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SERIES 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 7

CONTENTS

Mathematics

Julia Chaparova, Eli Kalcheva 7
Three approaches toward sublinear emden – fowler equation

Tihomir Gyulov, Gheorghe Moroşanu 14
On a fourth order differential inclusion involving p-biharmonic operator

Petar Rashkov 22
A note on averaging in differential equations with Hukuhara derivative and delay

Tsetska Rashkova, Antoaneta Mihova 30
Vishne identities for $M_2(G)$ and their computer realization by Mathematica

Informatics

Desislava Atanasova 38
An approach of visualization of belt pulley calculation results

Galina Atanasova 42
Conventional knowledge testing in comparison with intelligent test in algorithm area – an experimental study

Elena Evtimova, Galina Krumova 50
Learning development cycle applied to physics through adaptive learning methodology

Svetlin Stoyanov, Galina Krumova 57
Multilanguage and mobile training of physics with Moodle

Svetlin Stoyanov, Galina Krumova, Elena Evtimova 61
Information technologies for optimization of the learning process in the universities

Tzvetomir Vassilev, Stanislav Kostadinov 67
An approach to 3d on the web using Java OpenGL

Physics

František Látal, Renata Holubová 73
Remote experiments – new approaches to physical experimentation

Nadezhda Nancheva, Svetlin Stoyanov 79
M-learning of the superconductivity

Vladimir Voinov, Roza Voinova, Zlatka Mateva 84
An application of the extended mirror images method in constructing equivalent models for calculating the vector potential of the magnetic field produced from current fibre in inhomogeneous dielectric medium

ON A FOURTH ORDER DIFFERENTIAL INCLUSION INVOLVING *p*-BIHARMONIC OPERATOR

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Abstract: *A fourth order boundary value problem associated with the p-biharmonic operator is considered, where **p>1**. It models various problems in the theory of elasticity, e.g. the shape of an elastic beam where the bending moment depends on the curvature as a power function with exponent **p-1**. The results presented here concern the existence of solutions satisfying a quite general boundary condition.*

Keywords: *bi-p-Laplacian, differential inclusion, generalized Clarke gradient, non-smooth critical point theory.*

INTRODUCTION

In this paper we investigate the following fourth order differential inclusion

$$\left(|u''|^{p-2}u''\right)'' - a\left(|u'|^{p-2}u'\right)' + b|u|^{p-2}u \in \bar{\partial}F(t,u), \tag{1}$$

$$\begin{pmatrix} -\left(|u''|^{p-2}u''\right)'(0) + a|u'(0)|^{p-2}u'(0) \\ \left(|u''|^{p-2}u''\right)'(1) - a|u'(1)|^{p-2}u'(1) \\ |u''(0)|^{p-2}u''(0) \\ -|u''(1)|^{p-2}u''(1) \end{pmatrix} \in \partial j \begin{pmatrix} u(0) \\ u(1) \\ u'(0) \\ u'(1) \end{pmatrix}, \tag{2}$$

where a and b are given constants, $p > 1$ and functions F, j satisfy the following assumptions:

(H_1) $F(t,\xi):(0,1)\times \mathrm{R} \to \mathrm{R}$ is a Carathéodory mapping, satisfying in addition $F(t,0)=0$ for a.a. $t \in (0,1)$, as well as the Lipschitz condition: $\forall \rho > 0,$ there is an $\alpha_\rho \in L^1(0,1)$, such that, for a.a. $t \in (0,1)$ and all $x,\ y$ with $|x|,|y| \le \rho,$

$$|F(t,x) - F(t,y)| \le \alpha_\rho(t)|x-y|;$$

(H_2) Function $j:\mathrm{R}^4 \to (-\infty,+\infty]$ is proper, convex and lower semicontinuous (l.s.c), such that $(0,0,0,0)^T \in D(j)$. Here $\bar{\partial}F(t,\xi)$ denotes the generalized Clarke gradient (see [1]) of $F(t,\cdot)$ at $\xi \in \mathrm{R}$, while ∂j stands for the subdifferential of j.

