

PROCEEDINGS

of the Union of Scientists - Ruse

Book 5
**Mathematics, Informatics and
Physics**

Volume 8, 2011



RUSE

The Ruse Branch of the Union of Scientists in Bulgaria was founded in 1956. Its first Chairman was Prof. Stoyan Petrov. He was followed by Prof. Trifon Georgiev, Prof. Kolyo Vasilev, Prof. Georgi Popov, Prof. Mityo Kanev, Assoc. Prof. Boris Borisov, Prof. Emil Marinov. The individual members number nearly 300 recognized scientists from Ruse, organized in 13 scientific sections. There are several collective members too – organizations and companies from Ruse, known for their success in the field of science and higher education, or their applied research activities. The activities of the Union of Scientists – Ruse are numerous: scientific, educational and other humanitarian events directly related to hot issues in the development of Ruse region, including its infrastructure, environment, history and future development; commitment to the development of the scientific organizations in Ruse, the professional development and growth of the scientists and the protection of their individual rights.

The Union of Scientists – Ruse (US – Ruse) organizes publishing of scientific and popular informative literature, and since 1998 – the "Proceedings of the Union of Scientists- Ruse".

BOOK 5

**"MATHEMATICS,
INFORMATICS AND
PHYSICS"**

VOLUME 8

CONTENTS

Mathematics

<i>Meline Aprahamian</i>	7
Mean Value Theorems in Discrete Calculus	
<i>Antoaneta Mihova</i>	13
Polynomial Identities of the 2x2 Matrices over the Finite Dimensional Grassmann Algebra	
<i>Veselina Evtimova</i>	19
Analysis of the Impact of the Incoming Calls Flow Intensity on Some Basic Characteristics of an Emergency Aid Centre	
<i>Veselina Evtimova</i>	25
A Study on the Influence of Incoming Calls Flow Intensity on the Waiting Time Characteristics of an Emergency Medical Aid Centre	
<i>Ivanka Angelova</i>	31
Numerical Solution of the Two-Phase Stefan Problem for Sphere	
<i>Ivanka Angelova</i>	38
Mathematical Models of Interface Problems for Steady-Unsteady Heat Conduction	

Informatics

<i>Valentin Velikov</i>	44
Some Possibilities For Automatic Programs Generation	
<i>Margarita Teodosieva</i>	50
Information System for Medicines	
<i>Mihail Iliev</i>	55
Extending the Lifetime of Wireless Sensor Networks by Using a Modified Method for Hierarchical Organization of the System in Clusters with Unequal Number of Devices	
<i>Georgi Krastev, Tsvetozar Georgiev</i>	63
One Approach for Continuous Signals Representation	
<i>Viktoria Rashkova</i>	70
Design and Implementation of Knowledge Control Test System	

Physics

<i>Galina Krumova</i>	77
Calculations of Light, Medium and Heavy Neutron-Rich Nuclei Characteristics	
<i>Vladimir Voinov, Roza Voinova</i>	86
Calculation of the Characteristic Impedance of a Microstrip, Reversed Microstrip and Embedded Microstrip Lines	
<i>Galina Krumova</i>	93
Some Problems of Atomic and Nuclear Physics Teaching	
<i>Tsanko Karadzhov, Nikolay Angelov</i>	101
Determining the Lateral Oscillations Natural Frequency of a Beam Fixed at One End	

BOOK 5
**"MATHEMATICS,
 INFORMATICS AND
 PHYSICS"**
VOLUME 8

Education

Plamenka Hristova, Neli Maneva 106
 An Innovative Approach to Informatics Training for Children

Margarita Teodosieva 114
 Using Web Based Technologies on Training in XHTML

Desislava Atanasova, Plamenka Hristova 120
 Human Computer Interaction in Computer Science Education

Valentina Voinohovska 125
 Computer – based conceptual mapping for facilitation of
 creative and meaningful learning in the course of "Multimedia
 Systems and technologies"

Galina Atanasova, Katalina Grigorova 132
 An Educational Tool for Novice Programmers

Valentina Voinohovska 139
 A Course for Promoting Student's Visual Literacy

Magdalena Metodieva Petkova 145
 Teaching and Learning Mathematics Based on Geogebra Usage

Participation in International Projects

Nadezhda Nancheva 153
 Mosem 2 Project - Learning Electromagnetic Phenomena
 and Superconductivity by Integration of Data Acquisition,
 Data Video, Modelling, Simulation and Animation

CALCULATIONS OF LIGHT, MEDIUM AND HEAVY NEUTRON-RICH NUCLEI CHARACTERISTICS

Galina Krumova

Angel Kanchev University of Ruse

Abstract: Results of charge form factor calculations for several unstable neutron-rich isotopes of light, medium and heavy nuclei (He, Li, Ni, Kr, Sn) are presented and compared to those of stable isotopes in the same isotopic chain. Proton densities are compared to matter densities. Whenever possible a comparison of form factors and densities with available experimental data is also performed. The charge form factor of the ${}^6\text{Li}$ nucleus is obtained also on the basis of its cluster structure.

