

# PROCEEDINGS

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of the Union of Scientists - Ruse

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Book 5  
**Mathematics, Informatics and  
Physics**

Volume 9, 2012



RUSE

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was founded in 1956. Its first Chairman was Prof. Stoyan Petrov. He was followed by Prof. Trifon Georgiev, Prof. Kolyo Vasilev, Prof. Georgi Popov, Prof. Mityo Kanev, Assoc. Prof. Boris Borisov, Prof. Emil Marinov, Prof. Hristo Beloev. The individual members number nearly 300 recognized scientists from Ruse, organized in 13 scientific sections. There are several collective members too – organizations and companies from Ruse, known for their success in the field of science and higher education, or their applied research activities. The activities of the Union of Scientists – Ruse are numerous: scientific, educational and other humanitarian events directly related to hot issues in the development of Ruse region, including its infrastructure, environment, history and future development; commitment to the development of the scientific organizations in Ruse, the professional development and growth of the scientists and the protection of their individual rights.

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**BOOK 5**

**"MATHEMATICS,  
INFORMATICS AND  
PHYSICS"**

**VOLUME 9**

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## DEFORMATION EFFECTS ON DENSITY AND MOMENTUM DISTRIBUTIONS OF $^{98}\text{Kr}$ NUCLEUS

Galina Krumova

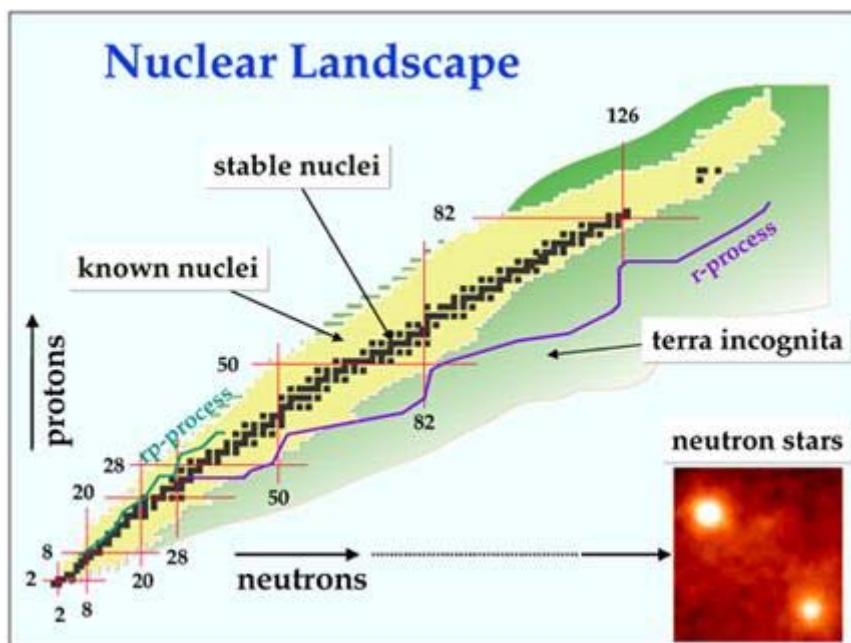
Angel Kanchev University of Ruse

**Abstract:** Neutron and proton density and momentum distributions of the  $^{98}\text{Kr}$  nucleus for two kinds of deformation are compared.