A classical fourth order equation arising in the beam-column theory is the following (see Timoshenko and Gere [10])

$$EI\frac{d^4u}{dx^4} + P\frac{d^2u}{dx^2} = q, \tag{3}$$

where u is the lateral deflection, q is the intensity of a distributed lateral load, P is the axial compressive force applied to the beam and EI represents the flexural rigidity in the plane of bending. Equation (3) is derived from the static equilibrium equations for any slice at distance x along the beam, namely the equilibrium of forces reads

$$q = -\frac{dV}{dx}, \tag{4}$$

where V is the shearing force and the equilibrium of moments is described by the equation

$$V=\frac{dM}{dx}-P\frac{du}{dx}, \tag{5}$$

where M denotes the bending moment. It is assumed that the bending moment depends linearly on the curvature. It can be expressed (if the higher order terms are neglected) as follows

$$EI\frac{d^2u}{dx^2}=-M. \tag{6}$$

Let a more general condition is assumed, that the bending moments depend as a power function of the curvature with exponent $p-1$, i.e.

$$M=-c\left|\frac{d^2u}{dx^2}\right|^{p-2}\frac{d^2u}{dx^2}, \tag{7}$$

where c is a constant. Then the presence of the term $\left(|u''|^{p-2}u''\right)''$ in (1) is justified if we assume (7) instead of (6) when equation (3) is derived. If $p=2$, then (7) coincides with (6) where $c=EI$. We obtain results that extend our previous ones presented in [5] for the case $p=2$.

Another equation that motivates the studied problem is the following one

$$Dw^{iv}+N_x w''+Eh\frac{w}{a^2}=q, \qquad t\in(0,1), \tag{8}$$

which models the radial deflection w for symmetrical buckling of a cylindrical shell under uniform axial compression N_x (see [10], p. 457, [9]).

The applied lateral load q in (3) or (8) may be presented as the reaction of a support, which generally depends nonlinearly on the deflection (see [2]-[5], [8]-[9]),

$$q(t)=f(t,u(t)),$$

or, more generally,

$$q(t)\in\bar{\partial}F(t,u(t)),$$

for some function F (when the nonlinearity in Eq. (1) has some jumps, e.g., the case of adhesive support, see [9]).

Condition (2) provides a general framework for different types of boundary conditions (see [6]). For example, the functional

$$j\left((x_1,x_2,x_3,x_4)^T\right):=\begin{cases}0, & x_1=x_2,\, x_3=x_4,\\ +\infty, & \text{otherwise},\end{cases}$$

corresponds to periodic conditions $u^{(i)}(0)=u^{(i)}(1), i=0,1,2,3,$ while the case of simply supported endpoints, i.e., $u(0)=u(1)=u''(0)=u''(1)=0$, can be obtained by choosing

$$j\left((x_1,x_2,x_3,x_4)^T\right):=\begin{cases}0, & x_1=x_2=0,\\ +\infty, & \text{otherwise}.\end{cases}$$

In this paper we investigate the existence of solutions to problem (1)-(2). By a *solution* of this problem we mean a function $u\in W^{2,p}(0,1)$ such that

$\left(|u''|^{p-2}u''\right)'\in \mathrm{AC}([0,1],\mathrm{R})$, and u satisfies (2) and for a.a. $t\in(0,1)$

$$\left(|u''(t)|^{p-2}u''(t)\right)'' - a\left(|u''(t)|^{p-2}u''(t)\right)' + b|u(t)|^{p-2}u(t) \in \overline{\partial}F(t,u). \tag{9}$$

Consider the set

$$\mathrm{D} = \left\{u : u \in W^{2,p}(0,1), (u(0),u(1),u'(0),u'(1))^T \in D(j)\right\},$$

and the functional $J : W^{2,p}(0,1) \to \mathrm{R} \cup \{+\infty\}$ defined by

$$J(u) := j\left((u(0),u(1),u'(0),u'(1))^T\right), \quad \forall u \in W^{2,p}(0,1),$$

whose effective domain is $D(J) = \mathrm{D}$.

Obviously, $\mathrm{D} \neq \varnothing$ since $(0,0,0,0)^T \in D(j)$, so J is proper, convex and l.s.c.