Keywords: Charge form factors, proton and matter density, cluster structure

INTRODUCTION

It has been found from analyses of total interaction cross sections of scattering of particles and ions from nuclei that weakly-bound neutron-rich light nuclei, e.g. ${}^{6,8}\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, ${}^{17,19}\text{B}$, have increased sizes that deviate substantially from the $R \sim A^{1/3}$ rule. It was realized that such a new phenomenon is due to the weak binding of the last few nucleons which form a diffuse nuclear cloud due to quantum-mechanical penetration (the so called "nuclear halo"). Another effect is that the nucleons can form a "neutron skin" when the neutrons are on average less bound than the protons.

Most exotic nuclei are so shortlived that they cannot be used as targets at rest. Instead, direct reactions with radioactive nuclear beams (RNB) can be investigated in inverse kinematics, where the roles of beam and target are interchanged. For example, proton elastic scattering angular distributions were measured at incident energies less than 100 MeV/nucleon for He and Li isotopes and even at energy of 700 MeV/nucleon for the same nuclei at GSI (Darmstadt). The charge and matter distributions of these nuclei were tested in analyses of differential and total reaction cross sections of the proton scattering on exotic nuclei using different phenomenological and theoretical methods. It was shown that elastic scattering of protons serves as a good tool to distinguish between different models of density distributions.

Concerning the charge distributions of nuclei, it is known that their most accurate determination can be obtained from electron-nucleus scattering. For the case of exotic nuclei the corresponding charge densities are planned to be obtained by colliding electrons with these nuclei in storage rings. It is important to study how the charge distribution as well as the radii and diffuseness evolve with increasing neutron number (or isospin) at fixed proton number. This point may be very important for understanding the neutron-proton interaction in the nuclear medium. To this end the preliminary theoretical calculations of the charge form factors of neutron-rich exotic nuclei can serve as a challenge for future experimental work and thus, for accurate determination of the charge distributions in these nuclei. This can be a test of the different theoretical models used for predicting charge distributions.

In recent years theoretical work has been done along these lines focusing on halo nuclei. Various existing theoretical predictions for the charge distributions in light exotic nuclei ${}^{6,8}\text{He}$, ${}^{11}\text{Li}$, ${}^{14}\text{Be}$, ${}^{17,19}\text{B}$ have been used for calculations of charge form factors - for instance, those of Tanihata *et al.* for He isotopes, the results of the cluster-orbital shell-model approximation (COSMA) for He and Li isotopes, the large-scale shell-model (LSSM) method (for He and Li isotopes) and that of Suzuki *et al.* for ${}^{14}\text{Be}$ and ${}^{17,19}\text{B}$ nuclei. The

charge form factors have been calculated within the plane wave Born approximation (PWBA). Calculations of form factors of heavier exotic nuclei within the PWBA are also available.

In [1,2] we extend the range of exotic nuclei for which charge form factors have been calculated earlier. Along with the new calculations for He and Li isotopes, we present results on charge form factors of several unstable isotopes of medium (Ni) and heavy (Kr and Sn) nuclei and compare them to those of stable isotopes in the same isotopic chain. We also give the charge densities and compare them to matter density distributions. The calculated proton, neutron, charge and matter rms radii are compared with those for $^{4,6,8}\text{He}$ and $^{6,11}\text{Li}$ deduced from the proton scattering experiments at GSI and from the total interaction cross sections obtained from the measurements of Tanihata *et al.* and from the re-analysis of the same data. In our calculations for the He and Li isotopes we use the LSSM proton and neutron densities obtained in calculations based on the set of wave functions with exponential asymptotic behaviour. For the isotopes of heavier nuclei Ni, Kr and Sn we use proton and neutron densities which are obtained from self-consistent mean-field (HF+BCS, shortly HFB) calculations with density-dependent Skyrme effective interactions in a large harmonic-oscillator (HO) basis. Secondly, we calculate the charge form factors not only within the PWBA but also in the distorted wave Born approximation (DWBA) by numerical solution of Dirac equation for electron scattering in the Coulomb potential of the charge distribution of a given nucleus.

A theoretical scheme for calculations of the charge density distribution and form factor of ^6Li in the framework of the α -d cluster model of this nucleus is suggested in [3,4].

CHARGE FORM FACTOR AND DENSITY DISTRIBUTION CALCULATIONS

The nuclear charge form factor $F_{ch}(q)$ has been calculated as follows:

$$(1) \quad F_{ch}(q) = [F_{point;p}(q) G_{Ep}(q) + (N/Z) F_{point;n}(q) G_{En}(q)] F_{c.m.}(q),$$

where $F_{point;p}(q)$ and $F_{point;n}(q)$ are the form factors which are related to the point-like proton and neutron densities $\rho_{point;p}(r)$ and $\rho_{point;n}(r)$, respectively. These densities correspond to wave functions in which the positions \mathbf{r} of the nucleons are defined with respect to the centre of the potential related to the laboratory system. In PWBA these form factors are Fourier transforms of the corresponding point-like densities normalized to Z and N . In order that $F_{ch}(q)$ corresponds to density distributions in the centre-of-mass coordinate system, a factor $F_{c.m.}(q)$ is introduced in the standard way [$F_{c.m.}(q) = \exp(q^2/4A^{2/3})$]. In Eq. (1) $G_{Ep}(q)$ and $G_{En}(q)$ are the Sachs proton and neutron electric form factors, correspondingly, and they are taken from one of the most recent phenomenological parametrizations.

