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Book 5

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 9

CONTENTS

Mathematics

Tsetska Rashkova 7
Grassmann algebra’s PI-properties in matrix algebras with Grassmann entries

Antoaneta Mihova 15
A comparison of two methods for calculation with Grassmann numbers

Mihail Kirilov 21
δ - Characteristic sets for finite state acceptor

Mihail Kirilov 25
λ - Characteristic sets for finite Mealy automaton

Veselina Evtimova 29
Research on the utilization of transport vehicles in an emergency medical care center

Valerij Djurov, Milena Kostova, Ivan Georgiev 35
A mathematical model system for radiolocational image reconstruction of dynamic object with low radiolocational visibility

Informatics

Tzvetomir Vassilev 41
Soft shadows for GPU based ray-tracing

Rumen Rusev, Ana Kaneva 47
Software module for spectral analysis of audio signals

Galina Atanasova, Plamenka Hristova, Katalina Grigorova 52
An approach to flow charts comparing

Valentin Velikov 60
Computer viruses and effectively protection of the home users

Georgi Krastev 66
Nonhierarchical method for clustering

Valentina Voinohovska, Svetlozar Tsankov 70
Corporate presence web site for dental clinic

Metodi Dimitrov 76
Global repository for sequences of robots instructions

Svetlozar Tsankov, Valentina Voinohovska 79
(X)HTML E-handbook in the discipline "Multimedia systems and technologies" for teaching and learning purposes

Physics

Galina Krumova 85
Momentum distributions of medium and heavy neutron-rich nuclei

Galina Krumova 92
Deformation effects on density and momentum distributions of 98kr nucleus

MOMENTUM DISTRIBUTIONS OF MEDIUM AND HEAVY NEUTRON-RICH NUCLEI

Galina Krumova

Angel Kanchev University of Ruse

Abstract: *Calculated neutron and proton momentum distributions for several unstable neutron-rich isotopes of medium and heavy nuclei are presented and compared to those of stable isotopes in the same isotopic chain.*

Keywords: *Exotic nuclei, momentum distributions*

INTRODUCTION

The physics of nuclei in exotic states is one of the most important and rapidly developing areas in nuclear physics. Researchers can now produce nuclei in extreme states – such as nuclei with high angular momentum (rapidly rotating nuclei), high excitation energy (“hot" nuclei), highly deformed nuclei (nuclei with unusual, super- and hyper-deformed shapes), nuclei with very large numbers of neutrons or protons (neutron-rich and proton-rich nuclei) and superheavy nuclei with a proton number Z above 110. The research work of nuclear matter in such extreme states provides important information about the properties of the microcosmos and makes possible the modelling of a variety of processes taking place in the Universe [1].

The four largest centres involved in the investigation of exotic nuclear states are: the GSI Helmholtzzentrum für Schwerionenforschung in Germany, the research centre RIKEN in Japan, JINR in Russia and the Grand Accélérateur National d’Ions Lourds (GANIL) in France. The accelerator systems can create exotic nuclei with extremely high proton-to-neutron ratios. The investigation of the nuclear “terra incognita” [2] is giving us deeper insight into the origin and stability of the matter in our Universe.

MOMENTUM DISTRIBUTIONS IN NUCLEI

The nucleon momentum distribution (NMD) $n(k)$ is an important characteristic of the nuclear ground state. The scaling analyses of inclusive electron scattering from a large variety of nuclei showed evidence of the existence of high-momentum components of NMD at momenta $k > 2\ fm^{-1}$. It has been shown that it is due to the presence of nucleon-nucleon (NN) correlations in nuclei. It has been pointed out that this specific feature of $n(k)/A$ is similar for all nuclei and that it is a physical reason for the scaling and superscaling phenomena in nuclei. As known, the mean-field approximation (MFA) is unable to describe simultaneously the two important characteristics of the nuclear ground state: the density and momentum distribution. Therefore, a consistent analysis of the effects of the NN correlations on both quantities is required, using theoretical methods beyond the MFA in the description of relevant phenomena, e.g., the scaling ones. Particular attention has been paid to the NMD in a given single-particle state analyzing the *(e,e'p)* reactions in nuclei. The self-consistent density-dependent Hartree-Fock (DDHF) approximation has been applied to calculate NMD in spherical and deformed Nd isotopes, studying the effects of deformation, as well as those of pairing and of dynamical short-range NN correlations.