The following two constants,

$$\lambda_1 := \inf\left\{\frac{\int_0^1\left(|u''|^p + a|u'|^p + b|u|^p\right)dt}{\|u\|_{L^p}^p} : u \in \mathrm{D} \setminus \{0\}\right\}, \tag{10}$$

and

$$\overline{\lambda}_1 := \lim_{s\to\infty}\inf_{\substack{ru\in \mathrm{D} \\ r\geq s}}\left\{\int_0^1\left(|u''|^p + a|u'|^p + b|u|^p\right)dt + \frac{pJ(ru)}{r^p} : \|u\|_{L^p}^p = 1\right\}, \tag{11}$$

will be very important in the sequel. It is easily seen that $\lambda_1 \leq \overline{\lambda}_1$, but in general $\lambda_1 < \overline{\lambda}_1$.

We are going to apply the variational approach developed in Motreanu and Panagiotopoulos [9] to the functional

$$I(u) := \frac{1}{p}\int_0^1\left(|u''|^p + a|u'|^p + b|u|^p\right)dt - \int_0^1 F(t,u)dt + J(u), \tag{12}$$

to obtain existence of solutions to problem (1)-(2).

Our main results are included in the following two theorems.

Theorem 1.1. *Assume (H_1) and (H_2). If, in addition,*

(L_∞)
$$\limsup_{|x|\to\infty}\frac{F(t,x)}{|x|^p} < \frac{\lambda_1}{p}, \tag{13}$$

uniformly for a.a. $t \in (0,1)$, then problem (1)-(2) has at least a solution.

Next, suppose that function F satisfies

(L_0)
$$\limsup_{x\to 0}\frac{F(t,x)}{|x|^p} < \frac{\lambda_1}{p},$$

uniformly for a.e. $t \in (0,1)$. Then, $0 \in \overline{\partial}F(t,0)$ for a.a. $t \in (0,1)$, so in this case $u(t) \equiv 0$ is a solution of problem (1)-(2). Thus it is reasonable to investigate the existence of nonzero solutions of problem (1)-(2). We have

Theorem 1.2. *Assume that $\lambda_1 > 0$ and that (L_0), (H_1), and (H_2) are fulfilled. Suppose, in addition, that $D(j)$ is closed, $(0,0,0,0)^T \in \partial j\left((0,0,0,0)^T\right)$, and either (G_θ) or (G_p)– $(\overline{L}_\infty)$ hold, where*

(G_θ) *there exist constants $\theta > p$, and $k, M > 0$, such that*

$$j'(z;z) \le \theta j(z) + k, \qquad \forall z \in D(j), \tag{14}$$

$$0 < \theta F(t,x) \le \xi x, \quad \forall \xi \in \bar{\partial} F(t,x), \tag{15}$$

for all $|x| > M$, and a.a. $t \in (0,1)$, (G_p) *there exist positive constants* c, k, M *such that*

$$j'(z;z) \le pj(z) + k, \qquad \forall z \in D(j), \tag{16}$$

$$0 < \left(p + \frac{c}{|x|^{p-1}} \right) F(t,x) \le \xi x, \quad \forall \xi \in \bar{\partial} F(t,x), \tag{17}$$

for all $|x| > M$, *and a.a.* $t \in (0,1)$, *and*

$(\bar{L}_\infty)$
$$\liminf_{|x| \to \infty} \frac{F(t,x)}{|x|^p} > \frac{\bar{\lambda}_1}{p}, \tag{18}$$

uniformly for a.e. $t \in (0,1)$.

Then problem (1)-(2) has a nonzero solution.