**Keywords:** Neutron skin, deformed nuclei, density and momentum distributions

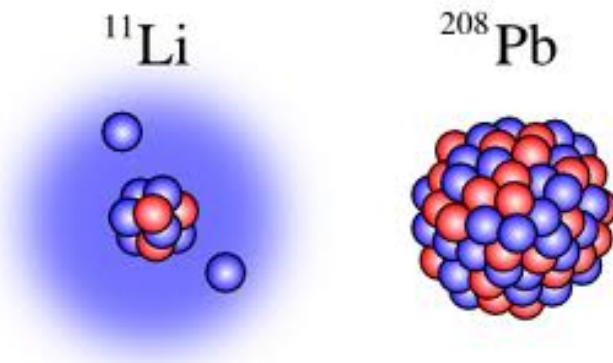
### INTRODUCTION

Exotic nuclei are nuclei with an extraordinary ratio of protons and neutrons. Typically, they are very unstable and decay into more stable nuclei. They can either be produced in compound nucleus reactions where two nuclei are fused at rather low energies, or they can be made by projectile fragmentation at medium energies [1]. The exotic nuclei tend to decay. The following picture shows the number of protons  $Z$  versus the number of neutrons  $N$  for the known nuclei [2]. The binding energy of each nucleus is color-coded with the lighter color indicating less bound nuclei. The region of possibly synthesizable nuclei extends both to the proton-rich as well as the neutron-rich side.

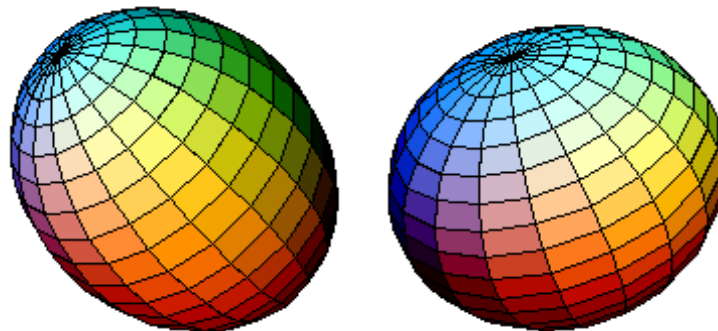


Up to now, the standard model of nuclear structure – the nuclear shell model – has been tested only in regions close to the valley of stability in the nuclear chart. While leaving this region of stable nuclei new phenomena are expected and new insights into the complex nuclear many-body problem should be obtainable. The detailed understanding of the structure of exotic nuclei is strongly connected to astrophysics, since the nucleosynthesis - which usually happens inside massive stars – is typically taking place in regions of extreme neutron to proton ratios.

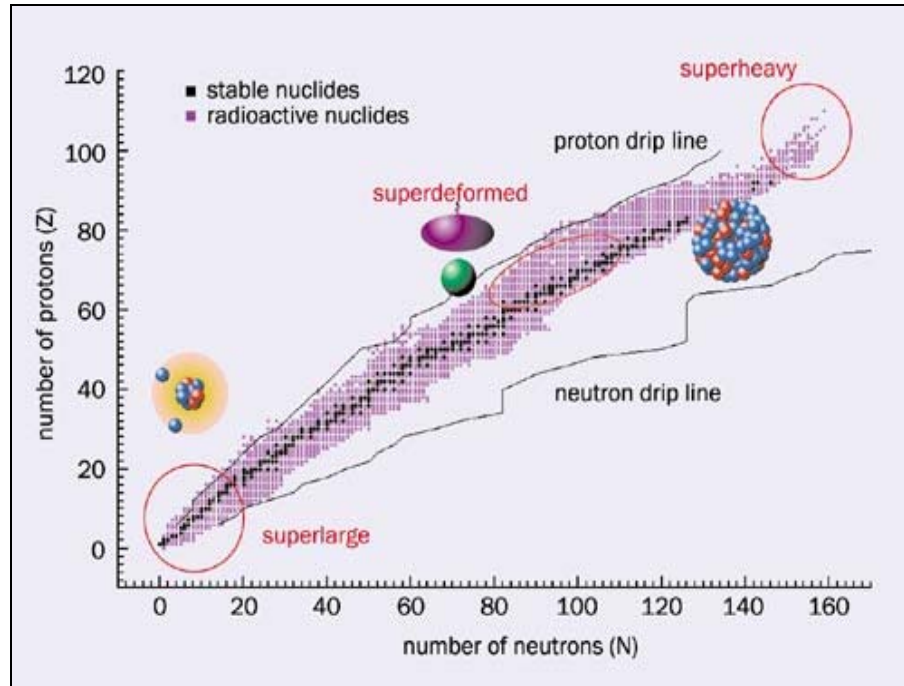
The study of nuclei close to the nuclear drip line and even beyond it has been greatly extended in recent years. This increased interest is based on new phenomena that already have been observed or that are predicted to occur in these nuclei. Since the first experiments, it has been found from analyses of total interaction cross sections that weakly bound neutron-rich nuclei, e.g.,  ${}^6,8\text{He}$ ,  ${}^{11}\text{Li}$ ,  ${}^{14}\text{Be}$ ,  ${}^{17,19}\text{B}$  have increased sizes that deviate substantially from the  $R \sim A^{1/3}$  rule. It was realized that such a new phenomenon is due to the weak binding of the last few nucleons that form a diffuse nuclear cloud (the “nuclear halo”) due to quantum-mechanical penetration. Another effect is that the nucleons can form a “neutron skin” when the neutrons are on average less bound than the protons. The origin of the skin lies in the large difference of the Fermi energy levels of protons and neutrons so that the neutron wave function extends beyond the effectively more bound proton wave functions (see the following pictures, showing the “halo” nucleus  ${}^{11}\text{Li}$  and the “neutron skin”  ${}^{208}\text{Pb}$  nucleus).



