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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

This is the jubilee 10-th volume of book 5 Mathematics, Informatics and Physics. The beginning was in Spring, 2001, when the colleagues of the former section Mathematics and Physics decided to start publishing our own book of the Proceedings of the Union of Scientists – Ruse. The first volume included 24 papers. Through the years there have been authors not only from the Angel Kanchev University of Ruse but as well as from universities of Gabrovo, Varna, Veliko Tarnovo and abroad – Russia, Greece and USA.

Since the 6-th volume the preparation and publishing of the papers began to be done in English.

The new 10-th volume of book 5 Mathematics, Informatics and Physics includes papers in Mathematics, Informatics and Information Technologies, Physics and materials from the Scientific Conference 'Information Technologies in Education' (ITE), held at the University of Ruse in November 2012 in the frame of Project 2012-FNSE-02.

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USING THE MAPLE SOFTWARE PRODUCT IN STUDYING FUNCTIONS

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***Abstract**: The paper discusses the use of the MAPLE software product in teaching the subject Higher Mathematics Part 1, and more specifically in working with functions. A sequence of operations is presented for collecting the necessary information about the behavior of the function and for drawing its graph. One fractional-rational and one exponential function are studied. By drawing the graph of each function, the results obtained at earlier stages are confirmed. The visual representation of how a given problem is solved helps students to more easily comprehend and better analyze the results.*

***Keywords**: Calculus, MAPLE, Study of a function.*

The use of software products in the training of university students is becoming increasingly widespread in the educational process. Higher Mathematics Part 1 is taught to students of the *Computer Systems and* Technologies and *Telecommunication Systems* courses by applying the MAPLE software program.

The present paper examines the use of the product to study a function of one variable. One fractional-rational and one exponential function are considered.

The study of these functions is accomplished by applying the following sequence of actions intended to gather the necessary information about the behavior of the function and to plot its graph:

1. Determining the domain of the function.
2. Checking if the function is even, odd, or periodic.
3. Studying the behavior of the function at the ends of the domain intervals. Finding the horizontal and vertical asymptotes.
4. Finding the oblique asymptotes.
5. Finding intersections with the *Ox* and *Oy* axes and the intervals in which the function keeps a constant sign.
6. Finding the intervals of monotonicity of the function and its extrema.
7. Finding the intervals of convexity and concavity of the function and its inflection points.
8. Summarizing the results in a table and plotting a graph of the function.

Function 1

The first function chosen to be investigated is $y = \frac{x^3}{2*(x+1)^2}$.

1. The function is entered in the working area of the MAPLE application.

```
> y1:=x**3/2/((x+1)**2);
```

$$y1 := \frac{1}{2}\frac{x^3}{(x+1)^2}$$

In determining its domain it is necessary to exclude those values of x, which lead to division by zero. Then the domain will be $x \in (-\infty,-1)\bigcup(-1,+\infty)$.

2. The function is neither even nor odd as $f(-x)=\dfrac{(-x)^3}{2*(-x+1)^2}\neq \pm f(x)$. Therefore, no symmetry can be expected to exist in the graph of the function with respect either to an axis, or to a point.

The studied function is not periodic, either, because when solving the equation $f(x+T)=f(x)$, i.e. $\dfrac{(x+T)^3}{2*(x+T+1)^2}=\dfrac{x^3}{2*(x+1)^2}$ no value for *T* is obtained that is a positive number not depending on *x.*

```
> solve((x+T)**3/2/(x+T+1)**2-x**3/2/(x+1)**2=0,T);
```

$$0, \frac{1}{2}\frac{(-2x^2-6x-3+\sqrt{-4x-3})x}{x^2+2x+1}, \frac{1}{2}\frac{(-2x^2-6x-3-\sqrt{-4x-3})x}{x^2+2x+1}$$

3. Now we find the corresponding limits:

```
> a1:=limit(y1, x=-infinity);
```

$$a1 := -\infty$$

```
> a2:=limit(y1, x=-1, left);
```

$$a2 := -\infty$$

```
> a3:=limit(y1, x=-1, right);
```

$$a3 := -\infty$$

```
> a4:=limit(y1, x=infinity);
```

$$a4 := \infty$$

The obtained values of the limits *a1* and *a4* give reason to conclude that there are no horizontal asymptotes on the function graph and limits *a2* and *a3* lead to the conclusion that the line $x=-1$ is a vertical asymptote on the graph of the function being examined.