PRELIMINARIES

Let X be a Banach space and let Φ be a locally Lipschitz function defined on X. Denote by $\Phi^0(u;v)$ the generalized directional derivative of Φ at point $u \in X$ in the direction $v \in X$,

$$\Phi^0(u;v) := \limsup_{w \to u, s \downarrow 0} \frac{\Phi(w + sv) - \Phi(w)}{s},$$

and by $\bar{\partial}\Phi(u)$ the generalized gradient of Clarke,

$$\bar{\partial}\Phi(u) = \{\eta \in X^* : \Phi^0(u;v) \ge \langle \eta, v \rangle, \forall v \in X\}.$$

Let Φ be as above and let $\psi : X \to (-\infty, +\infty]$ be a proper, convex and l.s.c. function. We recall that an element $u \in X$ is a *critical point* of a functional I of the form

$$I = \Phi + \psi,$$

if the following inequality holds

$$\Phi^0(u; v - u) + \psi(v) - \psi(u) \ge 0, \quad \forall v \in X.$$

A number $c \in \mathbb{R}$ such that $I^{-1}(c)$ contains a critical point is called a *critical value* of I.

Definition 2.1. *The functional* I *is said to satisfy the Palais-Smale condition if every sequence* $\{u_n\} \subset X$ *for which* $I(u_n)$ *is bounded and*

$$\Phi^0(u_n; v - u_n) + \psi(v) - \psi(u_n) \ge -\varepsilon_n \|v - u_n\|, \quad \forall v \in X, \tag{19}$$

for a sequence $\{\varepsilon_n\} \subset \mathbb{R}^+$ *with* $\varepsilon_n \to 0$, *possesses a convergent subsequence.*

We use the following generalized mountain pass result which can be found in [9] (see also [7]).

Theorem 2.1. (Mountain Pass) *Suppose that* I *satisfies the Palais-Smale condition,* $I(0) = 0$ *and*

(i) there exist α, $\rho > 0$ such that $I(u) \ge \alpha$ if $\|u\| = \rho$,

(ii) $I(e)\le 0$ for some $e\in X$, with $\|e\|>\rho$.

Then the number

$$c=\inf_{\gamma\in\Gamma}\sup_{t\in[0,1]} I(\gamma(t)),$$

where

$$\Gamma=\{\gamma\in C([0,1],X):\gamma(0)=0,\,\gamma(1)=e\},$$

is a critical value of I with $c\ge\alpha$.

In what follows, our space will be $X=W^{2,p}(0,1)$. Define the convex functional

$$\psi(u):=\frac{1}{p}\int_0^1\left(|u''|^p+|u'|^p+|u|^p\right)dt+J(u)=\frac{1}{p}\|u\|^p_{W^{2,p}(0,1)}+J(u),$$

whose effective domain is $D(\psi)=\mathrm{D}$, and

$$\Phi(u):=-\int_0^1 F(t,u)dt+\varphi(u),\quad u\in W^{2,p}(0,1), \tag{20}$$

where

$$\varphi(u):=\frac{1}{p}\int_0^1\left((a-1)|u'(t)|^p+(b-1)|u(t)|^p\right)dt.$$

Obviously, ψ is proper, convex and l.s.c., while $\varphi\in C^1\left(W^{2,p}(0,1),\mathrm{R}\right)$, and

$$\langle\varphi'(u),v\rangle=\int_0^1\left((a-1)|u'(t)|^{p-2}u'(t)v'(t)+(b-1)|u(t)|^{p-2}u(t)v(t)\right)dt.$$

We first state without proof some auxiliary results.

Proposition 2.1. *Assume that $F:[0,1]\times\mathrm{R}\to\mathrm{R}$ satisfies (H_1) and let Φ be defined by (20). Then $\Phi(\cdot)$ is locally Lipschitz. Moreover, if $u\in W^{2,p}(0,1)$ and $l\in\bar\partial\Phi(u)$ then there is some $u_l\in L^1(0,1)$ such that $u_l(t)\in\bar\partial F(t,u(t))$ for a.a. $t\in(0,1)$, and*

$$\langle l,v\rangle=\int_0^1\left(-u_l(t)v(t)+(a-1)|u'(t)|^{p-2}u'(t)v'(t)+(b-1)|u(t)|^{p-2}u(t)v(t)\right)dt, \tag{21}$$

for all $v\in W^{2,p}(0,1)$. Here $\langle\cdot,\cdot\rangle$ denotes the duality pairing in $W^{2,p}(0,1)$.

Now, let the functional I be defined by (12). Obviously, $I(u)=\Phi(u)+\psi(u)$.