It is important to study the NMD not only in stable but also in exotic nuclei. We know that in the reactions with exotic nuclei the momentum distribution of a core fragment of the projectile reflects the momentum distribution of the valence nucleons in the projectile near the surface. Many experimental works have been carried out to study the momentum distribution from the breakup of the projectile.

THEORETICAL APPROACH

The main aim of our work in [3,4] is to calculate the NMD for the same isotopic chains of neutron-rich nuclei (Ni, Kr and Sn) for which we have studied charge densities, radii, form factors, halo, and skin in our previous works. The mean-field contributions to *n(k)* in these nuclei are calculated within the same self-consistent approach applied there, in which the one-body energy density functional is obtained starting from a two-body density-dependent Skyrme interaction and a pairing interaction that is treated in the Bardeen-Cooper-Schrieffer (BCS) limit. The HF equations are solved for the (*N,Z*) nucleus, using a deformed harmonic-oscillator basis in cylindrical coordinates with oscillator lengths used as variational parameters. The BCS equations are solved at each HF iteration and the occupation numbers are used to construct the density-dependent mean field for the next HF iteration. We refer to this mean-field approach as DDHF+BCS. The remaining effects of the NN interactions, to which we refer as NN correlations, are considered in two of the correlations approaches, namely in the approach of our group using the light-front dynamics (LFD) method and in one based on the local density approximation (LDA). Several questions are investigated, such as the sensitivity of *n(k)* to all details of the calculations, e.g., (i) to different types of Skyrme forces; (ii) to the pairing correlation effects; (iii) to the effects of nuclear deformation; and (iv) to the strength of the NN correlations included in the LFD and LDA approaches (respectively, to the values of the correlations strength parameters β and γ). Special attention is paid to the isotopic and isotonic sensitivity of the proton and neutron momentum distributions. The results for *n(k)* in the exotic nuclei are compared with that in nuclear matter (NM).

Briefly, our calculations are based on:

A. Deformed Skyrme HF+BCS formalism

Some of the results have been obtained from self-consistent deformed Hartree-Fock calculations with density-dependent Skyrme interaction and pairing correlations. Pairing between like nucleons has been included by solving the BCS equations at each iteration either with a fixed pairing-gap parameter (determined from the odd-even experimental mass differences) or with a fixed pairing-strength parameter. We consider the Skyrme force SLy4 that gives an appropriate description of bulk properties of spherical and deformed nuclei.

B. Methods going beyond the mean-field approximation

It is well known that the methods within the MFA (e.g., shell-model, Hartree-Fock and others) can describe the nucleon momentum distribution *n(k)* only for momentum values $k < 1.5\ fm^{-1}$ and are unable to explain *n(k)* for larger *k*. The high-momentum components of *n(k)* ($k > 1.5\ fm^{-1}$) are due to the specific forces between the nucleons near the nuclear core ($r_c \approx 0.4\ fm$) that are the reasons for the short-range and tensor NN correlations. The differences between the values of *n(k)* for large *k* obtained within the correlation methods reach orders of magnitude. We consider the effects of NN correlations included in two correlation methods on the high-momentum contributions to the nucleon momentum distribution:

- Theoretical approach based on the light-front dynamics method (**LFD**);
- Theoretical approach based on the local density approximation (**LDA**).

RESULTS AND DISCUSSION

Figs. 1-6 display some of the obtained results.