4. Finding the other asympotes is done in the following way:

```
> k1:=limit(y1/x, x=-infinity);
```

$$k1 := \frac{1}{2}$$

```
> n1:=limit(y1-k1*x, x=-infinity);
```

$$n1 := -1$$

```
> Y1:=k1*x+n1;
```

$$Y1 := \frac{1}{2}x-1$$

```
> k2:=limit(y1/x, x=infinity);
```

$$k2 := \frac{1}{2}$$

```
> n2:=limit(y1-k1*x, x=infinity);
```

$$n2 := -1$$

```
> Y2:=k2*x+n2;
```

$$Y2 := \frac{1}{2}x-1$$

Since the results of the calculation of the limits *к1* and *к2* and those of *n1* and *n2* coincide, it follows that the equations of the lines $Y1 := \frac{1}{2}x - 1$ and *Y2*, which are oblique asymptotes on the graph of *y1* with $x \to \pm\infty$ will coincide, too.

5. The next two commands find the intersections of the function with the axes *Ox* and *Oy*:

```
> solve(y1=0,x);
```

$$0, 0, 0$$

```
> y11:=1/2*0**3/((0+1)**2);
```

$$y11 := 0$$

There is only one point of intersection and it is the origin of the coordinates, point *O(0,0)*.

6. Finding the intervals of monotonicity of the function and its extrema is connected with finding the first derivative which is accomplished with the next command. The second command is used to simplify the first derivative expression.

```
> y1prim:=diff(y1,x);
```

$$y1prim := \frac{3}{2}\frac{x^2}{(x+1)^2} - \frac{x^3}{(x+1)^3}$$

```
> simplify(%);
```

$$\frac{1}{2}\frac{x^2(x+3)}{(x+1)^3}$$

To find the points of a hypothesised extremum, the first derivative is equated to zero and points are searched for in the area where it does not exist. These must be points belonging to the function domain.

```
> solve(y1prim=0,x);
```

$$-3, 0, 0$$

The result obtained after completing this command shows that the points of a hypothesised extremum of the examined function are $x = -3$ and $x = 0$. The first derivative does not exist for $x = -1$, but this is not a point in the function domain. Therefore, it does not appear to be a hypothesised extremum point.

The sign of y' is studied with the help of the next two commands:

```
> solve(y1prim>0,x);
```

$$\text{RealRange}(-\infty, \text{Open}(-3)), \text{RealRange}(\text{Open}(-1), \text{Open}(0)), \text{RealRange}(\text{Open}(0), \infty)$$

```
> solve(y1prim<0,x);
```

$$\text{RealRange}(\text{Open}(-3), \text{Open}(-1))$$

```
> y1max:=1/2*(-3)**3/((-3+1)**2);
```

$$y1max := \frac{-27}{8}$$

The results show that *y1* is increasing for $x \in (-\infty,-3) \cup (-1,0) \cup (0,+\infty)$ and decreasing for $x \in (-3,-1)$. At point $x = -3$ *y1* reaches a maximum $y1_{max} = \frac{-27}{8}$. In the neighbourhood of *x=0* the sign of y' doesn't change. Therefore there is no extremum at this point of the function.

7. The expression $y'' = (y')'$ is derived and simplified with the following two commands:

```
> y1sec:=diff(y1prim,x);
```

$$y1sec := 3\frac{x}{(x+1)^2} - \frac{6x^2}{(x+1)^3} + \frac{3x^3}{(x+1)^4}$$

```
> simplify(%);
```

$$3\frac{x}{(x+1)^4}$$

The following three commands are used to look for the points of a hypothesized inflexion.

```
> solve(y1sec=0,x);
```

$$0$$

```
> solve(y1sec>0,x);
```

$$\text{RealRange}(\text{Open}(0), \infty)$$

```
> solve(y1sec<0,x);
```

$$\text{RealRange}(-\infty, \text{Open}(-1)), \text{RealRange}(\text{Open}(-1), \text{Open}(0))$$

The results show that $x = 0$ is a points of the hypothesized inflexion. The function is convex downward for $x \in (0,\infty)$ and convex upward for $x \in (-\infty,-1) \cup (-1,0)$. In the neighbourhood of $x = 0$ y'' changes its sign. Therefore, point *O(0,0)* is an inflexion point for the function under study.