In [1], in addition to PWBA, we also perform DWBA calculations solving the Dirac equation which contains the central potential arising from the proton ground-state distribution. We use two codes for numerical calculations of the form factors and the results of both calculations are in good agreement.

The theoretical predictions for the point-like proton and neutron nuclear densities of the light exotic nuclei $^{6,8}\text{He}$ and ^{11}Li , as well as of the corresponding stable isotopes ^4He and ^6Li are taken from the LSSM calculations. For $^{4,6,8}\text{He}$ nuclei they are obtained in a complete $4\hbar\omega$ shell-model space. The LSSM calculations use a Woods-Saxon single-particle wave function basis for ^6He and ^8He and HO one for ^4He . For comparison we use also the "experimental" charge density for ^4He and, i.e. the so-called "model-independent" shape of the density. The proton and neutron densities of ^6Li are obtained within the LSSM in a complete $4\hbar\omega$ shell-model space and of ^{11}Li - in complete $2\hbar\omega$ shell-model

calculations. For ${}^6\text{Li}$ the single-particle HO wave functions have been used in the LSSM calculations and Woods-Saxon ones for ${}^{11}\text{Li}$. For ${}^6\text{Li}$ we also use the point-proton nuclear density distribution which leads to the "experimental" charge distribution with rms radius equal to 2.57 fm.

The point proton and neutron density distributions of Ni, Kr and Sn isotopes are taken from deformed self-consistent HFB calculations with density-dependent SG2 effective interactions using a large HO basis with 11 major shells.

Figs. 1-8 display some of the obtained results.

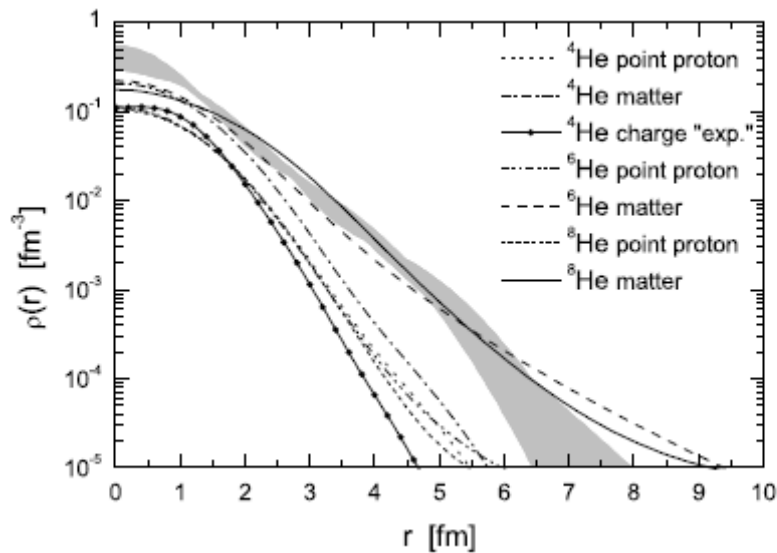


Fig. 1: Thin lines are LSSM point proton densities of ${}^{4,6,8}\text{He}$ compared to the "experimental" charge density for ${}^4\text{He}$ from "model-independent" analyses. Thick lines are LSSM matter densities of ${}^{4,6,8}\text{He}$ compared to matter density of ${}^8\text{He}$ deduced from the experimental proton scattering cross section data (grey area).

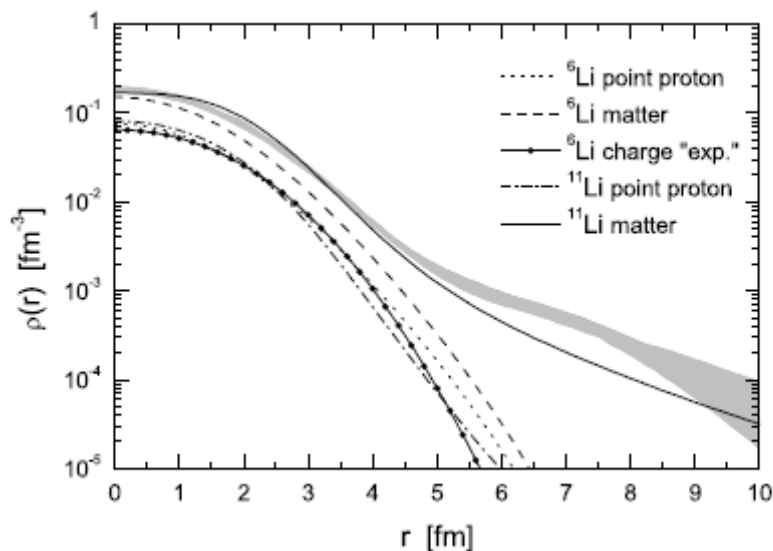


Fig. 2: Thin lines are LSSM point proton densities of ${}^{6,11}\text{Li}$ compared to the point-proton density of ${}^6\text{Li}$ extracted from the "experimental" charge density in a "model-independent" analysis. Thick lines are LSSM matter densities of ${}^{6,11}\text{Li}$ compared to matter density of ${}^{11}\text{Li}$ deduced from the experimental proton scattering cross section data (grey area).