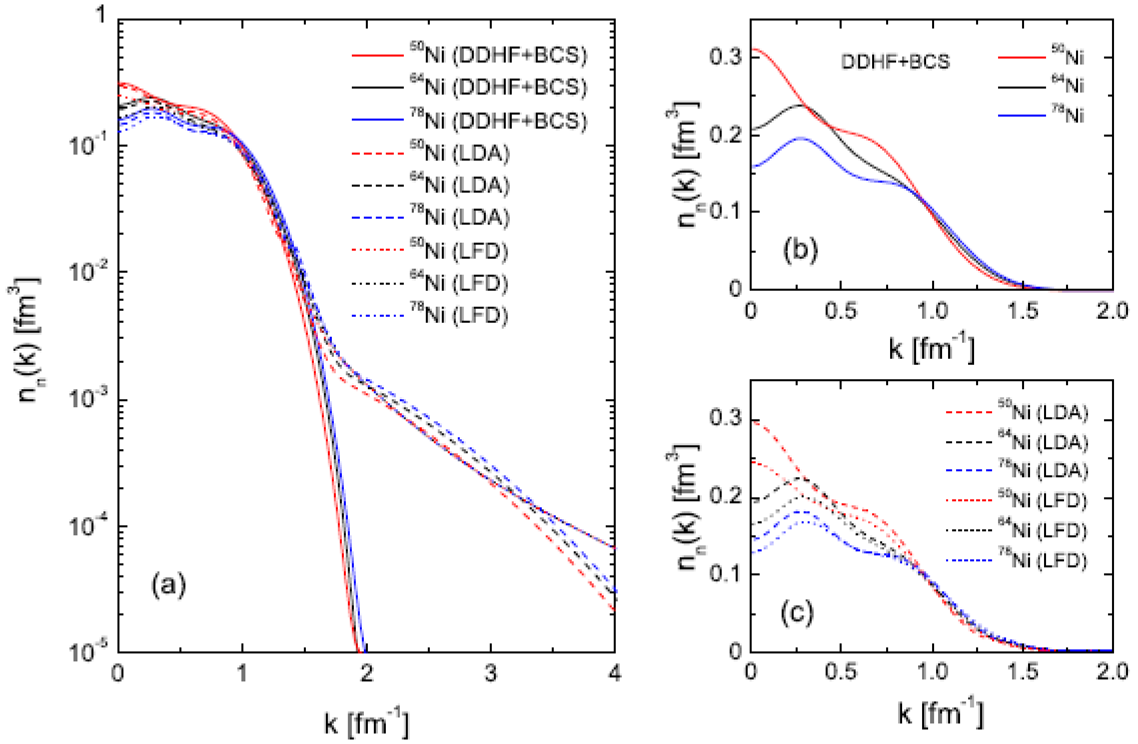

Fig. 1: (a) Neutron momentum distributions obtained within the DDHF+BCS (solid line), LFD (dotted line), and LDA (dashed line) methods for ^{50}Ni, ^{64}Ni, and ^{78}Ni isotopes. The normalization is: $\int n_n(\mathbf{k})d^3k = 1$. The DDHF+BCS results, as well as the LFD and LDA results are separately shown in a linear scale in **(b)** and **(c)**, respectively.