8. The graph of the function *y1* and its oblique asymptote *Y1* are plotted (Fig.1) and confirmation is found for the results obtained above.

```
> plot({y1,Y1},x=-10..20,y=-20..30);
```

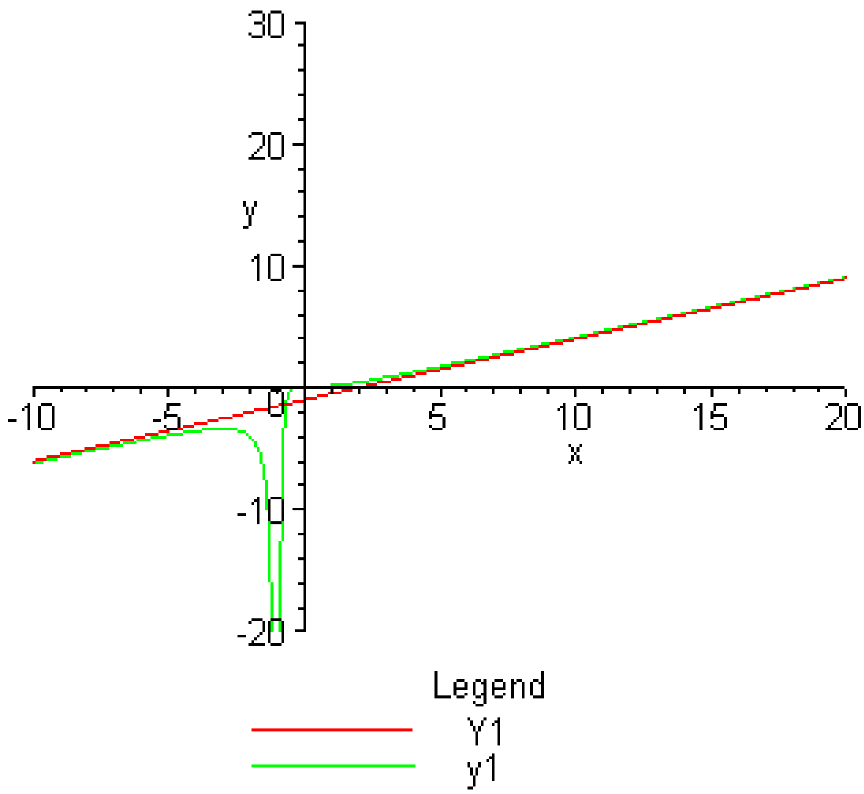

Figure 1. Function 1

Function 2

1. The function is entered in the working area of the MAPLE application.

> **y2:=(x+2)*exp(1/x);**

$$y2 := (x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}$$

In determining its domain it is necessary to exclude those values of x, which lead to division by zero. Then the domain will be $x \in (-\infty, 0) \bigcup (0, +\infty)$.

2. The function is neither even nor odd because $f(-x) = (-x+2) * e^{\frac{1}{(-x)}} \neq \pm f(x)$. Therefore, no symmetry can be expected to exist in the graph of the function with respect either to an axis, or to a point.

The studied function is not periodic, either, because when solving the equation $f(x+T) = f(x)$, i.e. $(x+T+2) * e^{\frac{1}{x+T}} = (x+2) * e^{\frac{1}{x}}$ no value for *T* is obtained that is a positive number not depending on *x.*

3. Now we find the corresponding limits:

> **b1:=limit(y2,x=-infinity);**

$$b1 := -\infty$$

> **b2:=limit(y2,x=0, left);**

$$b2 := 0$$

> **b3:=limit(y2, x=0, right);**

$$b3 := \infty$$

```
> b4:=limit(y2, x=-infinity);
```

$$b4 := -\infty$$

The obtained values of the limits *b1* and *b4* give reason to conclude that there are no horizontal asymptotes on the function graph and limits *b2* and *b3* lead to the conclusion that the line $x = 0$ is a vertical asymptote on the graph of the function being examined.

