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"MATHEMATICS, INFORMATICS AND PHYSICS"

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"MATHEMATICS, INFORMATICS AND PHYSICS"

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ONE APPROACH FOR CONTINUOUS SIGNALS REPRESENTATION

Georgi Krastev, Tsvetozar Georgiev

Angel Kanchev University of Ruse

Abstract: *The purpose of this article is to describe how to provide signals geometrically as vectors. Discussed are the theoretical foundations of this approach and a concrete example in Matlab environment is presented.*

Keywords: *Signal Processing, Vectors, Vector spaces.*

INTRODUCTION

There is an extraordinary diversity in the classification of signals and methods of their presentation or description [1, 2, 3, 5]. The larger or smaller success in applying one method or another depends on and is determined mainly thereof how the contained in the signal information will be used. The mathematical apparatus of the functional analysis makes it possible to conduct full and sufficient universal research in this regard.

Nowadays, the scientific-technical bodies show increased interest in summarizing the many known ways and methods for analyzing the signals so that they can be treated by a single mathematical positions, from the one new common base. It is believed that implementing this approach different methods can be used for more general and effective application in solving various technical problems.

LAYOUT

1. Vector representation and description of continuous signals

It is not difficult to understand that the task of complex signals decomposition to simple, elementary signals is similar to the task of ordinary vector decomposition in three-dimensional space to its components of basic unit vectors, $\vec{i}$, $\vec{j}$, $\vec{k}$ - Fig.1.

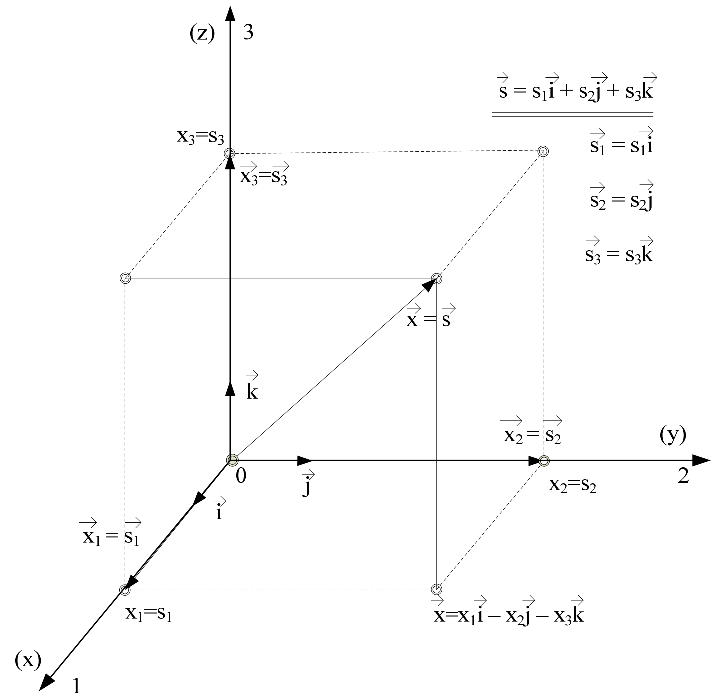

Fig.1. Vector in three dimensional space

Presentation of a signal - "vector" in the form of a vector sum

$\vec{s}(t) = \vec{i}s_1(t) + \vec{j}.s_2(t) + \vec{k}.s_3(t);$

$$\vec{s} = \vec{i}s_1 + \vec{j}.s_2 + \vec{k}.s_3 \quad (1)$$

is called the decomposition of the vector $\vec{s}$(t)=$\vec{s}$ to orthogonal unit vectors (orti) $\vec{i}$, $\vec{j}$, $\vec{k}$. Because the "vectors" are elements of decomposed "vector" it is perceived to call them components of the vector $\vec{s}$ at the basic $\vec{i}$, $\vec{j}$, $\vec{k}$. We see that the coefficients s_1, s_2 and s_3 represent himself as projections of the vector $\vec{s}$ on the axes $\vec{i}$, $\vec{j}$, $\vec{k}$, i.e. they are coordinates of the vector $\vec{s}$. Stated otherwise, this means that the vector of three dimensional space is completely determined by the sum of its coordinates.

$$\vec{s}(t) = \{s_1(t), s_2(t), s_3(t)\}; \; \vec{S} = \{s_1, s_2, s_3\}. \quad (2)$$