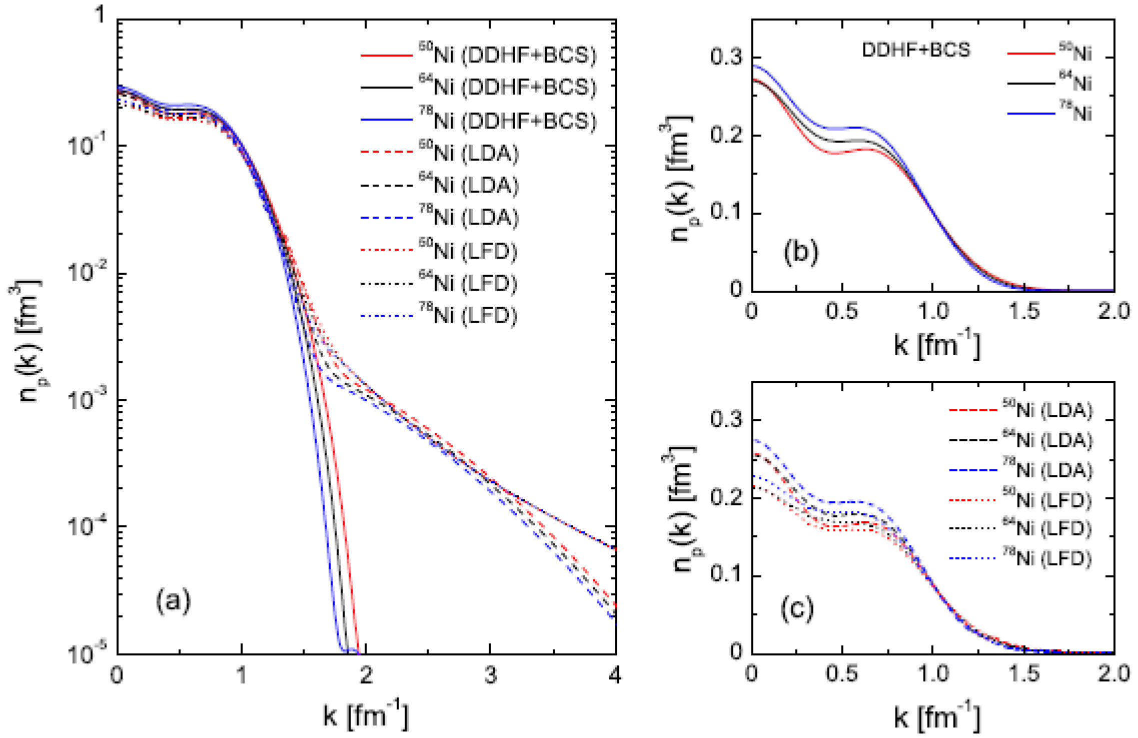

Fig. 2: The same as in **Fig. 1**, but for the proton momentum distributions, $\int n_p(\mathbf{k})d^3k = 1$.