Another example of nuclear extremes concerns shape and is a consequence of nuclear deformation – the deviation from a normal, spherical shape. Some nuclides, particularly those with unbalanced proton-to-neutron ratios, are more “comfortable” assuming some type of deformation, typically a cigar shape (**prolate**) or a disc shape (**oblate**) (see the next pictures).



A recent trend in experiments in nuclear physics is to create nuclei with the highest possible angular momentum. The consequence of so much spin is a nucleus that is so distorted that it is classified as “superdeformed”. Such nuclei were studied by a very special type of spectroscopic footprint: a comb-like energy spectrum produced by a series of cascading gamma-ray decays as the whirling nucleus shed its enormous excess of rotational energy. The table of known stable and radioactive nuclides, showing the regions that are inhabited by the superheavy, superdeformed and superlarge nuclei, as well as the drip lines, which have been calculated using the Hartree-Fock-Bogoliubov plus Gogny method, is given below [3].



### NEUTRON AND PROTON DENSITIES OF A DEFORMED NUCLEUS

The effects of deformation on the neutron skins in even-even deformed nuclei far from the stability line have been obtained and discussed in [4]. The self-consistent deformed Hartree-Fock plus Bardeen-Cooper-Schrieffer (HF+BCS) formalism has been used to give an answer to some questions concerning the neutron skin in deformed nuclei. The density dependent Skyrme interactions (SG2, Sk3 and SLy4) have been used and pairing correlations have been taken into account. Pairing between like nucleons has been included by solving the BCS equations at each iteration either with a fixed pairing strength parameter or with a fixed pairing gap parameter (determined from the odd-even experimental mass differences).

