


**Fig. 5:** Left: Proton $R_p$ (solid line) and neutron $R_n$ (dashed line) radius of $^{124}Sn$, $^{132}Sn$, and $^{152}Sn$ isotopes with respect to the temperature $T$ calculated with **SLy4** interaction. Right: Neutron skin thickness $\Delta R$ for the same *Sn* isotopes as a function of $T$.

If we consider infinite nuclear matter, the nuclear symmetry energy is the energy cost in converting asymmetric nuclear matter to a symmetric one. Then it is defined as

$$e_{sym}(\rho,\delta) = e(\rho,\delta) - e(\rho,\delta = 0),$$

where $e$ is the energy per nucleon of nuclear matter, $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$ is the nuclear asymmetry, and $\rho_n$ and $\rho_p$ are the neutron and proton densities with $\rho_n + \rho_p = \rho$. Expanding $e(\rho,\delta)$ into powers of $\delta$ around $\delta = 0$ and keeping only the even powers of $\delta$ (because of charge symmetry), one has

$$e_{sym}(\rho,\delta) = a_2\delta^2 + a_4\delta^4 + a_6\delta^6 + \cdots.$$

In [1] in the framework of **EOS** constructed from a momentum and density dependent finite-range two-body effective interaction, the quantitative magnitudes of the different symmetry elements of infinite nuclear matter are explored. The parameters of this interaction are determined from well-accepted characteristic constants associated with homogeneous nuclear matter. The symmetry energy coefficient $a_2$, its density slope $L_0$, the symmetry incompressibility $K_\delta$ as well as the density dependent incompressibility $K(\rho)$ evaluated with this **EOS** are in good harmony with those obtained from other diverse perspectives. The emergence of the relative importance of the higher order coefficients with increasing density is shown in **Fig. 6**. The growing difference of the total symmetry energy $e_{sym}$ (which is the sum of all orders of the symmetry coefficients) from $a_2 + a_4 + a_6$ with density shows that still higher order terms need to be taken into consideration at very high densities and asymmetries prevalent near the core of the neutron star. The relatively smaller values of the higher order symmetry coefficients at low densities and their growing importance with increasing density are in fair agreement with those obtained from both non-relativistic and relativistic calculations [1].

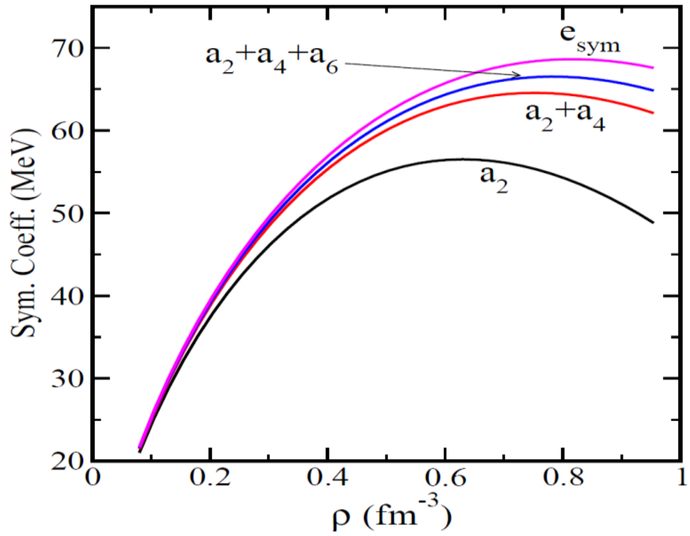


**Fig. 6:** The contributions of different orders of symmetry energy coefficients $a_2, a_4, a_6$ to the total symmetry energy coefficient $e_{sym}$ shown as a function of density.

The model dependence in the correlations of the neutron-skin thickness in heavy nuclei with various symmetry energy parameters is analysed in [10] by using several families of systematically varied microscopic mean field models. Relations between the nuclear symmetry energy coefficient and its density derivatives are derived in [11]. The relations hold for a class of interactions with quadratic momentum dependence and a power-law density dependence. The structural connection between the different symmetry energy elements as obtained seems to be followed by almost all reasonable nuclear energy density functionals, both relativistic and non-relativistic, suggesting a universality in the correlation structure.