4. Finding the other asympotes is done in the following way:

```
> k3:=limit(y2/x,x=-infinity);
```

$$k3 := 1$$

```
> n3:=limit(y2-k3*x, x=-infinity);
```

$$n3 := 3$$

```
> Y3:=k3*x+n3;
```

$$Y3 := x + 3$$

```
> k4:=limit(y2/x,x=infinity);
```

$$k4 := 1$$

```
> n4:=limit(y2-k3*x, x=infinity);
```

$$n4 := 3$$

```
> Y4:=k4*x+n4;
```

$$Y4 := x + 3$$

Since the results of the calculation of the limits *к3* and *к4* and those of *n3* and *n4* coincide, it follows that the equations of the lines $Y3 := x + 3$ and *Y4*, which are oblique asymptotes on the graph of *y2* with $x \to \pm\infty$ will coincide, too.

5. With the next command, the intersection is found of the function with the *Ox* axes $x = -2$.

The function under study has no point of intersection with the *Oy* axis since point $x = 0$ does not belong to the function domain.

```
> solve(y2=0,x);
```

$$-2$$

6. Finding the intervals of monotonicity of the function and its extrema is connected with finding the first derivative which is accomplished with the next command. The second command is used to simplify the first derivative expression.

```
> y2prim:=diff(y2,x);
```

$$y2prim := e^{\left(\frac{1}{x}\right)} - \frac{(x+2)\, e^{\left(\frac{1}{x}\right)}}{x^2}$$

```
> simplify(%);
```

$$\frac{e^{\left(\frac{1}{x}\right)} (x^2 - x - 2)}{x^2}$$

To find the points of a hypothesised extremum, the first derivative is equated to zero and points are searched for in the area where it does not exist. These must be points belonging to the function domain.

> **solve(y2prim=0,x);**

2, -1

The result obtained after completing this command shows that the points of a hypothesised extremum of the examined function are $x=-1$ and $x=2$. The first derivative does not exist for $x=0$, but this is not a point in the function domain.

The sign of y' is studied with the help of the next two commands.

> **solve(y2prim>0,x);**

$$\text{RealRange}(-\infty, \text{Open}(-1)), \text{RealRange}(\text{Open}(2), \infty)$$

> **solve(y2prim<0,x);**

$$\text{RealRange}(\text{Open}(-1), \text{Open}(0)), \text{RealRange}(\text{Open}(0), \text{Open}(2))$$

> **y2max:=(-1+2)*exp(1/(-1));**

$$y2max := \mathbf{e}^{(-1)}$$

> **y2min:=(2+2)*exp(1/2);**

$$y2min := 4\,\mathbf{e}^{(1/2)}$$

The results show that *y2* is increasing for $x \in (-\infty,-1) \cup (2,\infty)$ and decreasing for $x \in (-1,0) \cup (0,2)$. At point $x=-1$ *y2* reaches a maximum $y2_{\max} = e^{(-1)}$. The minimum is at point $x=2$ and $y2_{\min} = 4 * e^{\frac{1}{2}}$.

7. The expression $y'' = (y')'$ is derived and simplified with the following two commands:

> **y2sec:=diff(y2prim,x);**

$$y2sec := -2\,\frac{\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^2} + \frac{2\,(x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^3} + \frac{(x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^4}$$

> **simplify(%);**

$$\frac{\mathbf{e}^{\left(\frac{1}{x}\right)}\,(5\,x+2)}{x^4}$$

The following three commands are used to look for the points of a hypothesized inflexion.