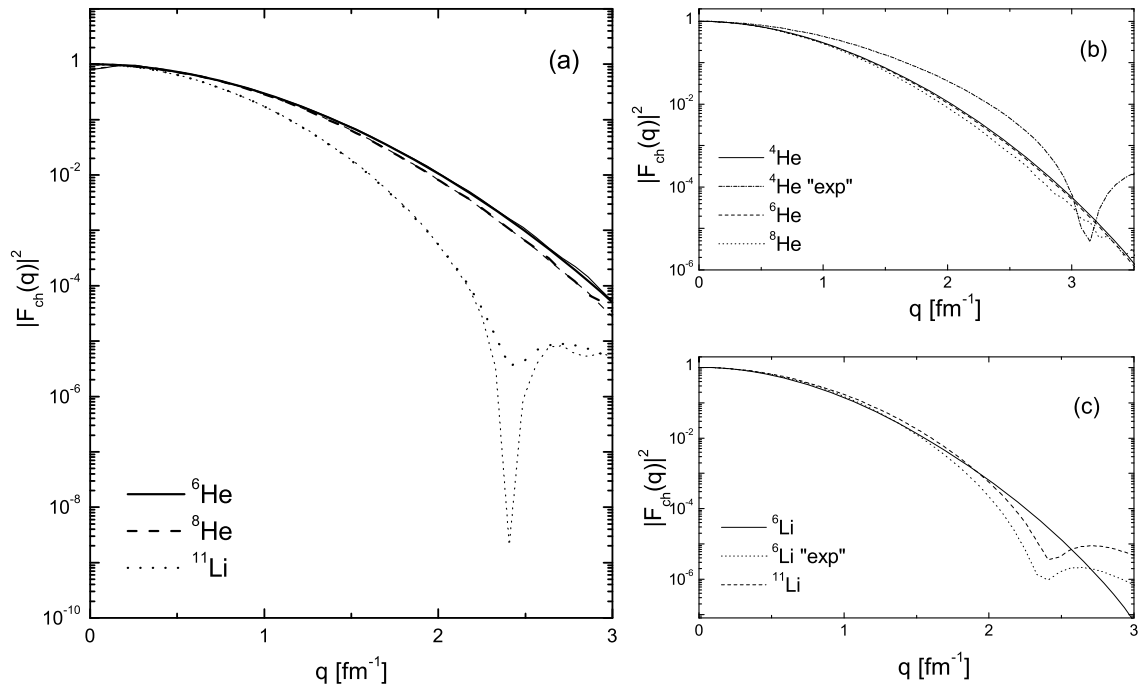


Fig. 3: (a) Charge form factors of ${}^6\text{He}$, ${}^8\text{He}$ and ${}^{11}\text{Li}$ calculated in BA (Born approximation) (thin lines) and SDE (solving the Dirac equation) (thick lines) using LSSM densities; (b) charge form factors in SDE for ${}^4\text{He}$ (calculated by using “experimental” charge density and the LSSM density) and of ${}^6,8\text{He}$ (using the LSSM densities); (c) charge form factor in SDE for ${}^6\text{Li}$ (using the “experimental” charge density and the LSSM densities) and for ${}^{11}\text{Li}$ (using the LSSM densities).