It is of certain interest to quantify the statistical correlations among the neutron skin thickness of atomic nuclei and the slope of the symmetry energy in the infinite nuclear medium. A new statistical tool is proposed in [12] and applied to two doubly magic nuclei - $^{132}$***Sn*** and $^{100}$***Sn***, observing a relatively high statistical correlation between these quantities in $^{132}$***Sn***. The authors plan to apply such a technique to a large variety of nuclei to assess the evolution of the statistical correlation as a function of the isospin asymmetry.

After recalling basic phenomenological features of isospin asymmetric nuclear matter, the author of [13] reviews predictions for the interaction part of the symmetry energy obtained from different microscopic approaches. These predictions are compared to updated constraints extracted from heavy-ion reaction observables of a recent **GSI** experiment. The discussion is extended to the neutron skin thickness in $^{208}$***Pb*** and its relation to the density derivative of the symmetry energy. It is stressed that heavy-ion reactions, experiments at radioactive beam facilities, and measurements of the weak charge density in nuclei from the electroweak program at **Jefferson laboratory** are expected to provide new data on neutron-rich systems. This information will improve current understanding of the **EOS** and symmetry energy, and may potentially reach out to astrophysical systems, such as neutron stars.

Effects of retarded electrical fields on observables sensitive to the high-density behavior of nuclear symmetry energy in heavy-ion collisions at intermediate energies are considered in [14]. The obtained results show that the retarded electric fields affect neutrons and protons differently. The energetic nucleons are affected differently depending on the stiffness parameter ***x*** of nuclear symmetry energy. This parameter is introduced to mimic the different forms of the symmetry energy predicted by various many-body theories without changing any property of symmetric nuclear matter and the value of symmetry energy at saturation density. The density dependence of nuclear symmetry energy with different ***x*** parameters is shown in **Fig. 7**.

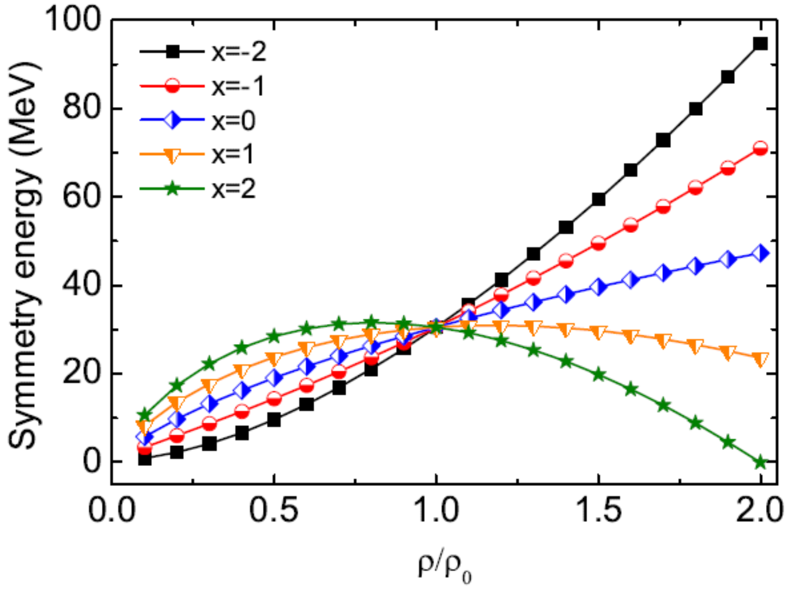


**Fig. 7:** The density dependence of nuclear symmetry energy $\boldsymbol{E_{sym}(\rho)}$ with different values of the stiffness parameter ***x***.

The results of recently performed calculations based on a set of relativistic energy density functionals that span a wide region of values of the symmetry energy slope ***L*** [15] confirm that the difference in charge radii of various neutron-deficient ***Ni*** isotopes and their corresponding mirror nuclei is highly correlated to both nucleon-skin thickness and ***L***. As various neutron star properties are also sensitive to ***L***, a data-to-data relation emerges between the charge radii difference of mirror nuclei and the radius of low-mass neutron stars. Enormous technical advances have resulted in pioneering measurements of the charge radii of unstable neutron-rich isotopes at such facilities as **ISOLDE-CERN** and **RIKEN-SCRIT**. The authors are confident that these techniques may also be used to measure the charge-radius of the neutron-deficient isotopes under investigation in their work.

