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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

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SOME STUDIES ON THE POSSIBILITIES TO PROVIDE EMERGENCY MEDICAL AID CENTRES WITH NEW TRANSPORT VEHICLES

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Abstract: *The present paper studies the process of buying and decommissioning new transport vehicles in the emergency medical aid centres in the Republic of Bulgaria. The process has been modelled by using an appropriate mathematical apparatus. The characteristics of the random process have been found. The actual number of ambulances in operation has been estimated. The probability has been determined for the number of vehicles to be no less than 1600 for the whole period of supplying ambulances to emergency medical centres, assuming that new cars are not taken out of operation. Conclusions are drawn, respectively.*

Keywords: *Mathematical modelling, Random processes, Markov's processes, Distribution laws, Emergency medical aid.*

INTRODUCTION

The purchase and utilization of transport vehicles in the emergency medical aid centres (EMA) in Bulgaria is an important part of the overall patient care. The formation of the teams includes a medical practitioner (doctor) and a vehicle (ambulance) by means of which the team moves to the patients in need of emergency care. The transport service accounts for about 30% of the budget of these centres. Ambulances are expensive vehicles and superfluous ones put an extra burden on the budget [3].

In the present study we set ourselves the task to examine the process of transport vehicles' entry into operation and withdrawal from service in the EMA centers in our country through modeling this process with the appropriate mathematical tools.

METHODOLOGY

TASK ONE

Let us suppose that the purchase of new vehicles is done with intensity $\lambda(t) = at$. We shall assume that the intensity of withdrawal of vehicles from the service is $\mu = const$. We shall be looking for the characteristics of the random process $X(t)$ - the number of ambulances in operation at the point of time *t*, if $X(0) = 0$.

It is shown in the reference literature [6] that with vehicle purchasing intensity of $\lambda(t)$ and intensity of withdrawal from service $\mu(t)$, the mathematical expectation of the random process $X(t)$ will be calculated using formula (1):

$$m_x(t) = e^{-\int_0^t \mu(\tau)d\tau} \left(\int_0^t \lambda(x).e^{\int_0^x \mu(\tau)d\tau} dx + m_x(0) \right) \tag{1}$$

For the case under investigation ($X(0) = 0$ and $m_x(0) = 0$), with the given intensity of ambulances being put into and withdrawn from service, equation (1) will acquire the form (2):

$$m_x(t) = e^{-\mu t} \int_0^t ax.e^{\mu x} dx = \frac{a}{\mu^2}\left(\mu t - 1 + e^{-\mu t}\right). \tag{2}$$

The graph of the relationship (2) is shown in Fig. 1 where $a = 40$ and $\mu = 0.3$. The asymptote to the curve is drawn on the same figure, too.

$$Y = kt + n = \frac{a}{\mu}t - \frac{a}{\mu^2} = \frac{a}{\mu}\left(t - \frac{1}{\mu}\right) \tag{3}$$

It is evident from Fig. 1 ($m_x(t)$ denoted as *mx*) that after a period of time $\tau > \frac{3}{\mu}$ has passed since the beginning of the process under study, the addend, containing $e^{-\mu t}$ will tend to zero and then the graph of the dependence $m_x(t)$ will become linear.

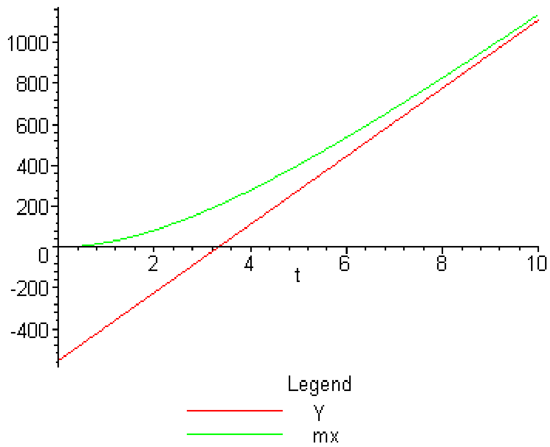

Fig. 1. Dependence of the mathematical expectation on time

As the considered random process is of Poisson type [6], it follows that $m_x(t) = D_x(t)$ and

$$m_x(t) = D_x(t) \approx \frac{a}{\mu}\left(t - \frac{1}{\mu}\right), \quad \left(t > \frac{3}{\mu}\right) \tag{4}$$