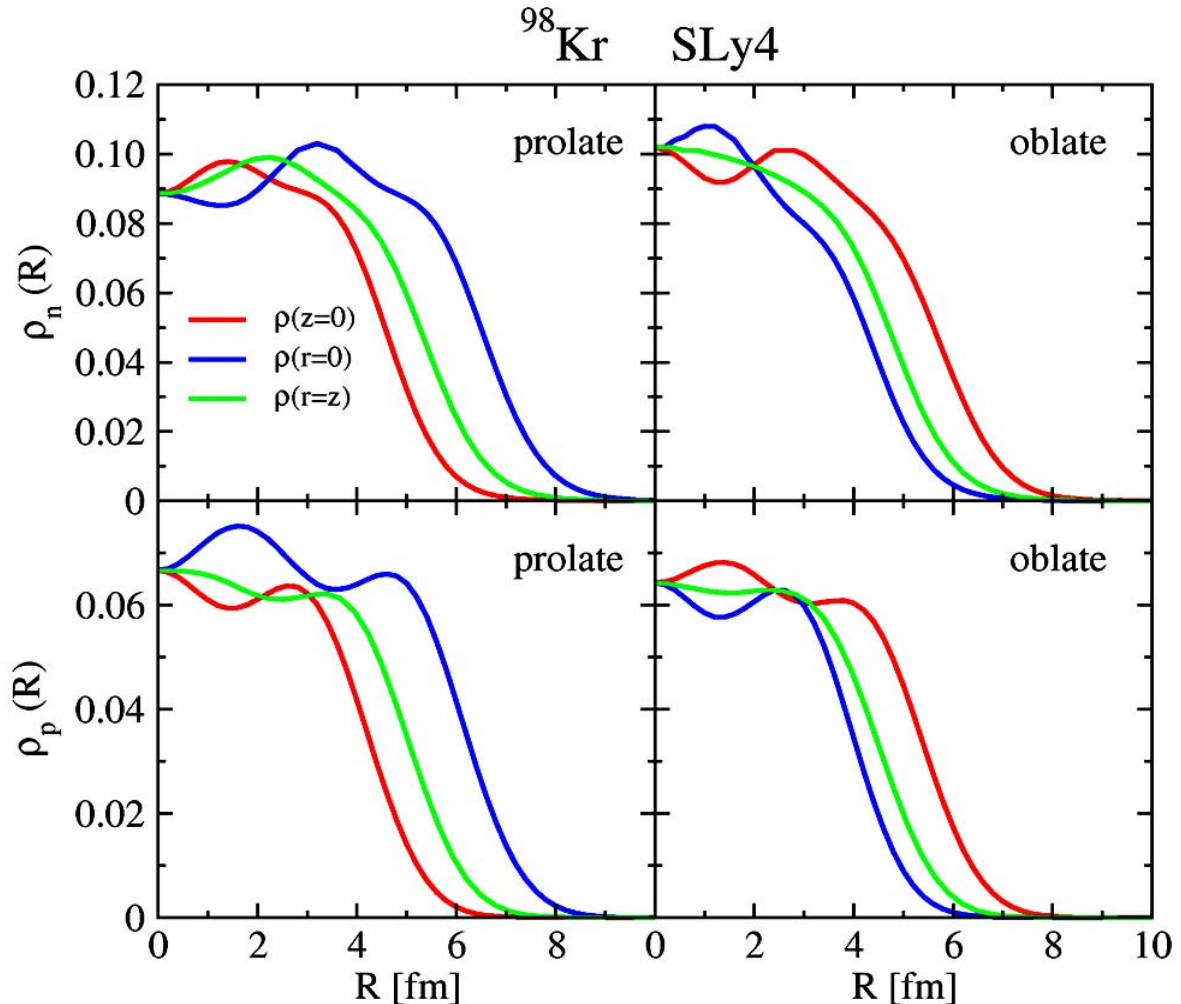
The thickness of the neutron skin might depend on the direction for deformed nuclei. It is an interesting and natural question to ask whether the deformed densities give rise to a different skin size in the different directions. It is also interesting to know whether the emergence of the skin may be influenced by the nuclear shape. In [4] is studied such a dependence on the example of Kr isotopes – well deformed nuclei characterized by a large variety of competing nuclear shapes. Former constraint HF+BCS calculations have shown the possibility of shape coexistence in these nuclei.

The results for the binding energy of the three selected even-even Kr isotopes –  $^{70}\text{Kr}$ ,  $^{84}\text{Kr}$  and  $^{98}\text{Kr}$  as a function of the quadrupole parameter  $\beta$  show that both prolate and oblate shapes produce minima very close in energy. The Skyrme effective force SLy4 has been used.

In the same work the neutron rich isotope  $^{98}\text{Kr}$  has been chosen to study the sensitivity of the neutron skin thickness to the various directions in the two shapes.