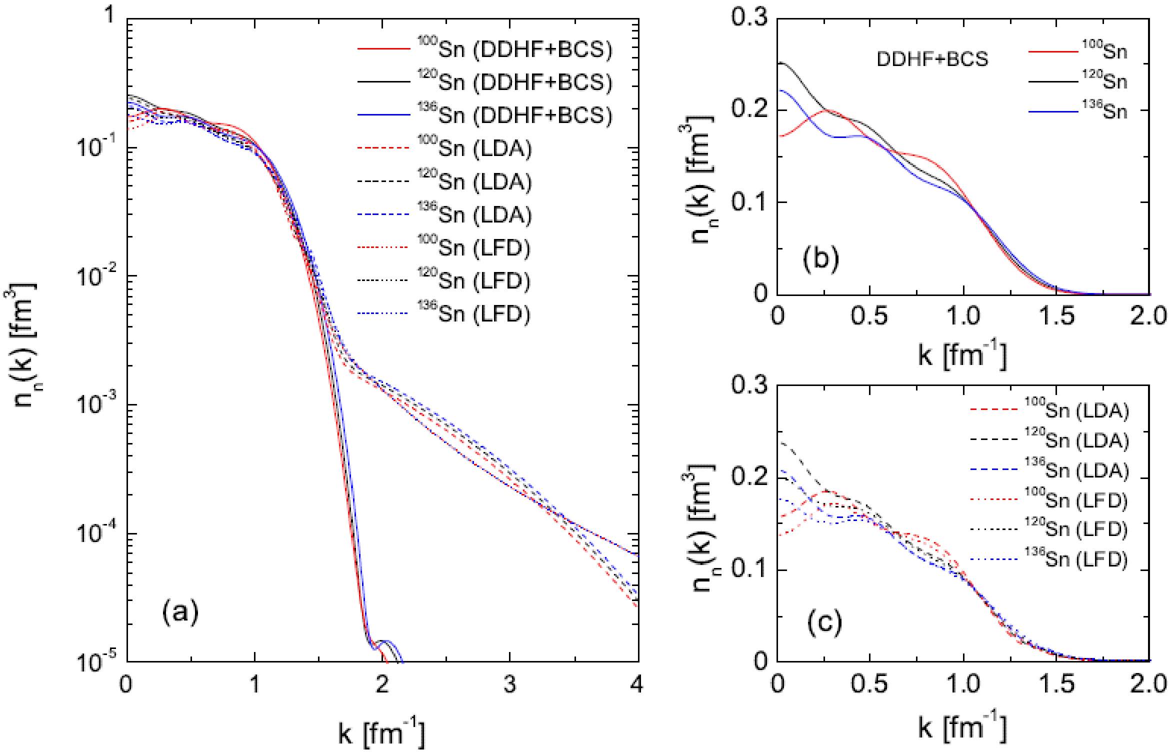

Fig. 3: The same as in **Fig. 1**, but for the ^{100}Sn, ^{120}Sn, and ^{136}Sn isotopes.

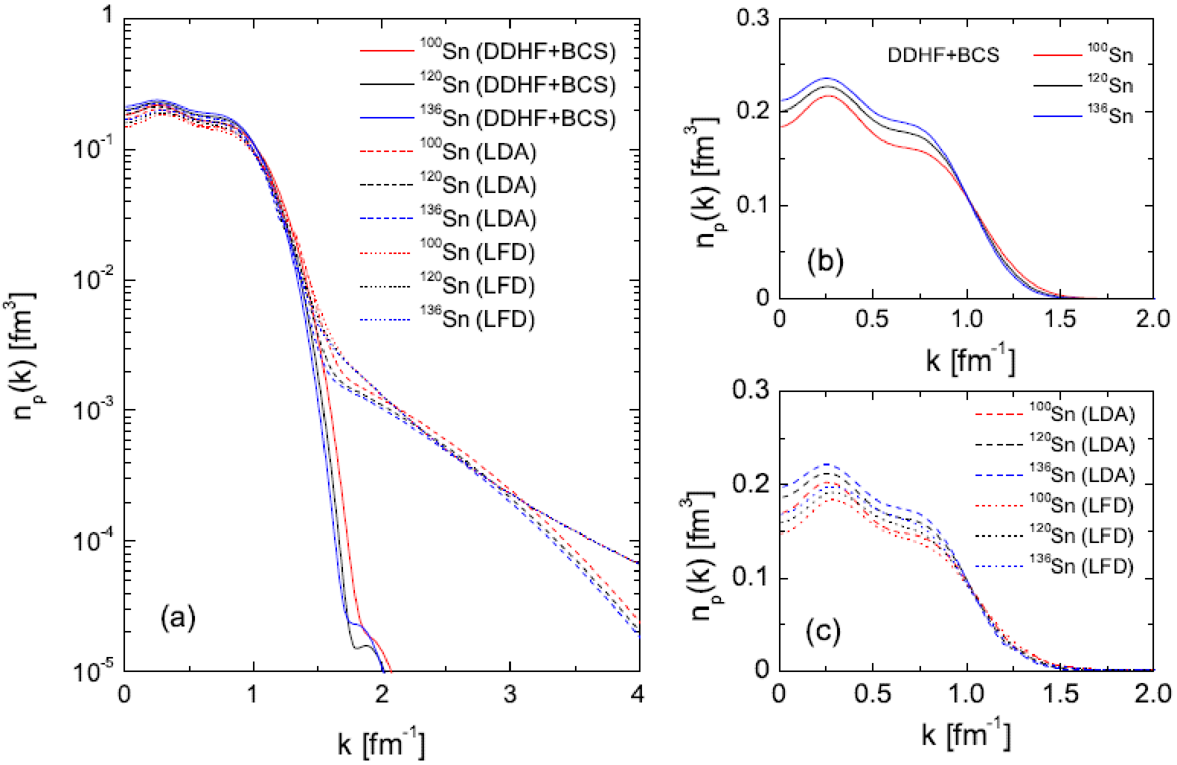

Fig. 4: The same as in **Fig. 3**, but for the proton momentum distributions.

