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RUSE

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 9

CONTENTS

Mathematics

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RESEARCH ON THE UTILIZATION OF TRANSPORT VEHICLES IN AN EMERGENCY MEDICAL CARE CENTER

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Abstract: *The present paper considers the process of utilizing transport vehicles in an emergency medical care centre. New ambulances enter its fleet with intensity λ(t) and are taken out of service with intensity μ(t). The mathematical expectation and dispersion characteristics have been found of a random process X(t) - the number of ambulances in service at a point in time t. The appropriate modes of operation of the system have been defined. Conclusions have been made.*

Keywords: *Mathematical Modelling, Emergency Medical Care, Theory of the Stochastic Processes*

The expeditious service of patients is a fundamental principle in the operation of the emergency medical care centres. This service is performed by teams of medical personnel and vehicles (ambulances and reanimobiles). Teams, when sufficient in number, will provide service to any call with virtually no waiting in queue in the system, which is the favourable situation for a patient, ensuing that they will receive the necessary emergency aid as soon as possible [3, 4, 6].

Let us consider the process of utilizing transport vehicles in the emergency medical care centre (EMCC) fleet in Ruse. The process of new ambulances entering the fleet is accomplished as Poisson's flow with intensity $\lambda(t)$. The flow of transport vehicle break-downs is of Poisson's type and they are taken out of service with intensity $\mu(t)$. We set ourselves the task to find the mathematical expectation and dispersion characteristics of a random process $X(t)$ - the number of ambulances in the fleet in service at a point in time t, provided there is practically no limit to the number of vehicles in the EMCC fleet and $X(0)=0$ in the initial moment.

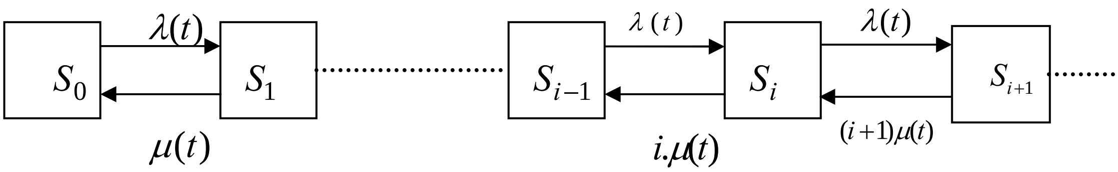

Fig.1. Graph of the states of the system

The graph of the states of the process is shown in Fig. 1. Let the following be satisfied:

$$\lambda_i(t)=\lambda(t), \qquad \mu_i(t)=i.\mu(t). \tag{1}$$

The equation for the mathematical expectation [8] $m_x(t)$ of the considered process $X(t)$ will take the form

$$\frac{dm_x(t)}{dt}=\sum_{i=0}^{\infty}(\lambda(t)-i.\mu(t)).p_i(t)= \tag{2}$$

$$= \lambda(t) - \mu(t).\sum_{i=0}^{\infty} i.p_i(t) = \lambda(t) - \mu(t).m_x(t) .$$

The differential equation (2) is linear. Its general solution for initial condition $m_x(0)$ will have the form:

$$m_x(t) = e^{-\int_0^t \mu(\tau)d\tau} \left[\int_0^t \lambda(x).e^{\int_0^x \mu(\tau)d\tau} dx + m_x(0) \right]. \quad (3)$$

In accordance with the task set, equation (3) can be solved with initial conditions

$$m_x(0) = X(0) = 0 . \quad (4)$$

When we take into account the conditions (4), equation (3) becomes:

$$m_x(t) = e^{-\int_0^t \mu(\tau)d\tau} \int_0^t \lambda(x).e^{\int_0^x \mu(\tau)d\tau} dx . \quad (5)$$

At constant intensities $\lambda = const$ and $\mu = const$ the solution of the equation (3) will take the following form:

$$m_x(t) = \frac{\lambda}{\mu}.\left(1 - e^{-\mu t}\right) + m_x(0).e^{-\mu t} , \quad (6)$$