From the spatial distributions for neutrons and protons in three different directions:  $z$ -direction ( $r = 0$ ),  $r$ -direction ( $z = 0$ ), and  $r = z$  direction one can observe that the profiles of the densities as well as the spatial extensions change with the direction. Clearly, the densities are more extended in the  $z$ -direction in the case of prolate shapes. The opposite is true in the case of oblate shapes. The case  $r = z$  gives always intermediate densities [4].

The dependence of the neutron and proton densities  $\rho_n(R)$  and  $\rho_p(R)$  on the different directions can be seen in **Fig. 1**, where as an example are plotted the neutron and proton densities of  $^{98}\text{Kr}$  nucleus in the three directions mentioned above for prolate (left) and oblate (right) shapes in the same plane. It is obvious that the extension of the density in the  $z$ -direction (labeled  $r = 0$ ) is the largest for the prolate shape and the shortest for the oblate shape.



**Fig. 1:** Neutron and proton density distributions  $\rho_n(R)$  and  $\rho_p(R)$  corresponding to different directions for prolate (left panel) and oblate (right panel) shape of  $^{98}\text{Kr}$  [4,5] obtained with Skyrme force SLy4.

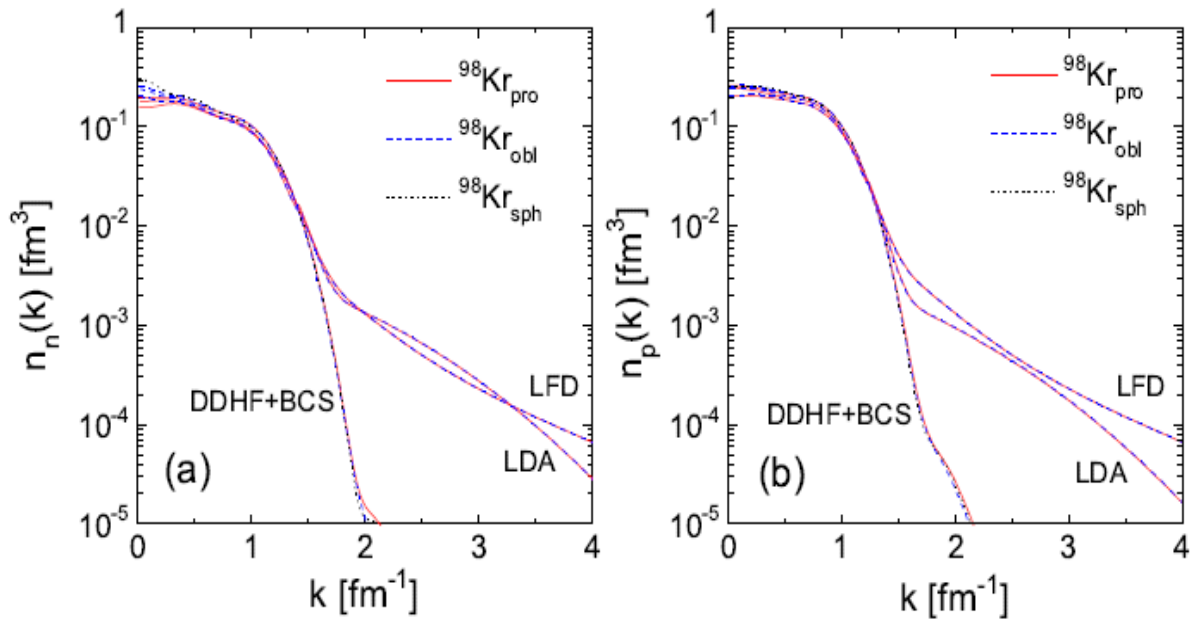
The overall skin thickness is similar in the oblate and prolate equilibrium shapes. From this example we may conclude that the skin thickness does not depend much on the oblate or prolate character of the deformation. This is in line with the conclusions reached on the example of Dy isotopes, where it was shown that the neutron skin is nearly independent of the size of deformation (spherical, deformed or superdeformed) [4].