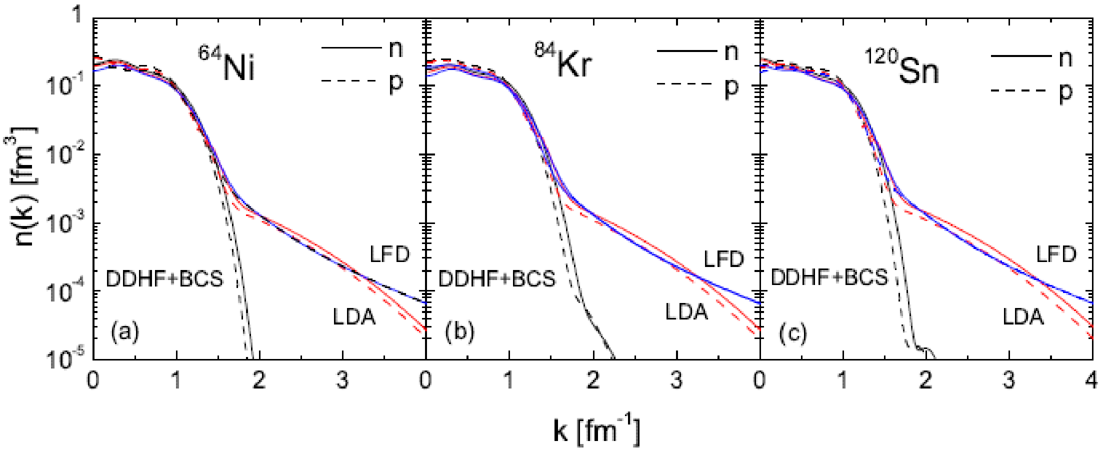

Fig. 5: Neutron (solid line) and proton (dashed line) momentum distributions obtained within the DDHF+BCS (black), LFD (blue), and LDA (red) methods for ^{64}Ni **(a)**, ^{84}Kr **(b)**, and ^{120}Sn **(c)** nuclei. The normalization is: $\int n_{n(p)}(\mathbf{k})d^3k = 1$.

