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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

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λ-CHARACTERISTIC SETS FOR A FINITE MEALY AUTOMATON

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Abstract: *A characteristic set for a word and a finite Mealy automaton have been defined. The output function λ has been considered instead of the state transition function δ.*

Keywords: *finite Mealy automaton, output function, characteristic set.*

INTRODUCTION

Automata theory is the study of mathematical objects called abstract machines or automata and the computational problems, that can be solved using automata. Automata play a huge role in the theory of computation, compilers design, formal verification. In biology and artificial intelligence research automata are used to describe neurological systems.

EXPOSITION

On the base of the unified approach to the dynamical systems and the finite automata [2, 3, 4, 5], considering the summary papers [2, 3, 4, 5, 7], in the paper the λ-characteristic sets for a word and a finite state Mealy automaton have been examined.

Let's recall some fundamental definitions, modifying some of them and introducing certain special notations and suitable general replacements, following [1], [5], [6].

Definition 1. A finite Mealy automaton is a 6-tuple $M = \{Q, V_1, V_2, \delta, \lambda, q_0\}$ where $Q = \{q_0, q_1, \dots, q_m\}$ is a finite set of control states; V_1 and V_2 are the starting and the exit alphabet respectively; $\delta : QxV_1 \to Q$ – the state transition function; $\lambda : QxV_1 \to V_2$ – the outset function; $q_0 \in Q$ is the initial state.

Put $\omega(\alpha, n)$ to be the string of the first *n* letters of the word α.

Definition 2. The set $C_\alpha(n) = \{\tau \in N \mid \delta(q, \omega(\alpha, n)) = \delta(q, \omega(\alpha, n+\tau))\}$ is called δ-characteristic set of a semi-dynamical system, defined by the finite automaton.

Definition 3. The set $P(\alpha, n) = \{\tau \in N, \lambda(q, \omega(h, n+\tau)) = \lambda(q, \omega(h, n))\}$ is called λ-characteristic set of a semi-dynamical system.

Example: The finite Mealy automaton $M = \{Q, V_1, V_2, \delta, \lambda, q_m\}$ is defined as follows: $Q = \{1,2,3,4\}$, $V_1 = \{a, b, c\}$, $V_2 = \{x, y, z\}$, $q_0 = 1$, $q_m = 4$ and its functions δ and λ are given in fig.1

M	1 (↓)	2	3	4
a	2,y	3,z	1,y	2,x
b	1,z	4,x	2,z	3,y
c	4,x	1,y	4,x	1,z

Fig.1

Experimental results over three words are done. The first one consists of two different words: *aabcac* and *abcac*, used a number of times. The second one consists only of the word *cabc* selected random times, and the symbol b between any two series of words *cabc*. The third one consists of symbols selected at random.

Experimental results 1:
As a result of the transformation, generated by the automaton *M* and the word
u = aabcacaabcacabcacaabcacabcacabcacaa…,
the following control states were passed over:

1→23212123242124121232121241212142123…
and the word *u* has transformed into the word
$v = yzzyyyyzzxxyyxzyyyzzyyyyxzyyyzxxyyz\ldots$

Therefore
$P_u(1) = \{3,4,5,6,11,12,15,16,17,20,21,22,23,26,27,28,32,33,...\}$;
$P_u(2) = \{1,6,7,13,17,18,24,28,33,...\}$;
$P_u(3) = \{5,6,12,16,17,23,27,32,...\}$;
$P_u(4) = \{1,2,3,8,9,12,13,14,17,18,19,20,23,24,25,29,30,...\}$;
$P_u(5) = \{1,2,7,8,11,12,13,16,17,18,19,22,23,24,28,29,...\}$;
$P_u(6) = \{1,6,7,10,11,12,15,16,17,18,21,22,23,27,28,...\}$;
$P_u(7) = \{5,6,9,10,11,14,15,16,17,20,21,22,26,27,...\}$;
$P_u(8) = \{1,7,11,12,18,22,27,...\}$;
$P_u(9) = \{6,10,11,17,21,26,...\}$;
$P_u(10) = \{1,4,15,21,22,...\}$;
$P_u(11) = \{3,14,20,21,...\}$;
$P_u(12) = \{1,4,5,6,9,10,11,12,15,16,17,21,22,\ldots\}$;
$P_u(13) = \{3,4,5,8,9,10,11,14,15,16,20,21,...\}$;
$P_u(14) = \{11,17,18,...\}$
…

Experimental results 2:
As a result of the transformation, generated by the automaton *M* and the word
u = cabcbcabccabcbcabccabccabcbcabccabccabccabcb…,
the following control states were passed over:

1→42411424142411424142414241142414241424142411…
and the word *u* has transformed into the word
$v = xxxzzxxxzxxxzzxxxzxxxzxxxzzxxxzxxxzxxxzxxxzz\ldots$

Therefore
$$P_u(1) = \begin{Bmatrix} 1,2,5,6,7,9,10,13,14,15,17,18,19,20,22,23,24,27,28,29,31,32, \\ 33,35,36,37,39,40,41,43,44,45,... \end{Bmatrix};$$
$$P_u(2) = \begin{Bmatrix} 1,4,5,6,8,1013,14,15,17,18,19,21,22,23,26,27,28,30,31,32,34, \\ 35,36,38,39,40,42,43,44,... \end{Bmatrix};$$
$$P_u(3) = \begin{Bmatrix} 3,4,5,7,8,9,12,13,14,16,17,18,20,21,22,25,26,27,29,30,31,33, \\ 34,35,37,38,39,41,42,43,... \end{Bmatrix};$$
$P_u(4) = \{1,5,9,10,14,18,22,23,27,31,35,39,40,...\}$;