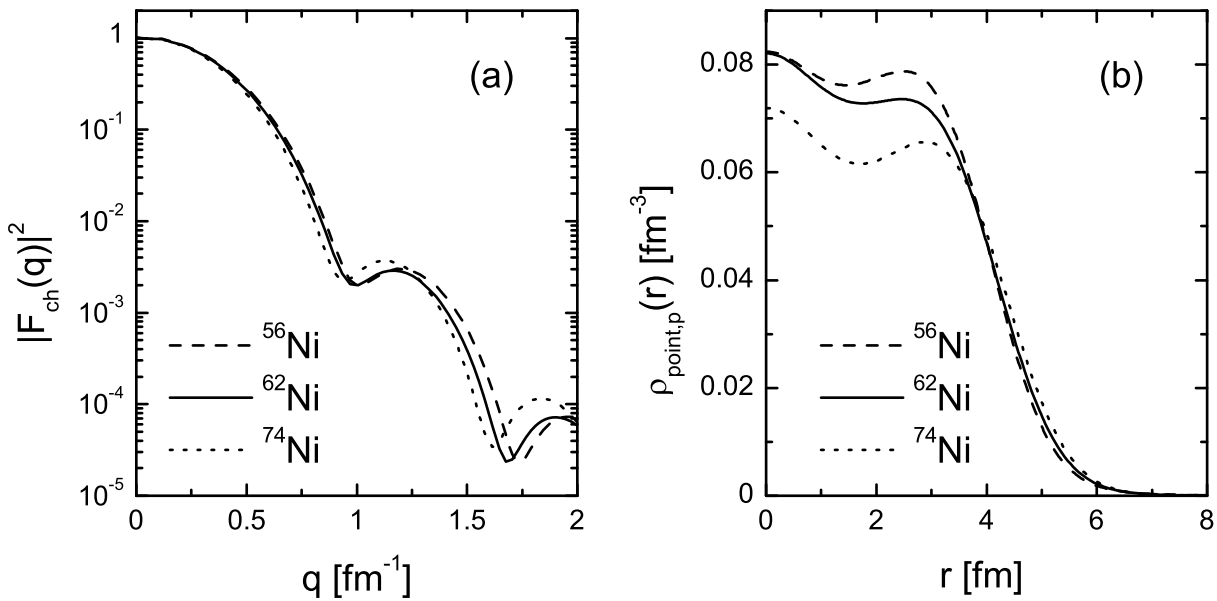


Fig. 4: (a) Charge form factors for the unstable doubly-magic ${}^{56}\text{Ni}$, stable ${}^{62}\text{Ni}$ and unstable ${}^{74}\text{Ni}$ isotopes calculated by using the HF+BCS densities and the DWBA; (b) HF+BCS proton densities of ${}^{56}\text{Ni}$, ${}^{62}\text{Ni}$ and ${}^{74}\text{Ni}$.

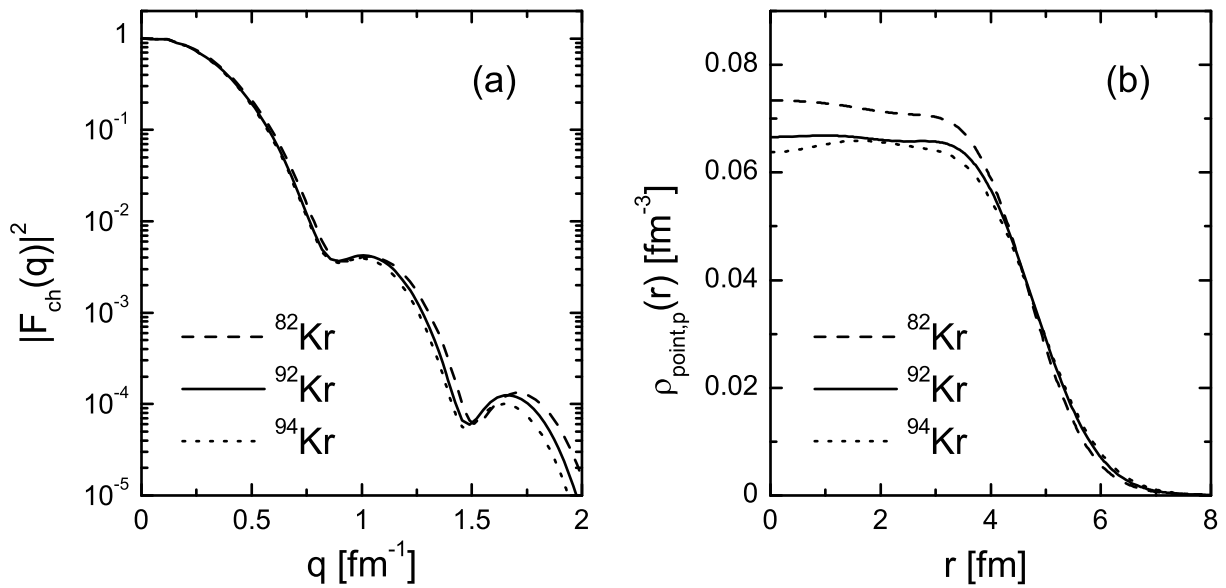


Fig. 5: (a) Charge form factors for the stable isotope ^{82}Kr and for the unstable ^{92}Kr and ^{94}Kr isotopes calculated by using the HF+BCS densities and the DWBA; (b) HF+BCS proton densities of ^{82}Kr , ^{92}Kr and ^{94}Kr .