The one-dimensional law of distribution of the random process $X(t)$ [1] will be the Poisson law:

$$P(X(t) = i) = p_i(t) = \frac{[m_x(t)]^i}{i!}e^{-m_x(t)}. \tag{5}$$

We will examine the process of providing the emergency medical aid centres with the necessary means of transport. We will presume that the process of introduction of new vehicles is of Poisson's type with intensity $\lambda(t)$ and it has the form:

$$\lambda(t) = \begin{cases} at & за\ 0 \le t \le t_1 \\ at_1 & за\ t_1 < t \le t_2 \\ 0 & за\ t > t_2 \end{cases} \tag{6}$$

Figure 2 shows the dependence of the intensity of introducing new ambulances on time. During the time interval $(0, t_1)$, the initial provision of the centres with new transport vehicles takes place; in the interval (t_1, t_2) the entry of vehicles is with a constant intensity at_1, and at point t_2 the supply of new ambulances is stopped for some reason (either their number is sufficient already, or the economic environment deteriorates and there are no funds for new ambulances). Each ambulance is used for a random period of time T (no longer than 15 years), distributed according to an exponential law with a parameter μ .

The problem we set to ourselves to solve is determining the characteristics of the random process $X(t)$ (the number of ambulances in service) – the mathematical expectation $m_x(t)$ and the variance $D_x(t)$ respectively. We will assume that at the initial moment of the examined process, the old ambulances are worn out and are permanently taken out of operation due to serious damage. Then, the following initial conditions will be fulfilled for the new vehicles: $m_x(0)=0,\ D_x(0)=0.$

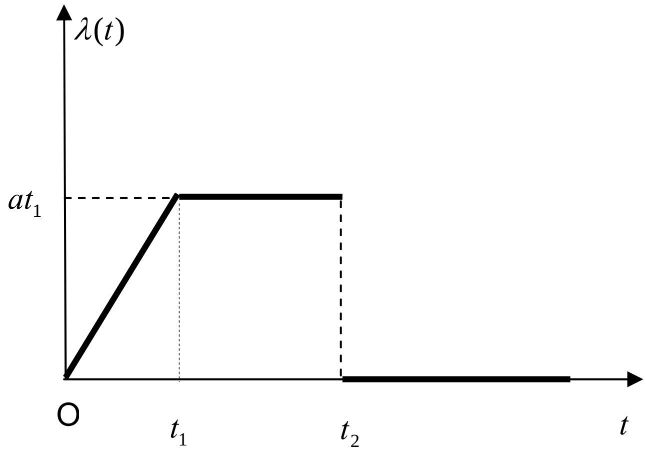

Fig. 2. Dependence of the intensity of introducing new ambulances on time

Taking into account dependence (6), formula (2) will be valid for the mathematical expectation and the variance of the random process $X(t)$ within the $(0,t_1)$ interval, i.e.:

$$m_x(t)=D_x(t)=\frac{a}{\mu^2}\left(\mu t-1+e^{-\mu t}\right). \tag{7}$$

To find the same characteristics in the (t_1,t_2) interval, it is necessary to introduce the variable $\tau=t-t_1$. For the initial condition at point t_1, from equation (2) we obtain

$$m_{x|\tau=0}=D_{x|\tau=0}=\frac{a}{\mu^2}\left(\mu t_1-1+e^{-\mu t_1}\right). \tag{8}$$

Since the intensities λ and μ are constant when $\tau>0$ (for the interval (t_1,t_2)), evidently equation (9) is derived from equation (1).

$$m_x(t)=\frac{\lambda}{\mu}\left(1-e^{-\mu t}\right)+m_x(0).e^{-\mu t}\ . \tag{9}$$

After substituting $\lambda(t)$ from (6) in (9), the following dependence is in effect:

$$m_x(\tau)=D_x(\tau)=\frac{at_1}{\mu}(1-e^{-\mu t})+m_{x|\tau=0}.e^{-\mu\tau}\ . \tag{10}$$

Therefore:

$$m_x(t)=D_x(t)=\frac{at_1}{\mu}\left(1-e^{-\mu(t-t_1)}\right)+\frac{a}{\mu^2}\left(\mu t_1-1+e^{-\mu t_1}\right)e^{-\mu(t-t_1)}=$$

$$= \frac{at_1}{\mu} - \frac{a}{\mu^2}\left(1 - e^{-\mu t_1}\right)e^{-\mu(t-t_1)}, \qquad \left(t_1 < t < t_2\right). \tag{11}$$