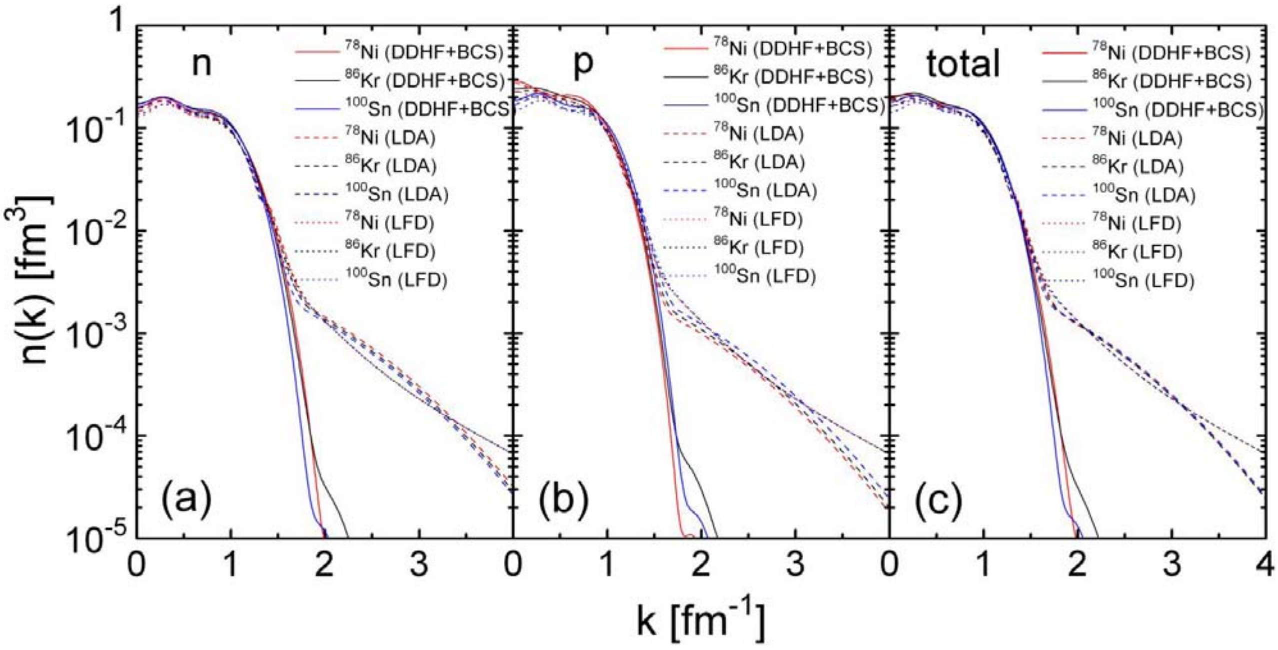

Fig. 6: Neutron **(a)**, proton (**b**), and total **(c)** momentum distributions obtained within the DDHF+BCS (solid line), LFD (dotted line), and LDA (dashed line) methods for ^{78}Ni, ^{86}Kr, and ^{100}Sn isotones. The normalization is: $\int n_{n(p)}(\mathbf{k})d^3k = \int n(\mathbf{k})d^3k = 1$.

It is seen from the figures that the evolution of the NMD's as we increase the number of neutrons consists of an increase of the high-momentum tails (for $k > 1.5\ fm^{-1}$) of $n_n(k)$, while the effect on $n_p(k)$ is opposite. However, the spreading of the tails corresponding to $n_p(k)$ of the considered isotopes is of the same order although the number of protons remains the same. In this respect, the results presented in **Figs. 2** and **4** are challenging because they show how proton momentum distributions "feel" the different number of neutrons in exotic nuclei. We would like also to emphasize that the LFD method does not show this isotopic sensitivity, in contrast to the HF and LDA methods which still demonstrate this trend. Concerning the low-momentum region it can be seen from **Figs. 1-4** that NMD's are very sensitive to the details of the calculations. In this region $n_n(k)$

decreases while, on the contrary, $n_p(k)$ increases with the increase of the number of neutrons N. This is a common feature of the calculated results obtained in all methods. Nevertheless, in this region the spreading is considerably reduced.

A comparison of the results for the neutron and proton momentum distributions of ^{64}Ni, ^{84}Kr, and ^{120}Sn nuclei obtained in the correlation methods is given in **Fig. 5** together with the HF momentum distributions. As can be seen, for all nuclei the inclusion of NN correlations strongly affects the high-momentum region of NMD. At $k > 1.5\ fm^{-1}$ both LFD and LDA momentum distributions start to deviate from the DDHF+BCS case. They behave rather similar in the interval $1.5 < k < 3\ fm^{-1}$. At $k > 3\ fm^{-1}$ the LFD method predicts systematically higher momentum components compared to LDA momentum distributions. This observation can be explained by the different extent to which NN correlations are taken into account in both approaches. Our results for the NMD's in the LFD method for large values of k ($k > 2\ fm^{-1}$) are similar to those obtained within the Jastrow correlation method and thus, the high-momentum tails of $n(k)$ are caused by the short-range NN correlation effects. The LDA approach through the nuclear matter dynamic effects and using the local Fermi momentum $k_F^{Z(N)}(r)$ calculated self-consistently by means of the HF density produces less pronounced high-momentum tail, but still the results are very close to those obtained in the LFD method. As was already shown, at $k > 1.5\ fm^{-1}$ the DDHF+HF momentum distributions fall off rapidly by several orders of magnitude in contrast to the correlated NMD's. In addition, we observe that: (i) the results shown above are similar for all nuclei in a given isotopic chain and going from Ni to Sn isotopes, as well; (ii) the behavior of $n(k)$ is similar for protons and neutrons; (iii) at high k the proton and neutron NMD's obtained within the LFD method cannot be distinguished from each other because the high-momentum tails in this approach are determined by the high-momentum component of the nucleons in the deuteron; (iv) concerning the NMD's calculated in the LDA approach, some difference between $n(k)$ for protons and neutrons can be observed due to $Z(N)$-dependence of the local Fermi momentum k_F.