and for an initial condition $m_x(0) = 0$ we obtain

$$m_x(t) = \frac{\lambda}{\mu}.\left(1 - e^{-\mu t}\right). \quad (7)$$

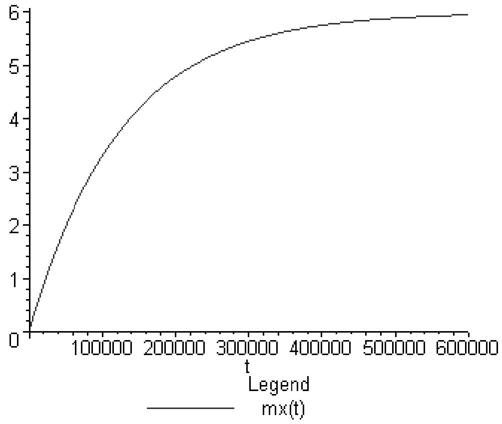

Fig. 2. Dependence of the mathematical expectation of the process on time

In Figure 2 the dependence of the mathematical expectation (7) of the process $X(t)$ is shown on the time during which the process goes on. Since the period of use of an ambulance is not more than 15 years [1], i.e. about 130 000 hours, it follows that $\mu \approx 8.10^{-6}$ ambulances per hour are taken out of service. We have assumed that the purchase of new ambulances occurs with an intensity of $\lambda = 48.10^{-6}$ a/h (ambulances per hour). According to data from the discussed emergency medical care centre in Ruse, in order to provide service to patients with virtually no waiting in queue in the system, and given the existing intensity of incoming calls and the average time for their service, it is necessary for the center to have four ambulances available at any point in time [7]. As ambulances undergo current and planned repairs [2], there should be at least five ambulances in operable condition available. Only after 200,000 hours of operation of the center for emergency medical care the requirement will be met that will enable the system to work close to stationary mode with mathematical expectation of 5, i.e. there will be at least five ambulances in working condition in the system. The enlargement of the fleet should be done with new ambulances and not with used ones, transferred from another centre, in order to minimize the intensity of failures and enable the system to reach faster its required mode of operation and fulfill its purpose.

At these intensities of hourly number of ambulances entering the EMCC in Ruse $\lambda = 48.10^{-6}$ and hourly number of ambulances are taken out of service $\mu = 8.10^{-6}$ Fig.2 shows that reaching the critical number of 4 ambulances, assuring timely service to the patients, is achieved only if the system has reached 140 000 hours of operation from the start of the study, when $X(0) = 0$. At the EMCC in Ruse when the number is below that limit, it is sometimes necessary for the patients to wait in the queue of the system, which may prove fatal to the life and health for those in need of emergency medical attention. In cases of heart attacks and strokes it is vital for the patients to receive the necessary medical aid within the so called “golden hour”, when the probability of their life being saved is the highest.

For the dispersion equation of the random process $X(t)$ we obtain:

$$\frac{dD_x(t)}{dt} = \sum_{i=0}^{\infty} \left[\lambda(t) + i.\mu(t) + 2.(i - m_x(t)).(\lambda(t) - i.\mu(t))\right] p_i(t) = \\ = \lambda(t) + \mu(t).m_x(t) - 2.\mu(t).D_x(t). \tag{8}$$

The general solution of equation (8) with initial condition $D_x(0)$ will have the form:

$$D_x(t) = e^{-\int_0^t 2.\mu(\tau)d\tau} \left[\int_0^t (\lambda(x) + \mu(x).m_x(x)).e^{\int_0^x 2.\mu(\tau).d\tau} dx + D_x(0) \right]. \tag{9}$$

In accordance with the outlined conditions of the task under consideration, the equation (9) will be solved with initial condition

$$D_x(0) = D[X(0)] = D[0] = 0. \tag{10}$$

The equation (9) will take the form:

$$D_x(t) = e^{-\int_0^t 2.\mu(\tau)d\tau} \int_0^t (\lambda(x) + \mu(x).m_x(x)).e^{\int_0^x 2.\mu(\tau)d\tau} dx. \tag{11}$$

If the intensities of both flows - the acquisition of new vehicles $\lambda = const$ and taking out of service $\mu = const$ are constant and the initial condition is $D_x(0) = m_x(0) = 0$, then the following result is obtained

$$D_x(t) = \frac{\lambda}{\mu}.\left(1 - e^{-\mu t}\right) = m_x(t). \tag{12}$$

Therefore, the mathematical expectation of the random process $X(t)$ is equal to its dispersion.

It has been shown in [5] that in stationary mode (with $t \to \infty$) the likelihood for the random process being analysed $X(t)$ to take a value of *i* (i.e. at a given moment of time, the number of functioning cars in the fleet of the EMCC is *i*) is determined by formula (13):

$$\lim_{t \to \infty} p_i(t) = p_i = \frac{\alpha^i}{i!}.e^{-\alpha}, \tag{13}$$

where $\alpha = \frac{\lambda}{\mu}$. Therefore, in stationary mode the distribution law of the random process $X(t)$ is a Poisson law with a parameter $\alpha = \frac{\lambda}{\mu}$ for which the mathematical expectation is equal to the dispersion.