For the generalization of the vector of three dimensional space concept in the case of n-dimensional space a specific example can be used. In Fig. 2 is shown a graph of a time signal *s(t)*, which represents a continuous time function in the final interval (0,*T*). The idea of this signal can be obtained in the case where an aggregate of its values in the points t_1, t_2, t_3, ..., t_k, ..., t_n are given. These points are separated one another with a small interval

$$\Delta t = \frac{T}{n}, \quad (3)$$

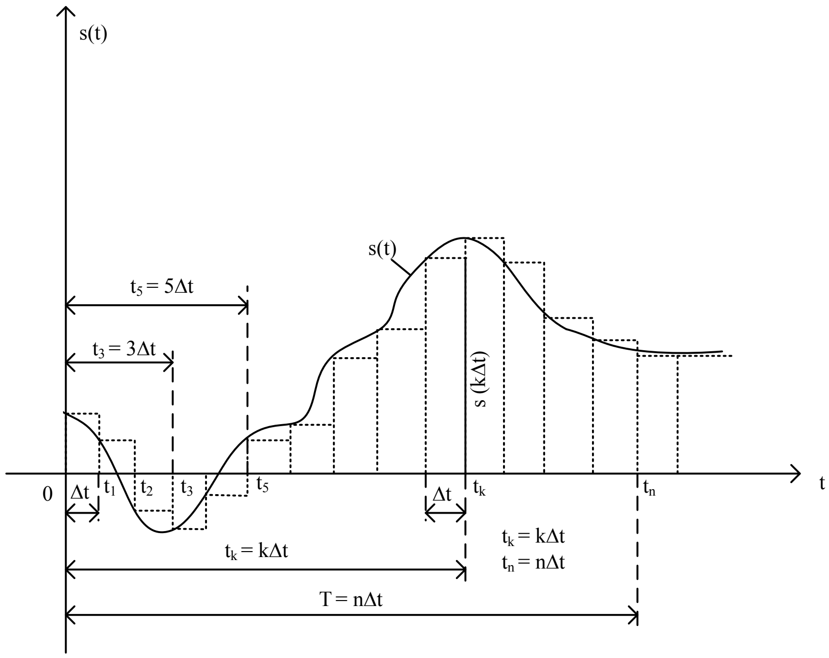

Fig.2. Vector representation of continuous signals

The values of the signal in these discrete points are respectively

$$s_1 = s(\Delta t), s_2 = s(2\Delta t), \ldots, s_k = s(k\Delta t), \ldots, s_n = s(n\Delta t) \,. \quad (4)$$

Assuming that this signal can be presented with a conditional "vector", it is not difficult to understand that for its full determination are needed

$$n = \frac{T}{\Delta t} \quad (5)$$

coordinates which represent samples of instant values of the signal at the moments t_1, t_2, t_3, ..., t_k, ..., t_n.

For major coordinate determinants which conditionally to play the role of a single vectors, can be considered rectangular pulses with a height of 1 - "one", which are shift against each other at an interval *t*, defined by (5). The whole system of these *n* coordinate determinants pulses with a height 1 can be represented "analytical" with the following singular function:

$$\psi_k(t) = \psi_k(t - k\Delta t) = \{1, t_k \le t \le t_{k+1}; 0, t_k > t > t_{k+1}\} \; k=1,2,\ldots,n. \qquad (6)$$

Based on all the above arrangements and acceptances, the signal *s(t)* can be approximately represented in the following component form (linear combination of orthogonal functions), similar to (1):

$$s(t) = s_1\psi_1(t) + s_2\psi_2(t) + \ldots + s_n\psi_n(t) = \sum_{k=1}^{n} s_k\psi_k(t). \qquad (7)$$

The expression (7) represents the signal *s(t)* in the form which is obtained as the result of signal decomposition in elementary coordinate signal-functions defined by the expression (6), with coefficients (4). By the analogy to (2) the signal *s(t)* can be represented in a more compact form - in the form of vector, given by its *n* coordinates:

$$\vec{s} = (s_1, s_2, \ldots, s_n) \qquad (8)$$

This type of recording the signal *s(t)* means that it corresponds to a vector (8) in *n*-dimensional space. Any random signal may be presented in the form of a sum analogous to (7).