The interplay of short-range correlations (**SRC**) and nuclear symmetry energy $\boldsymbol{E_{sym}(\rho)}$ in hard photon productions from heavy-ion reactions at **Fermi** energies is investigated in [16]. Interesting results on hard photon spectra in collisions of several ***Ca*** isotopes on $^{112}$***Sn*** and $^{124}$***Sn*** targets at a beam energy of ***45 MeV/nucleon*** are obtained. Over the whole energy range of hard photons considered, effects of the **SRC** overwhelm those due to the $\boldsymbol{E_{sym}(\rho)}$. From theoretical point of view this fact can be explained. The authors are looking forward to comparing their calculations with the forthcoming experimental data.

In 'Global analysis of Skyrme forces and the high-order density dependence' [17] the authors accept that a single density term in the Skyrme forces may be insufficient and a higher order density dependent term is added. Its influence on certain nuclear quantities is investigated. Global descriptions of nuclear masses and charge radii are presented. The influence on fission barriers and **EOS** are investigated. **Fig. 8** displays the symmetry energy of symmetric nuclear matter obtained by ***SkM**** as a function of density, compared to the same quantity determined by using two extended Skyrme forces. Generally, one can see that the symmetry energies of the extended forces decrease in high density region compared to the original force. This is also consistent with the previous results based on ***SLy4***. The perspectives to improve Skyrme forces are discussed, including global center-of-mass and pairing corrections. This global analysis should be useful for future development of high-precision nuclear energy-density functionals.

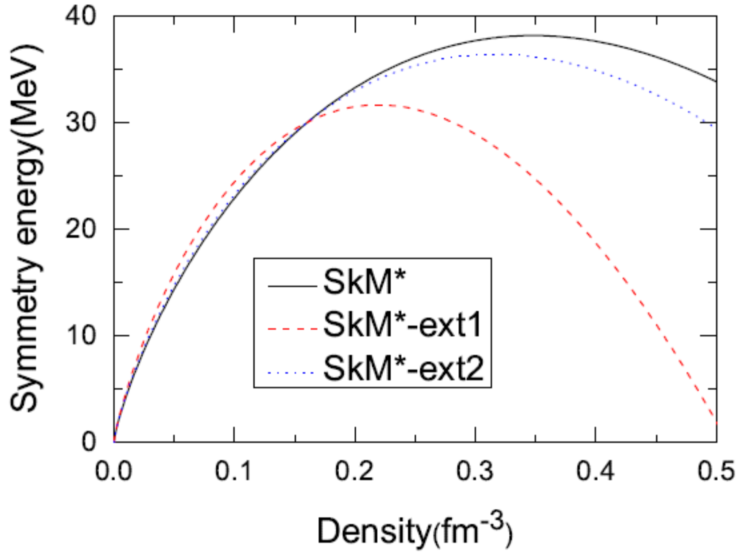


**Fig. 8:** The symmetry energy of the symmetric nuclear matter as a function of densities obtained with ***SkM*, SkM****$_{ext1}$ and ***SkM****$_{ext2}$ forces.

The density dependence of nuclear matter symmetry energy $\boldsymbol{E_{sym}(\rho)}$, which characterizes the isospin dependent part of the equation of state (**EOS**) of asymmetric nuclear matter, attracts much interest in current research frontiers of nuclear physics and astrophysics. The exact information on nuclear matter symmetry energy is critically important for understanding many challenging questions ranging from the structure of radioactive nuclei, the reaction dynamics induced by rare isotopes, the liquid-gas phase transition in asymmetric nuclear matter and the isospin dependence of **QCD** phase diagram to the location of neutron drip line and ***r***-process paths in the nuclear landscape, the properties of neutron stars and the explosion mechanism of supernovae as well as the frequency and strain amplitude of gravitational waves from in-spiralling neutron star binaries. In addition, some interesting issue of possible new physics beyond the standard model may also be related to the symmetry energy. All this determines the enormous interest in performing complicated theoretical and experimental researches.

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## СЪВРЕМЕННИ ТЕНДЕНЦИИ В ИЗСЛЕДВАНЕТО НА ЯДРЕНАТА ЕНЕРГИЯ НА СИМЕТРИЯ

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***Резюме:*** *Ядрената енергия на симетрия е един от най-важните физични параметри, които засягат много ядрени процеси и явления. Настоящата статия представя някои от получените наскоро резултати, свързани с различни аспекти на изследването на енергията на симетрия.*

***Ключови думи:*** *Ядрена енергия на симетрия, Енергия на свързване.*

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