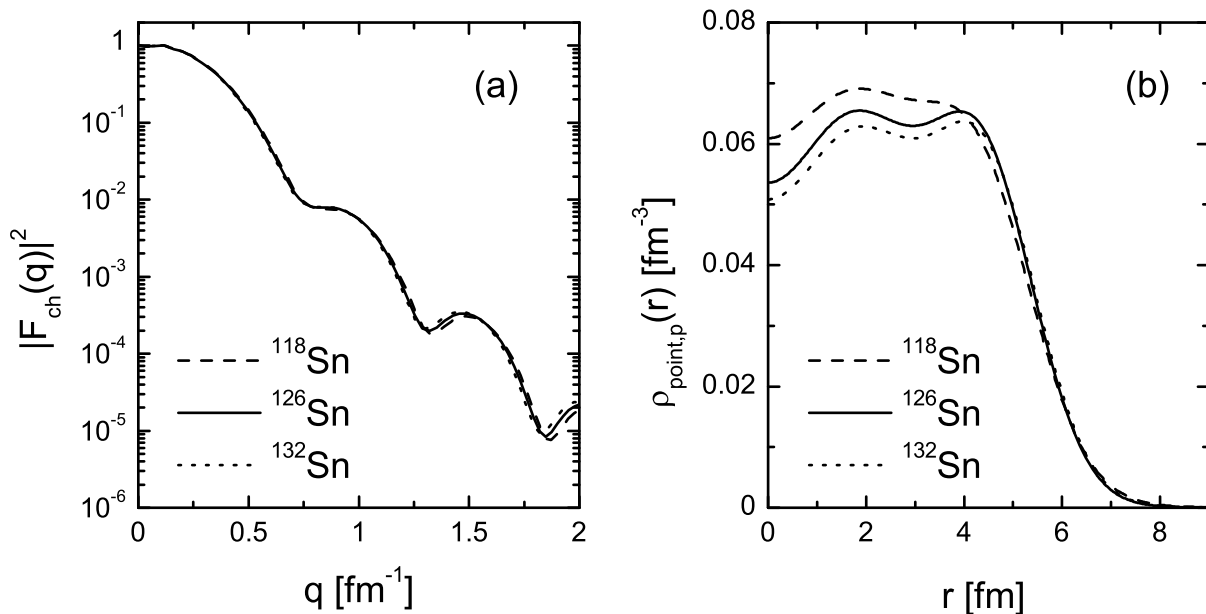


Fig. 6: (a) Charge form factors for the stable isotope ^{118}Sn , unstable ^{126}Sn and unstable doubly-magic ^{132}Sn isotopes calculated by using the HF+BCS densities and the DWBA; (b) HF+BCS proton densities of ^{118}Sn , ^{126}Sn and ^{132}Sn .

In addition, the charge form factor of ^6Li nucleus is considered on the basis of its cluster structure [3]. The charge density of ^6Li is presented as a superposition of two terms. One of them is a folded density and the second one is a sum of ^4He and the deuteron densities. Using the available experimental data for ^4He and deuteron charge form factors, a satisfactory agreement of the calculations within the suggested scheme is obtained with the experimental data for the charge form factor of ^6Li , including those in the region of large transferred momenta (see Fig. 9).

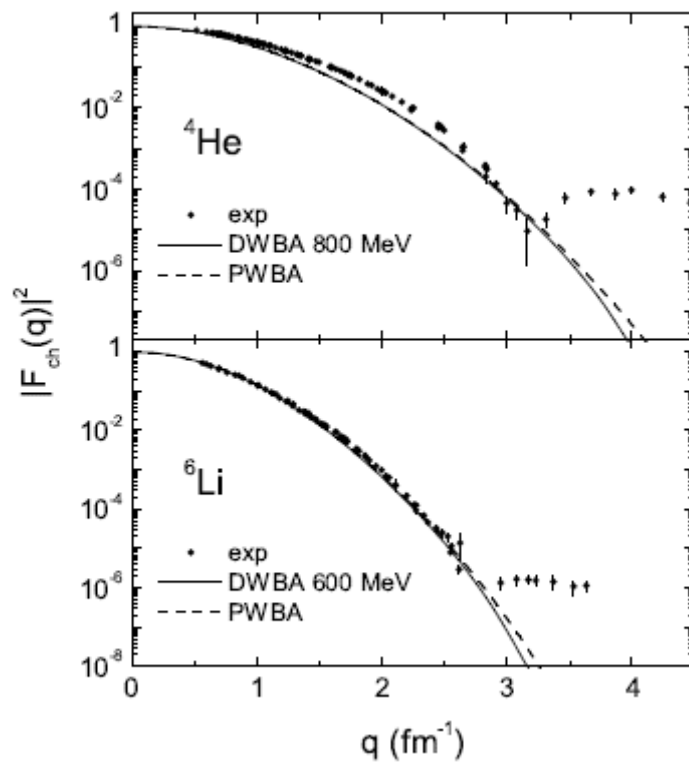


Fig. 7: Charge form factors for the stable isotopes ^4He and ^6Li calculated using LSSM densities in PWBA and in DWBA in comparison with the experimental data.

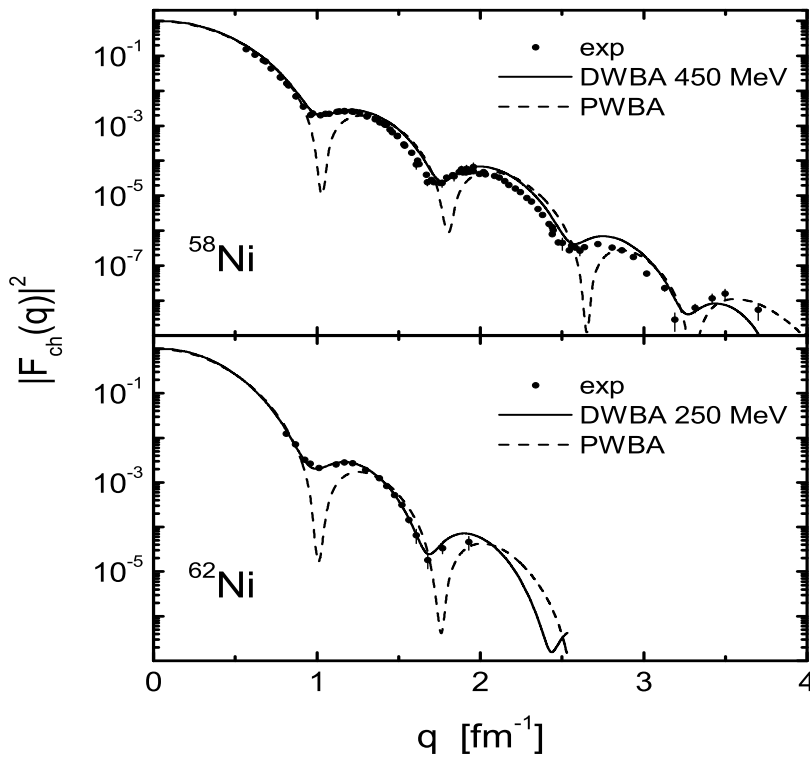


Fig. 8: Charge form factors for the stable isotopes ^{58}Ni and ^{62}Ni calculated by using the HF+BCS densities and the PWBA and DWBA in comparison with the experimental data.