2. Presentation of the signals as elements of linear spaces

Once accepted conditionally, that the signal in some preliminary arrangements may be regarded as a vector in *n*-dimensional space, by analogy of the vector in three-dimensional space the term "length" or "norm" of the signal may be defined, as follows:

$$\|\vec{s}\| = \sqrt{s_1^2 + s_2^2 + s_3^2 + \ldots + s_n^2}. \qquad (9)$$

If there are two signals *s(t)* and *ν(t)*, then the norm of their difference $\|s(t) - \nu(t)\|$ will be clearly defined by the expression

$$d\|s(t) - \nu(t)\| = \sqrt{\int [s(t) - \nu(t)]^2 dt} = d(s,\nu). \qquad (10)$$

It is not hard to see that (10) characterized actually mean square deviation of the signal *s(t)* from the signal *v(t)*. It is a geometric analogue of the distance between the vectors $\vec{s} = (s_1, s_2, \ldots, s_n)$ and $\vec{\nu} = (\nu_1, \nu_2, \ldots, \nu_n)$, i.e.

$$d(\vec{s}, \vec{\nu}) = \sqrt{\sum_{k=1}^{n} (s_k - \nu_k)^2} = \|\vec{s} - \vec{\nu}\|. \qquad (11)$$

Another important concept is the concept of "scalar product" of the signals *s(t)* and *ν(t)*, defined by the expression

$$\int s(t)\nu(t)dt = (s,\nu)\,. \qquad (12)$$

This definition of the scalar product of two signals *s(t)* and *ν(t)* is a generalization of the scalar produce of two vectors $\vec{s}$ and $\vec{\nu}$, i.e.

$$(\vec{s},\vec{\nu}) = \sum_{k=1}^{n} s_k \nu_k\,. \qquad (13)$$

It is necessary to pay attention to the following interesting and important fact. In the space of continuous signals *s(t)*, *ν(t)* with the scalar product of the type (12), the norm of the no interrupted signal is defined by the expression

$$\|s(t)\| = \sqrt{(s,s)} = \sqrt{\int s^2(t)dt} \qquad (14)$$

It is obtained from (12) when substitute *ν(t)* with *s(t)*.

Space in which the norm of the signal (function) is given with the scalar product (12) is called Hilbert. It is indicated by the symbol L^2.

In the *n*-dimensional space the norm of the vector is defined also with the scalar product

$$\|\vec{s}\| = \sqrt{(\vec{s},\vec{s})} = \sqrt{\sum_{k=1}^{n} s_k^2} \qquad (15)$$

Space in which the norm is given by the scalar product (13) is called Euclidean. It is indicated by the symbol R_n.

The method of determining the rate and the distance is called a metric of the space.

In terms of the above situations should be mentioned very briefly some other more fundamental correlations. They are given below.

From the vector calculus (respectively, from linear algebra) it is known that the absolute value of the scalar product of two vectors can not be greater than the product of their norms, which means that inequality is always true:

$$|(\vec{s},\vec{\nu})| \le \|\vec{s}\|.\|\vec{\nu}\|\,. \qquad (16)$$

Cosine of the angle between two vectors $\vec{s}$ and $\vec{\nu}$ is given by the expression

$$\cos\varphi_{s\nu} = \frac{(\vec{s},\vec{\nu})}{\|\vec{s}\|.\|\vec{\nu}\|}\,. \qquad (17)$$

The absolute value of (17) can never be greater than one.

Cosine of the angle between unit vectors $\vec{\psi}$ and $\vec{\eta}$ is obviously equal to their scalar product, i.e. $\cos\varphi_{\psi\eta} = (\vec{\psi},\vec{\eta})$. The cosine of the angle between normalization signals (functions) $[\|\psi(t)\| = \|\eta(t)\| = 1]$ - is true: $\cos\varphi_{\psi\eta} = (\vec{\psi},\vec{\eta}) = \int \psi(t)\eta(t)dt$.

On Fig. 3 are shown eight experimental curves after an experiment with a mobile research laboratory [4]. The minimum, average and maximum values are depicted on Fig. 4.

On Fig.5 are shown the results obtained in the Cartesian coordinate system, applying the discussed method.