### MOMENTUM DISTRIBUTIONS OF A DEFORMED NUCLEUS

The role of the deformation of the mean field on the nucleon momentum distributions (NMD) is studied on the example of the same  $^{98}\text{Kr}$  isotope in [6] within frameworks of the three theoretical approaches: density-dependent Hartree-Fock in the Bardeen-Cooper-

Schrieffer limit (DDHF+BCS), light-front dynamics (LFD) method and local density approximation (LDA) as described in the previous work here. We consider the Skyrme force SLy4 which gives an appropriate description of bulk properties of spherical and deformed nuclei.

We show in **Fig. 2** the intrinsic momentum distributions for neutrons and protons  $n_n(k)$  and  $n_p(k)$  and for prolate and oblate shapes of  $^{98}\text{Kr}$ . The results of the three methods considered in our work are given and compared together with the result for the spherical case obtained within the HF method. The differences between the momentum distributions calculated within a given theoretical method are negligible (especially at high  $k$ ) and practically can not be distinguished. Thus, we find almost no dependence of the  $n_n(k)$  and  $n_p(k)$  on the character of deformation.



**Fig.2:** Neutron (a) and proton (b) momentum distributions obtained within the DDHF+BCS, LFD, and LDA methods corresponding to prolate (solid line) and oblate (dashed line) shape of  $^{98}\text{Kr}$ . For comparison, the spherical case (dotted line) within the DDHF+BCS method is also given. The normalization is:  $\int n_{n(p)}(\mathbf{k})d^3k = 1$ . The results are obtained with Skyrme force SLy4.

## CONCLUSION

The role of the deformation on the neutron and proton density and momentum distributions is discussed in the present work on the example of the neutron rich  $^{98}\text{Kr}$  isotope. As it has been previously found the isotropy of the total momentum distribution is a property of the nucleus at equilibrium. Our results for the neutron and proton momentum distributions of this nucleus show small changes in the overall behavior for the oblate and prolate shapes. Although the neutron and proton densities change with deformation, the momentum distributions demonstrate a very weak dependence on the character of deformation. This is valid for all three theoretical approaches explored in our work. At the same time, the neutron skin thickness remains almost equal along the different directions perpendicular to the surface.



## REFERENCES

- [1] <http://www.hzdr.de/FWK/MITARB/wagnera/exotics/exotic-nuclei.html>  
[2] [http://fribusers.org/2\\_INFO/2\\_crucial.html](http://fribusers.org/2_INFO/2_crucial.html)  
[3] <http://cerncourier.com/cws/article/cern/29077>  
[4] Sarriguren, P., M. K. Gaidarov, E. Moya de Guerra, and A. N. Antonov. Nuclear Skin Emergence in Skyrme Deformed Hartree-Fock Calculations. arXiv: **0710.0542** [nucl-th], Phys. Rev. C**76**, 2007, 044322.  
[5] Гайдаров, М. К. Екзотични ядра: ядрена структура и ядрени реакции. <http://indico.cern.ch/getFile.py/access?contribId=17&resId=1&materialId=slides&confId=91262>  
[6] Gaidarov, M. K., G. Z. Krumova, P. Sarriguren, A. N. Antonov, M. V. Ivanov, and E. Moya de Guerra. Momentum Distributions in Medium and Heavy Exotic Nuclei. arXiv: **0911.2363v1** [nucl-th], Phys. Rev. C**80**, 2009, 054305.

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**ЕФЕКТИ НА ДЕФОРМАЦИЯТА ВЪРХУ ПЛЪТНОСТНИТЕ И  
ИМПУЛСНИ РАЗПРЕДЕЛЕНИЯ НА ЯДРОТО НА  $^{98}\text{Kr}$** 

Галина Крумова

Русенски университет „Ангел Кънчев”

**Резюме:** Сравнени са неутронните и протонни плътностни и импулсни разпределения на ядрото на  $^{98}\text{Kr}$  при два вида деформация.

**Ключови думи:** неутронна кожа, деформирани ядра, плътностни и импулсни разпределения

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