For the part of the graph $t > t_2$ (when new ambulances are not provided and $\lambda = 0$), from (9) we obtain

$$m_x(t) = D_x(t) = m_x(t_2).e^{-\mu(t-t_2)}, \quad \left(t > t_2\right), \tag{12}$$

where $m_x\left(t_2\right) = \frac{at_1}{\mu} - \frac{a}{\mu^2}\left(1 - e^{-\mu t_1}\right).e^{-\mu(t_2-t_1)}$. (13)

The graph of the dependence $m_x(t) = D_x(t)$ where $t_1 = 5\ years$, $t_2 = 9\ years$ $a = 50\,(1/year^2)$, $\mu = 0.3\,amb./year$ is shown in Fig. 3.

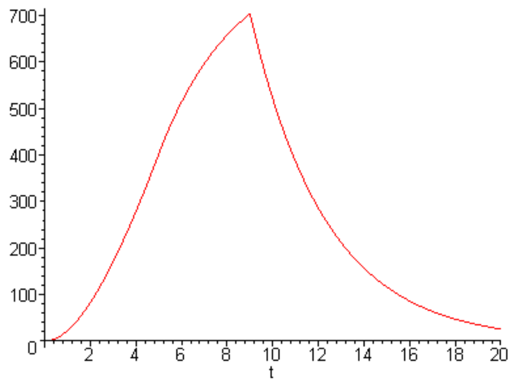

Fig. 3. Dependence of the mathematical expectation on time

The software product MAPLE [5] has been used to draw the graphs in Fig. 1. and Fig. 3.

When $m_x(t) > 20$, one can assert with accuracy sufficient for practical purposes [2] that the one-dimensional process $X(t)$ is normal with the found characteristics. If the average number of ambulances that serve patients in need is $m_x(t)$ and "the three-sigma rule" is applied [4], one can establish that the actual number of cars that are in service in the fleet of the emergency medical aid centres is within the limits

$$m_x(t) \pm 3\sigma(t) = m_x(t) \pm 3\sqrt{D_x(t)} = m_x(t) \pm 3\sqrt{m_x(t)} = m_x(t).\left(1 \pm \frac{3}{\sqrt{m_x(t)}}\right). \tag{14}$$

According to the Ministry of Health of the Republic of Bulgaria, the number of emergency medical aid centers is 186 with a total of 1600 ambulances in them. Then, if $m_x(t) = 1300$ ambulances on average are in service, the following limits will be obtained for the number of ambulances actually used:

$$1300.\left(1 \pm \frac{3}{\sqrt{1300}}\right) \approx 1300 \pm 108. \tag{15}$$

CONCLUSION

From the above results it can be concluded that the actual number of ambulances in operation fluctuates by 8% around their mean value.

TASK TWO

The second problem that the present study sets about to solve is finding the probability for the number of vehicles to be no less than x=1600 for the whole period of supplying ambulances to EMA centres, assuming that there are no withdrawals (taking out of operation) of newly bought ambulances, i.e. when $\mu_i(t) = 0, (i = 1,2,....)$.

In this case, a Markov's process of "pure" multiplication runs in the system and its characteristics are determined by the differential equations [6]

$$\frac{dm_x(t)}{dt} = \sum_{i=0}^{\infty} (\lambda_i(t) - \mu_i(t)).\, p_i(t) \qquad (16)$$

and $$\frac{dD[X(t)]}{dt} = \frac{dD_x(t)}{dt} = \sum_{i=0}^{\infty} [\lambda_i(t) + \mu_i(t) + 2(i - m_x(t)).\,(\lambda_i(t) - \mu_i(t))].\, p_i(t)\,. \qquad (17)$$

From (16) and (17) for $\mu_i(t) = 0, (i = 1,2,....)$ the following equations are obtained respectively

$$\frac{dm_x(t)}{dt} = \sum_{i=0}^{\infty} \lambda_i(t).\, p_i(t) \qquad (18)$$

and $$\frac{dD_x(t)}{dt} = \sum_{i=0}^{\infty} \lambda_i(t).\, [1 + 2.(i - m_x(t))].\, p_i(t). \qquad (19)$$