Theorem 2.2. *Assume that $F:[0,1]\times\mathrm{R}\to\mathrm{R}$ satisfies (H_1) and let $u\in W^{2,p}(0,1)$. If u is a critical point of functional I, then u is a solution of problem (1)-(2).*

Lemma 2.1 *Define λ_1 by (10). Then $\lambda_1>-\infty$. Moreover, if $\lambda_1>0$, then there exists a constant $m>0$ such that*

$$\int_0^1\left(|u''|^p+a|u'|^p+b|u|^p\right)dt\ge m\|u\|^p_{W^{2,p}(0,1)},\quad\forall u\in\mathrm{D}.$$

Lemma 2.2. *Assume (H_1) holds, $\lambda_1>0$, and either (G_θ) or (G_p) holds. If, in addition, $D(j)$ is closed, then functional I satisfies the Palais-Smale condition.*

Proof of Theorem 1.2. when case (G_θ) holds. We will apply Theorem 2.1. First of all, according to Lemma 2.2, I satisfies the Palais-Smale condition.

Next, we can assume without any loss of generality that $j\left((0,0,0,0)^T\right)=0$. Since $(0,0,0,0)^T \in \partial j\left((0,0,0,0)^T\right)$, we have $J(u)\geq J(0)=0$ and so in particular $I(0)=0$. Now, we will prove that there exist $\rho>0$ and $\alpha(\rho)>0$ such that $I(u)\geq \alpha$ for $\forall u\in W^{2,p}(0,1)$ with $\|u\|=\rho$, where $\|\cdot\|$ denotes the norm of $W^{2,p}(0,1)$. Indeed, if $\rho=\|u\|$ is small enough, then by $|u(t)|\leq d\|u\|$ for some constant d, and by

$$F(t,x)\leq \frac{\lambda_1-\sigma}{p}|x|^p, \qquad \forall |x|\leq \delta,$$

for some constants $\sigma>0$ and $\delta>0$, we have

$$I(u)\geq \frac{1}{p}\int_0^1\left(|u''|^p+a|u'|^p+b|u|^p\right)dt-\frac{\lambda_1-\sigma}{p}\int_0^1|u|^p\,dt\geq m\|u\|^p,$$

for some constant $m>0$. Here, we have used the inequality

$$\int_0^1\left(|u''|^p+a|u'|^p+(b-\lambda_1+\sigma)|u|^p\right)dt\geq \sigma\int_0^1|u|^p\,dt,$$

as well as Lemma 2.1 for b replaced with $b-\lambda_1+\sigma$.

Finally, we have to find an $e\in W^{2,p}(0,1)$ such that

$$I(e)\leq 0 \quad \text{and} \quad \|e\|>\rho. \tag{22}$$

The mapping $s\mapsto s^{-\theta}F(t,sx)$ is locally Lipschitz for a.a. $t\in(0,1)$, so we have for each $s>0$

$$\begin{aligned}\overline{\partial}_s\left(s^{-\theta}F(t,sx)\right)&\subset \overline{\partial}_s\left(s^{-\theta}\right)F(t,sx)+s^{-\theta}\,\overline{\partial}_s\left(F(t,sx)\right)\\ &=s^{-\theta-1}\left(-\theta F(t,sx)+sx\overline{\partial}F(t,sx)\right).\end{aligned}$$

Let $|x|>M$, where M is the constant which appear in the statement of the theorem. Given $1\leq r<s$, by Lebourg's mean value theorem and assumption (15), there exist $\tau\in(r,s)$ and $\xi\in\overline{\partial}_s\left(s^{-\theta}F(t,sx)\right)\big|_{s=\tau}$, $\xi\geq 0$, such that $s^{-\theta}F(t,sx)-r^{-\theta}F(t,rx)=\xi(s-r)\geq 0$, i.e.,

$$F(t,sx)\geq s^\theta F(t,x), \quad \text{for a.a. } t\in[0,1], \quad \forall |x|>M, \quad s\geq 1.$$