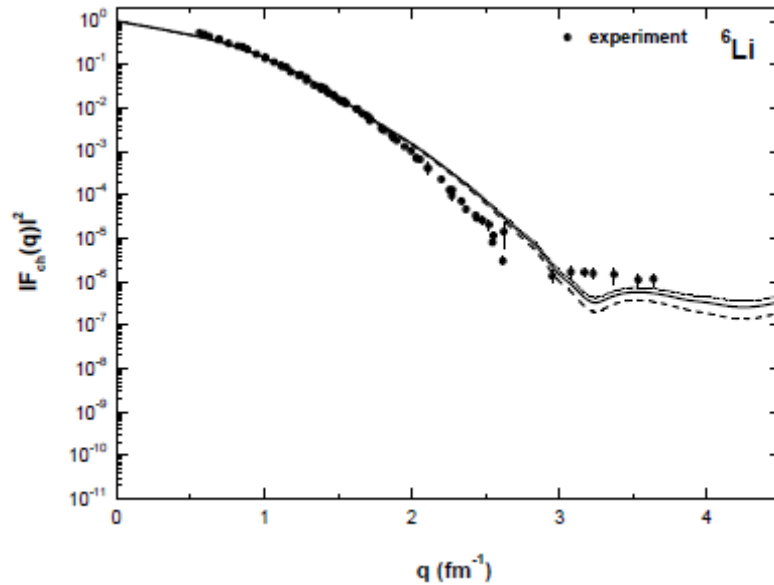


Fig. 9: The charge form factor of ${}^6\text{Li}$ calculated according to Eqs. (9)-(16) of [3] for $c_1=0.975$, $c_2=0.025$ (dotted line), $c_1=0.979$, $c_2=0.021$ (solid line), and $c_1=0.985$, $c_2=0.015$ (dashed line). The experimental data are as in Fig. 7.

CONCLUSIONS

The presented results can be summarized as follows:

1. The previous studies are extended to the proton, neutron and matter densities and related charge form factors from the light neutron-rich exotic nuclei ${}^6,8\text{He}$, ${}^{11}\text{Li}$ to examples of unstable medium (Ni) and heavy (Kr and Sn) isotopes in comparison with those of stable isotopes in the same isotopic chain. For He and Li isotopes are used proton and neutron densities obtained from realistic microscopic calculations within the Large-scale shell-model method. The densities of Ni, Kr and Sn isotopes are calculated within HF+BCS approach with a density-dependent effective interaction using a large harmonic-oscillator basis.

2. The proton and matter density distributions for He and Li isotopes are compared. The calculated matter distributions for the halo nuclei are much more extended than the proton ones. The comparison of the proton density distributions for the isotopes of He, Li, Ni, Kr and Sn reveals the differences of the proton densities in a given isotopic chain due to the presence of neutron excess. There is a decrease of the proton density in the nuclear interior and an increase of its tail at large r with neutron number increasing.

3. A comparison of the proton, neutron, charge and matter rms radii as well as of the corresponding diffuseness is performed for all isotopic chains under consideration. The general trend of the difference ΔR between the matter and proton rms radii is to increase with the number of neutrons but for the heavy isotopes this increase is moderate compared to that of the light ones.

4. The calculated matter densities for ${}^8\text{He}$ and ${}^{11}\text{Li}$ are in fair agreement with the experimental data obtained by proton scattering on these isotopes in GSI.

5. The charge form factors of He, Li, Ni, Kr and Sn isotopes are calculated by means of the densities mentioned above. The calculations are performed not only in PWBA but also in DWBA, solving the Dirac equation for electron scattering in the Coulomb potential of the charge distribution in a given nucleus. By accounting for the Coulomb distortion of the electron waves the form factors are shifted to smaller values of q which is clearly seen in the cases of the Ni, Kr and Sn isotopes where Z is large enough. This shift

is properly parametrized. In addition the charge distribution in the neutron itself is taken into account. The contributions of the neutrons to the charge form factors are less than 20 % up to $q \sim 2 \text{ fm}^{-1}$.

6. The differences between the charge form factors in a given isotopic chain are shown. A common feature of the charge form factors is the shift of their curves and minima to smaller values of q with the increase of the neutron number in a given isotopic chain. This is due to the corresponding enhancement of the proton tails in the peripheral region of the nuclei.

7. The performed theoretical analyses of the densities and charge form factors can be a step in the studies of the influence of the increasing neutron number on the proton and charge distributions in a given isotopic chain. This is important for understanding the neutron-proton interaction in the nuclear medium.