The isotonic sensitivity of the neutron, proton and total momentum distributions of ^{78}Ni, ^{86}Kr, and ^{100}Sn nuclei ($N=50$) is shown in **Fig. 6**. A small difference between the curves obtained in different approaches (DDHF+BCS or LDA) can be seen. The neutron momentum distributions $n_n(k)$ do not differ so much in comparison to the proton momentum distributions $n_p(k)$. The relative contributions of $n_n(k)$ and $n_p(k)$ to the total momentum distribution $n(k)$ lead to almost equal high-momentum tails of $n(k)$ for these isotones. This is in accordance with the well-known results showing that the tails of $n(k)/A$ are almost equal for all nuclei.

CONCLUSIONS

The presented results can be summarized as follows:

1. A theoretical investigation of the neutron, proton, and total momentum distributions of medium and heavy exotic nuclei, especially of Ni, Kr, and Sn even-even isotopes, was performed on the base of the mean-field method, as well as of two correlation methods taking into account the NN correlations at short distances.

2. The study of the isotopic sensitivity of various kinds of momentum distributions shows different trends. For a given isotopic chain, we find that in the high-momentum region ($k > 1.5\ fm^{-1}$) the high-momentum tails of the neutron momentum distributions $n_n(k)$ increase with the increase of the number of neutrons N, while the proton momentum distributions $n_p(k)$ exhibit an opposite effect. In the same region the LFD method does not show this isotopic sensitivity, in contrast to the DDHF+BCS and LDA methods. At low momenta $n_n(k)$ decreases while, on the contrary, $n_p(k)$ increases with the increase of N.

3. In addition to the isotopic sensitivity we studied how the momentum distributions of some isotones are modified keeping the neutron number constant. We find that the total momentum distributions of ^{78}Ni, ^{86}Kr, and ^{100}Sn nuclei (*N=50*) reveal the same high-momentum tails in all methods used.

4. A comparison of the obtained results for the neutron and proton momentum distributions of ^{64}Ni, ^{84}Kr, and ^{120}Sn shows that the inclusion of NN correlations strongly affects the high-momentum region of NMD.

5. We emphasize that in our work a possible practical way to make predictions for the momentum distributions of exotic nuclei far from the stability line is proposed. It provides a systematic description of $n(k)$ in medium-weight and heavy nuclei. The comparison of the predicted nucleon momentum distributions with the results of possible experiments using a colliding electron-exotic nuclei storage rings would show the effect of the neutron excess in these nuclei and will be also a test for various theoretical models of the structure of exotic nuclei.

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CONTACT ADDRESS

Assoc. Prof. Galina Zaharieva Krumova, PhD
Department of Physics, FEEEA
Angel Kanchev University of Ruse
7017 Ruse, Studentska Str. 8, Bulgaria
Phone: (++35982) 888 215
E-mail: gal@ uni-ruse.bg

ИМПУЛСНИ РАЗПРЕДЕЛЕНИЯ НА СРЕДНИ И ТЕЖКИ, БОГАТИ НА НЕУТРОНИ ЯДРА

Галина Крумова

Русенски университет „Ангел Кънчев"

Резюме: *Изложени са резултатите от пресмятанията на неутронните и протонни импулсни разпределения на някои нестабилни, богати на неутрони изотопи на средни и тежки ядра. Същите са сравнени с тези на стабилни изотопи от същата изотопична верига.*

Ключови думи: *екзотични ядра, импулсни разпределения*

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