Now, let $h\in C_0^\infty(0,1)$ be such that $|h|>M$ on a set with positive measure. Then,

$$\begin{aligned}\int_0^1 F(t,sh)dt&=\int_{\{|sh|>M\}}F(t,sh)dt+\int_{\{|sh|\leq M\}}F(t,sh)dt\\ &\geq s^\theta\int_{\{|h|>M\}}F(t,h)dt-M\int_0^1\alpha_M(t)dt,\end{aligned}$$

for all $s\geq 1$. We have $J(sh)=0$ for each s, thus

$$I(sh)=\frac{s^p}{p}\int_0^1\left(|h''|^p+a|h'|^p+b|h|^p\right)dt-\int_0^1F(t,sh)dt$$

$$\leq \frac{s^p}{p}\int_0^1 \left(|h''|^p + a|h'|^p + b|h|^p\right)dt - s^\theta \int_{\{|h|>M\}} F(t,h)dt + M\int_0^1 \alpha_M(t)dt$$

for all $s \geq 1,$ i.e.,

$$I(sh) \leq s^p k_1 - s^\theta k_2 + k_3 \to -\infty, \quad \text{as} \quad s \to \infty,$$

with $k_1, k_2, k_3 > 0.$ Therefore, we can choose s_0 sufficiently large such that $I(s_0 h) \leq 0$ and $\|s_0 h\| > \rho.$ Then $e := s_0 h$ satisfies conditions (22).

AN EXAMPLE

We present an example for the case $p = 2.$ Consider the boundary value problem

$$u^{iv} = F'(u), \tag{23}$$

$$u(0) = u'(0) = u(1) = u'(1) = 0, \tag{24}$$

where $F(x) = \frac{c}{2}x^2 e^{-\frac{1}{|x|}},\ c > 0.$ It is easily seen that

$$F'(x) = c\left(x + \frac{\text{sgn}x}{2}\right)e^{-\frac{1}{|x|}}$$

and we can choose $j\left((x_1, x_2, x_3, x_4)^T\right) = 0$, if $x_1 = x_2 = x_3 = x_4 = 0$, and $= +\infty$, otherwise.

Obviously, functions j and F satisfy assumptions (16), (17), and $\lim_{|x|\to\infty} \frac{F(x)}{x^2} = c/2$.

Note that $\lambda_1 = \overline{\lambda}_1 > 0$. In fact, λ_1 is the first eigenvalue of the clamped beam operator.

If $c \leq \lambda_1,$ then $\forall u \in D(J),$

$$I(u) = \frac{1}{2}\left(\int_0^1 u''^2 dt - c\int_0^1 u^2 e^{-\frac{1}{|u|}} dt\right),$$

and

$$0 = I'(u;u),$$

$$= \int_0^1 \left(u''^2 - \lambda_1 u^2\right)dt + \lambda_1 \int_0^1 u^2 dt - c\int_0^1 \left(u^2 + \frac{|u|}{2}\right)e^{-\frac{1}{|u|}} dt$$

$$\geq c\int_0^1 u^2 \left(1 - \left(1 + \frac{1}{2|u|}\right)e^{-\frac{1}{|u|}}\right)dt \geq 0,$$

where $I'(u;u)$ is the directional derivative of I at u in the direction u. It follows that problem (23)-(24) has only the null solution.

If $c > \lambda_1,$ then Theorem 1.2. guarantees the existence of at least one nonzero solution.

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ВЪРХУ ЕДНО ДИФЕРЕНЦИАЛНО ВКЛЮЧВАНЕ ОТ ЧЕТВЪРТИ РЕД С *p*-БИХАРМОНИЧЕН ОПЕРАТОР

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Резюме: *Разгледана е гранична задача от четвърти ред, включваща p-бихармоничен оператор, където $p>1$. Тя е мотивирана от различни проблеми в теорията на еластичността, например, модел на еластична греда, в случая когато моментът на огъване зависи нелинейно, като степенна функция, с показател $p-1$, от кривината. Представените резултати се отнасят до съществуване на решения, удовлетворяващи гранични условия от общ вид.*

Ключови думи: *би-p-Лапласиан, диференциално включване, обобщен градиент на Кларк, теория за критични точки на негладки функции.*

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