> **solve(y2sec=0,x);**

$$\frac{-2}{5}$$

> **solve(y2sec>0,x);**

$$\mathrm{RealRange}\left(\mathrm{Open}\left(\frac{-2}{5}\right), \mathrm{Open}(0)\right), \mathrm{RealRange}(\mathrm{Open}(0), \infty)$$

> **solve(y2sec<0,x);**

$$\mathrm{RealRange}\left(-\infty, \mathrm{Open}\left(\frac{-2}{5}\right)\right)$$

The results show that $x=-\dfrac{2}{5}$ is a points of the hypothesized inflexion. The function is convex downward for $x \in \left(-\dfrac{2}{5},0\right) \cup (0,\infty)$ and convex upward for $x \in \left(-\infty,-\dfrac{2}{5}\right)$. In the neighbourhood of $x=-\dfrac{2}{5}$ y'' changes its sign. Therefore, point $A\left(-\dfrac{2}{5}, \dfrac{8}{5} * e^{-\frac{5}{2}}\right)$ is an inflexion point for the function under study.

8. The graph of the function *y2* and its oblique asymptote *Y3* are plotted (Fig.2) and confirmation is found for the results obtained above.

> **plot({y2,Y3},x=-20..20, y=-10..20);**

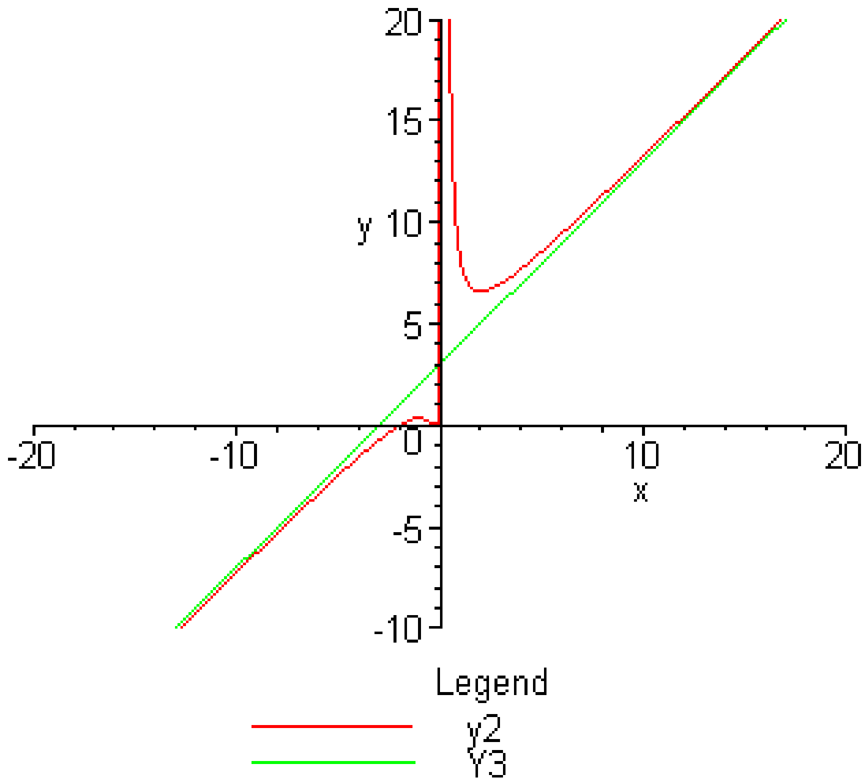

Figure 2. Function 2

Through the use of the MAPLE software product, visualization is achieved of how the given problems are solved which helps the students to comprehend and analyze the results better.

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ИЗПОЛЗВАНЕ НА ПРОГРАМНИЯ ПРОДУКТ MAPLE ПРИ ИЗСЛЕДВАНЕ НА ФУНКЦИЯ

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***Резюме**: В настоящата работа е разгледано използването на програмния продукт MAPLE в обучението по Висша математика 1 в частност при изследване на функция. Показана е последователността от действия за събирането на необходимата информация за поведението на функцията и изчертаване на нейната графика. Направено е изследване на една дробно-рационална и една експоненциална функция. При изчертаването на графиката на функцията е установено потвърждение на получените в предишните етапи резултати. Постигнато е онагледяване на решението на разглежданите задачи, което помага на студентите да осмислят и анализират по-добре резултатите.*

***Ключови думи**: Висша математика, MAPLE, Изследване на функция.*

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