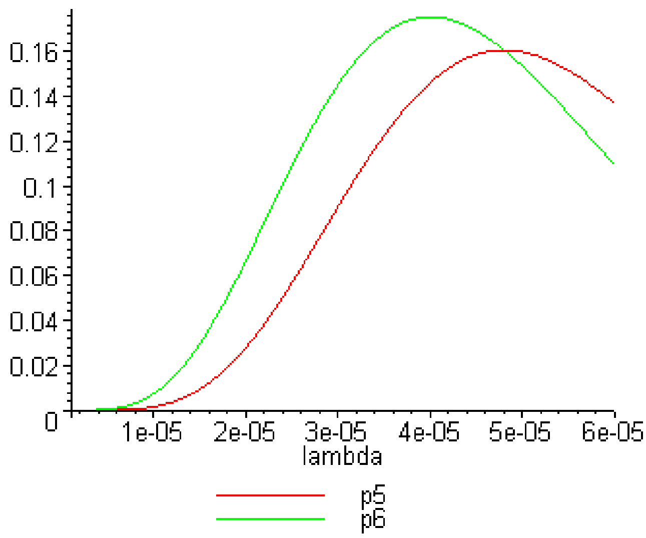

Fig.3. Dependence of probabilities p_5 and p_6 on the intensity of the incoming flow of ambulances purchased

The graphs in Figure 3 for the probabilities p_5 and p_6 are plotted for intensity of removal of ambulances from service of $\mu = 8.10^{-6}$ ambulances per hour.

The conclusions that can be drawn from the research are as follows:

In accordance with the above considerations regarding the requirements for serving patients in the EMCC in Ruse, probabilities p_5 and p_6 are of interest to us, i.e. when at least five operable ambulances are available in the system. The maximum for p_6 is reached at $\lambda = 48.10^{-6}$ a/h. Then the probability for five ambulances being available in the system is 0.16. The maximum for p_6 is reached at $\lambda = 40.10^{-6}$ a/h. The probability for having six operable ambulances in the system is 0.175.

As we are considering a case where at the starting point of the research the number of cars in the EMCC fleet is $X(0)=0$, the acquisition of new vehicles should be carried out with great intensity λ, so that the number of operable ambulance is sufficient and a timely service is provided to patients with virtually no waiting in queue in the system.

In the case being investigated, we have found that for a non-stationary mode of operation of the system of ambulances, the distribution law [8] of the random process $X(t)$ is also a Poisson's law with a parameter

$$m_x(t)=\frac{\lambda}{\mu}.\left(1-e^{-\mu t}\right) \tag{14}$$

with initial conditions (4) and (10) and at constant intensities of both flows $\lambda = const$ and $\mu = const$. If

$$p_i(t)=\frac{\left[m_x(t)\right]^i}{i!}.e^{-m_x(t)}\text{, then} \tag{15}$$

$$\frac{dp_i(t)}{dt}=\frac{i.m_x(t)^{i-1}.e^{-m_x(t)}.\frac{dm_x(t)}{dt}-\left[m_x(t)\right]^i.e^{-m_x(t)}.\frac{dm_x(t)}{dt}}{i!}= \tag{16}$$

$$=\frac{\left[m_x(t)\right]^{i-1}}{(i-1)!}.e^{-m_x(t)}.\frac{dm_x(t)}{dt}-\frac{\left[m_x(t)\right]^i}{i!}.e^{-m_x(t)}.\frac{dm_x(t)}{dt}=$$

$$=p_{i-1}(t).\frac{dm_x(t)}{dt}-p_i(t).\frac{dm_x(t)}{dt}=\left(p_{i-1}(t)-p_i(t)\right).\frac{dm_x(t)}{dt},\quad (i=1,2,......).$$

Therefore:

$$\frac{dm_x(t)}{dt}=\frac{d}{dt}\left[\frac{\lambda}{\mu}.\left(1-e^{-\mu t}\right)\right]=\lambda.e^{-\mu t}=\lambda-\mu.m_x(t). \tag{17}$$

Taking into account equality (17) and substituting it in (16), we then obtain the following result:

$$\frac{dp_i(t)}{dt}=\lambda.p_{i-1}(t)+(i+1).\mu.p_{i+1}(t)-\lambda.p_i(t)-i.\mu.p_i(t). \tag{18}$$

Equation (18) coincides with the equation for the derivative of the probability $p_i(t)$ (according to Kolmogorov equations) for the graph of the states of the system shown in Figure 1.

The results of the research show that equation (15) corresponds to the solution of the system of differential equations for the probabilities of the states, derived from the graph of the states of the system, shown in Figure 1; and the law of distribution of the random process $X(t)$ at constant $\lambda = const$ and $\mu = const$ and initial condition $X(0)=0$ is a Poisson's law with a parameter $m_x(t)$ defined by equation (7).

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ИЗСЛЕДВАНИЯ ВЪРХУ ЕКСПЛОАТАЦИЯТА НА ТРАНСПОРТНИТЕ СРЕДСТВА В ЦЕНТЪР ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ

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Резюме: *В настоящата работа е разгледан процесът на експлоатация на транспортните средства в център за спешна медицинска помощ. В него постъпват линейки с интензивност $\lambda(t)$ и се снемат от експлоатация с интензивност $\mu(t)$. Намерени са характеристиките математическо очакване и дисперсия на случайния процес $X(t)$ - брой линейки в експлоатация в момента от време t. Установени са подходящите режими на работа на системата. Направени са съответните изводи.*

Ключови думи: *Математическо моделиране, Спешна медицинска помощ, Теория на случайните процеси.*

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