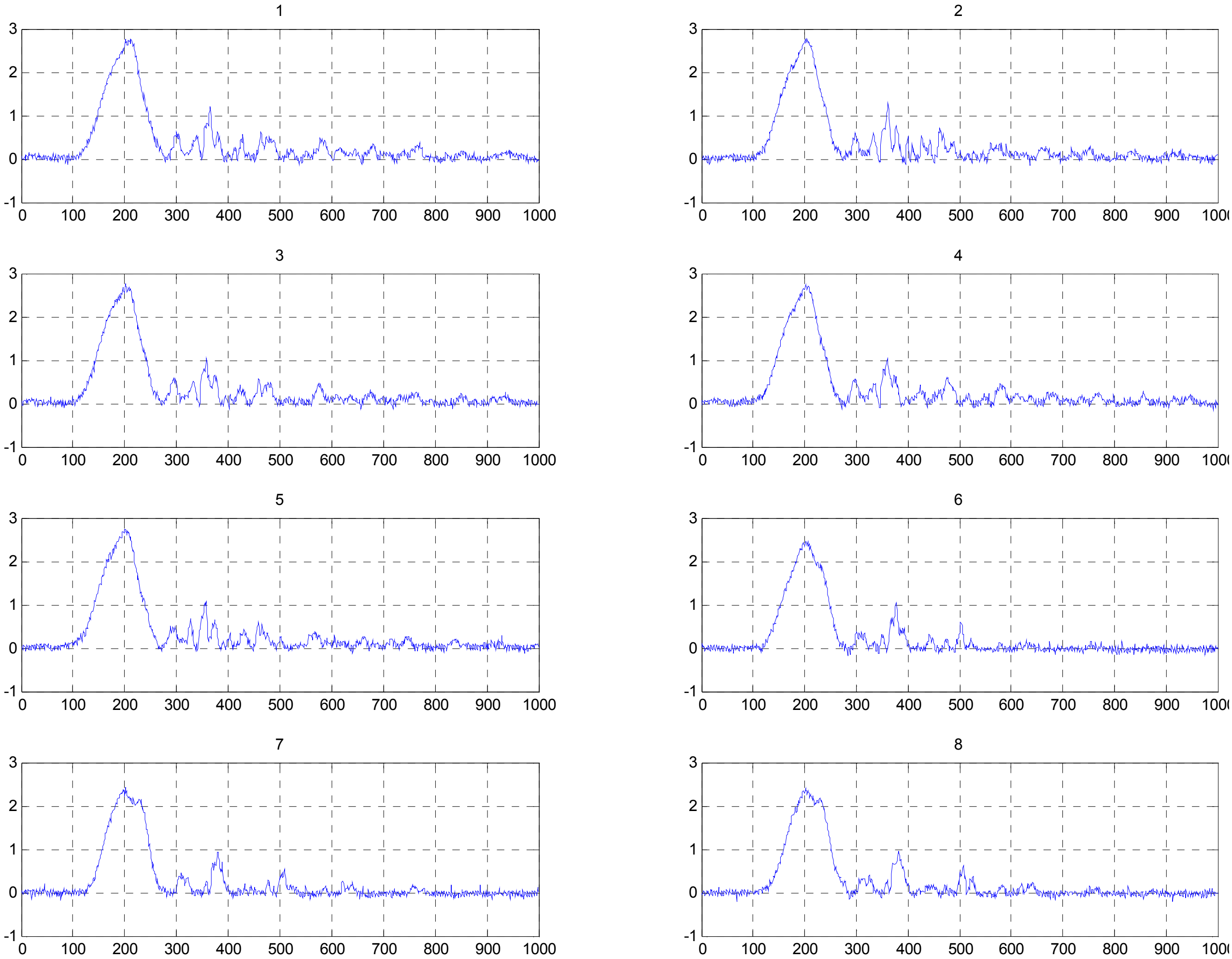

Fig.3. Examples of experimental realizations

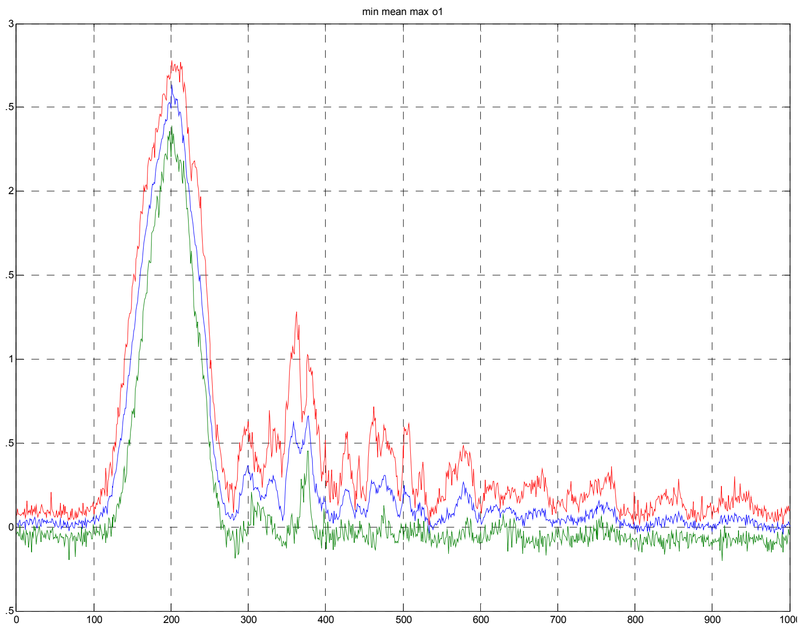

Fig.4. Minimum, average and maximum curves

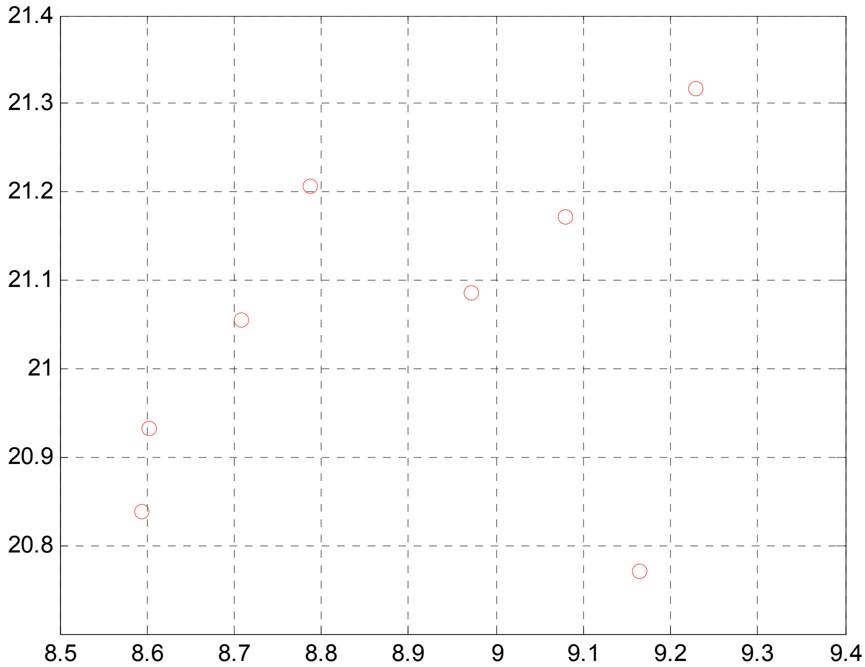

Fig.5. Obtained results

CONCLUSIONS AND FUTURE WORK

By nature for a man is easier, faster and better to understand the mathematical nature of things when they are illustrated geometrically. This feature is based on the idea of signals to be presented geometrically-dimensional and infinite-dimensional or final-vector space as vectors.

The discussed approach will be used for testing objects with continuous action.

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CONTACT ADDRESSES

Assoc. Prof. Georgi Krastev, PhD,
Angel Kanchev University of Ruse
Department of Computing,
Phone: +359 82 888 672,
E-mail: GKrastev@ecs.uni-ruse.bg

Assoc. Prof. Tsvetozar Georgiev, PhD,
Angel Kanchev University of Ruse
Department of Computing,
Phone: +359 82 888 827,
E-mail: TGeorgiev@ecs.uni-ruse.bg

ЕДИН ПОДХОД ЗА ПРЕДСТАВЯНЕ НА НЕПРЕКЪСНАТИ СИГНАЛИ

Георги Кръстев, Цветозар Георгиев

Русенски университет "Ангел Кънчев"

Резюме: *Целта на настоящата статия е сигналите да се представят геометрически като вектори. Разгледани са теоретичните основи на този подход и е представен конкретен пример в среда Matlab.*

Ключови думи: *Обработка на сигнали, Вектори, Векторни пространства*

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