8. A theoretical scheme for calculations of the charge density distribution and form factor of ${}^6\text{Li}$ in the framework of the α -d cluster model of this nucleus is suggested. The calculations show a reasonable description of the charge form factor of ${}^6\text{Li}$ on the basis of a superposition of two density distributions:

i) a folding density obtained from ${}^4\text{He}$ and the deuteron charge densities. Provided corresponding experimental data for both densities is used, the calculations show that a good agreement with the data can be obtained when the weight of this contribution is about 97.5÷98.5% and; ii) a sum of the ${}^4\text{He}$ and deuteron charge densities with a weight of this contribution of about 2.5÷1.5%.

This scheme has only one free parameter (c_1 or c_2) with a clear physical meaning, namely, it is the weight of the one of the contributions to the density of ${}^6\text{Li}$.

9. The behavior of the charge form factor of ${}^6\text{Li}$ for $0 < q \leq 2.7 \text{ fm}^{-1}$ is determined mainly by the folding contribution (of ${}^4\text{He}$ and the deuteron densities) to the charge density of ${}^6\text{Li}$.

10. The shell-model α -d cluster density of ${}^6\text{Li}$ (i.e. the sum of ${}^4\text{He}$ and the deuteron charge densities) is important (though with a small weight of about 2.5÷1.5%) in the central nuclear region and, correspondingly, it is responsible for the values of the charge form factor of ${}^6\text{Li}$ at large values of q ($q \geq 3 \text{ fm}^{-1}$).

11. The calculated within the suggested scheme charge rms radius of ${}^6\text{Li}$ agrees with the experimental estimations of this quantity.

12. The theoretical predictions for the charge form factors of exotic nuclei are a challenge for their measurements in the future experiments in GSI and RIKEN and thus, for obtaining detailed information on the charge distributions of these nuclei. The comparison of the calculated charge form factors with the future data will be a test of the corresponding theoretical models used for studies of the exotic nuclei structure.

REFERENCES

- [1] Antonov, A. N., D. N. Kadrev, M. K. Gaidarov, E. Moya de Guerra, P. Sarriguren, J. M. Udias, V. K. Lukyanov, E. V. Zemlyanaya, G. Z. Krumova. Charge and Matter Distributions and Form Factors of Light, Medium and Heavy Neutron-Rich Nuclei. *nucl – th / 0506056*, Phys. Rev. C72, 2005, 044307.
- [2] Antonov, A. N., D. N. Kadrev, M. K. Gaidarov, E. Moya de Guerra, P. Sarriguren, J. M. Udias, V. K. Lukyanov, E. V. Zemlyanaya, and G. Z. Krumova. Density distributions and form factors in neutron-rich nuclei. JAEA-Conf 2006-009 (JAEA, Japan, 2006) - Proceedings of the 2005 Symposium on Nuclear Data, February 2-3. JAEA, Tokai, Japan, Eds. Y. Tahara and T. Fukahori, 2006, pp. 163-168.

- [3] Krumova, Galina Z., Egle Tomasi-Gustafsson and Anton N. Antonov. Charge Form Factor and Cluster Structure of the ${}^6\text{Li}$ Nucleus. nucl - th / 0709.1016, Cent. Eur. J. Phys., v. 6, No 3, 2008, pp. 491-497.
- [4] Krumova, G. Z., E. Tomasi-Gustafsson, and A. N. Antonov. Charge Form Factor and Cluster Structure of ${}^6\text{Li}$ Nucleus. Nuclear Theory'26 - Proceedings of the XXVIth International Workshop on Nuclear Theory, Rila Mountains, Bulgaria, 25 – 30 June 2007. Sofia: BM Trade Ltd., 2007, pp. 259-269.

CONTACT ADDRESS

Assoc. Prof. Galina Zaharieva Krumova, PhD
Department of Physics
FEEEA
Angel Kanchev University of Ruse
Ruse - 7017
Bulgaria
Tel.: (+359 82) 888 215
E-mail: gal@uni-ruse.bg

**ПРЕСМЯТАНИЯ НА ХАРАКТЕРИСТИКИ НА ЛЕКИ, СРЕДНИ И ТЕЖКИ,
БОГАТИ НА НЕУТРОНИ ЯДРА**

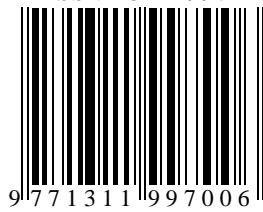
Галина Крумова

Русенски университет „Ангел Кънчев“

Резюме: В тази работа са изложени резултатите от пресмятанията на зарядовите форм-фактори на някои нестабилни, богати на неутрони изотопи на леки, средни и тежки ядра (He, Li, Ni, Kr, Sn). Същите са сравнени с тези на стабилни изотопи от същата изотопична верига. Сравнени са протонните и масовите плътностни разпределения. Направено е сравнение на форм-факторите и плътностните разпределения с наличните експериментални данни. Получен е зарядовият форм-фактор на ядрото на ${}^6\text{Li}$ на базата на неговата кластерна структура.

Ключови думи: зарядови форм-фактори, протонни и масови плътностни разпределения, кластерна структура

ISSN 1311-9974



9 771311 997006