If $\lambda_i(t) = \lambda(t)$ and at the initial point of time $t = 0$ the distribution of probabilities $p_i(0), (i = 0,1,2,3,.....)$ represents Poisson distribution with a parameter $m_x(0)$ and the equations below are fulfilled

$$p_i(0) = \frac{[m_x(0)]^i}{i!}.\, e^{-m_x(0)},\; i = 0,1,2........, \qquad (20)$$

then the one-dimensional law for distribution of the random process $X(t)$ represents Poisson law with a parameter $m_x(t)$, which can be defined by solving the equation

$$\frac{dm_x(t)}{dt} = \sum_{t=0}^{\infty} \lambda(t).\, p_i(t) = \lambda(t)\,. \qquad (21)$$

Therefore $$m_x(t) = \int_0^t \lambda(t)dt + m_x(0)\,. \qquad (22)$$

By taking into account the way of setting of the intensity of the Poisson flow of purchasing new transport vehicles for the EMA centres (6), then from the dependence (22) one can calculate the mathematical expectation of the total number of supplied cars on the condition that $m_x(0) = 0,$ i.e. at the initial point of the period for supplying the EMA centres, there isn't a single new ambulance there. Then:

$$m_x(\infty) = \int_0^{\infty} \lambda(t)dt = \int_0^{t_2} \lambda(t)dt = \int_0^{t_1} atdt + \int_{t_1}^{t_2} at_1 dt = \frac{at_1^2}{2} + at_1(t_2 - t_1). \qquad (23)$$

By substituting the values of the quantities t_1, t_2, a in (23), i.e. $t_1 = 5 years,\ t_2 = 9\ years,\ a = 50\ (1/\,year^2)$ with the ones above, we get:

$$m_x(\infty) = m_x(t_2) = 50.\frac{5^2}{2} + 50.5.(9-5) = 1625 \text{ ambulances.} \tag{24}$$

Because in our case the conditions of the Poisson distribution are fulfilled for the random process $X(t)$, then $m_x(0) = D_x(0) = 0,\ p_0(0) = 1$ and $m_x(t_2) = D_x(t_2) = 1625$, from which the result is obtained, namely

$$\sigma_x(t_2) = \sqrt{D_x(t_2)} = \sqrt{1625} = 40{,}31 \text{ ambulances.} \tag{25}$$

CONCLUSIONS

It is known from the literature [2] that when $m_x(t_2) > 20$, one can assert with accuracy sufficient for practical purposes, that the random quantity $X(t_2)$ distributed according to Poisson law, is if fact distributed according to a normal law with a parameter $m_x(t_2) = 1625$ ambulances. Then $\sigma_x(t_2) = 40{,}31$ ambulances. Therefore, the probability we are looking for is

$$\begin{aligned} P\{X(t_2) > 1600\} &= 0{,}5 - \Phi\left\{\frac{x - m_x(t_2)}{\sigma_x(t_2)}\right\} = 0{,}5 - \Phi\left\{\frac{1600-1625}{40{,}31}\right\} = \\ &= 0{,}5 - \Phi\{-0{,}61\} = 0{,}5 + 0{,}2291 = 0{,}7291 \end{aligned}. \tag{26}$$

By applying "the three-sigma rule", the average number of ambulances supplied for the duration of the period under investigation is in practice defined by the condition

$$m_x(t_2) \pm 3\sigma_x(t_2) = 1625 \pm 3.\sqrt{1625} = 1625 \pm 3.40{,}31 \approx 1625 \pm 120, \tag{27}$$

i.e. it varies between the limits $1505 < X(t_2) < 1745$.

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НЯКОИ ИЗСЛЕДВАНИЯ ВЪРХУ ВЪЗМОЖНОСТИТЕ ЗА ОБОРУДВАНЕ НА ЦЕНТРОВЕТЕ ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ С НОВИ ТРАНСПОРТНИ СРЕДСТВА

Веселина Евтимова

Русенски университет „Ангел Кънчев”

Резюме: *В настоящата работа е изследван процесът на закупуване и снемане от експлоатация на нови транспортни средства в центровете за спешна медицинска помощ в Република България. Процесът е моделиран като е използван подходящ математически апарат. Намерени са характеристиките на случайния процес. Определен е реалният брой на линейките в експлоатация. Намерена е вероятността за целия период на доставка на нови автомобили в центровете за спешна медицинска помощ броят на линейките да бъде не по-малко от 1600 като се счита, че няма откази на закупените нови автомобили. Направени са съответните изводи.*

Ключови думи: *Математическо моделиране, Случайни процеси, Марковски процеси, Закони на разпределение, Спешна медицинска помощ*

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