$P_u(5) = \{4, 8, 9, 13, 17, 21, 22, 26, 30, 34, 38, 39, ...\}$;
$P_u(6) = \{1, 2, 4, 5, 6, 9, 10, 11, 13, 14, 15, 17, 18, 19, 22, 23, 24, 26, 27, 28, 30, 31, 32, 34, 35, 36, ...\}$;
$P_u(7) = \{1, 3, 4, 5, 8, 9, 10, 12, 13, 14, 16, 17, 18, 21, 22, 23, 25, 26, 27, 29, 30, 31, 32, 34, 35, ...\}$;
$P_u(8) = \{2, 3, 4, 7, 8, 9, 11, 12, 13, 15, 17, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 33, 34, ...\}$;
$P_u(9) = \{4, 5, 9, 13, 17, 18, 22, 26, 30, 34, 35, ...\}$;
$P_u(10) = \{1, 2, 5, 6, 7, 9, 10, 11, 13, 14, 15, 18, 19, 20, 22, 23, 24, 26, 27, 28, 30, 31, 32, ...\}$;
$P_u(11) = \{1, 4, 5, 6, 8, 9, 10, 12, 13, 14, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 30, 31, ...\}$;
$P_u(12) = \{3, 4, 5, 7, 8, 9, 11, 12, 13, 16, 17, 18, 20, 21, 22, 24, 25, 26, 28, 31, \ldots\}$;
$P_u(13) = \{1, 5, 9, 13, 14, 18, 22, 26, 30, 31, ...\}$;
$P_u(14) = \{4, 8, 12, 13, 17, 21, 25, 29, 30, ...\}$
…

Experimental results 3:
As a result of the transformation, generated by the automaton *M* and the word
u = *bacabccabbcaacbcbaaccbcacaabaacacba…*,
the following control states were passed over:

1→121241424342343431214342123231421 12…
and the word *u* has transformed into the word
$v = zyyyxzxxxyxxzxyxyyyyxyxxyyzzzyxxyzy\ldots$

Therefore
$P_u(1) = \{5, 12, 26, 27, 28, ...\}$;
$P_u(2) = \{1, 2, 8, 13, 15, 1617, 18, 20, 23, 24, 28, 31, 33, ...\}$;
$P_u(3) = \{1, 7, 12, 14, 15, 16, 17, 19, 22, 23, 25, 30, 32, ...\}$;
$P_u(4) = \{6, 11, 13, 14, 15, 16, 18, 21, 22, 24, 29, 31, ...\}$;
$P_u(5) = \{2, 3, 4, 6, 7, 9, 11, 16, 18, 19, 26, 27, ...\}$;
$P_u(6) = \{7, 21, 22, 23, 28, ...\}$;
$P_u(7) = \{1, 2, 4, 5, 7, 8, 13, 15, 16, 23, 24, ...\}$;
$P_u(8) = \{1, 3, 4, 6, 8, 13, 15, 16, 23, 24, ...\}$;
$P_u(9) = \{2, 3, 5, 7, 12, 14, 15, 22, 23, ...\}$;
$P_u(10) = \{5, 7, 8, 9, 10, 12, 15, 16, 20, 23, 25, ...\}$;
$P_u(11) = \{1, 3, 5, 6, 7, 8, 10, 13, 14, 18, 21, 23, ...\}$;
$P_u(12) = \{2, 4, 9, 11, 12, 19, 20, \ldots\}$;
$P_u(13) = \{14, 15, 16, 21, ...\}$;
$P_u(14) = \{2, 4, 9, 11, 12, 19, 20, ...\}$
…

CONCLUSION

In this paper λ- characteristic set for a word *u* and finite Mealy automata has been considered. The next aim is to consider optimization problem for a word and an automat over the output function with weighting. The numerical experiments have been performed using suitable software, developed by the author [5].

REFERENCES

[1] Manev K., Introduction to Discrete Mathematics, KLMN, Sofia, 2003.
[2] Saperstone S. H., Semidynamical Systems in Infinite Dimensional Spaces, Applied mathematical systems, 1981.
[3] Dochev D. Tr., On some properties of discrete semidynamical systems generated by finite automatons, UAAM, Sofia, V.21, 1985, 53-64.
[4] Sibirskiy K., Introduction to Topological Dynamics, AN MSSP, Chisinau, 1970.
[5] Dochev, D.Tr., M.K. Kirilov, Fundamental properties of discrete semidynamical systems defined by finite automatons. Научни Трудове на Русенския Университет, V.46, B 6, 2007, 23-30.
[6] Kirilov, M.K. Splitting of the sets of words with fixed length. Научни Трудове на Русенския Университет, V.47, B 5.1, 2008, 51-56.
[7] Hopcroft J., Aho A., and Ullman J., Data Structures and Algorithms, Addison-Wesley Series in Computer Science and Information Processing, 1983.

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λ-ХАРАКТЕРИСТИЧНИ МНОЖЕСТВА ЗА КРАЕН АВТОМАТ НА МИЛИ

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Резюме: *Разгледани са характеристични множества за краен автомат на Мили свързани с дадена дума. Построените характеристични множества са получени в зависимост от функцията на изходите* λ *вместо функцията на преходите* δ *.*

Ключови думи: *краен автомат на Мили, функция на изходите, характеристични множества.*

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