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Book 5

Mathematics, Informatics and Physics

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

This is the jubilee 10-th volume of book 5 Mathematics, Informatics and Physics. The beginning was in Spring, 2001, when the colleagues of the former section Mathematics and Physics decided to start publishing our own book of the Proceedings of the Union of Scientists – Ruse. The first volume included 24 papers. Through the years there have been authors not only from the Angel Kanchev University of Ruse but as well as from universities of Gabrovo, Varna, Veliko Tarnovo and abroad – Russia, Greece and USA.

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PLANE IN SPACE WITH MATHEMATICAL SOFTWARE

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Abstract: *The article presents some* mathematical problems suitable *for training students on the topic "Plane in Space" included in the discipline Analytical Geometry. Analytical solutions are visualized by using mathematical software Maple 13. This helps students in better understanding the material taught in spatial patterns. Thus, the rate of success increases significantly.*
Keywords: *plane in the space, teaching and learning mathematics, mathematical software, Maple 13.*

INTRODUCTION

The rapid development of information technology and the large number of mathematical software (*Maple, Mathematica, MATLAB, GeoGebra* etc.), that are readily available to students, helps the interesting teaching of mathematics. Using mathematical software is activated and increases the cognitive activity of students. It is very important for the students to have a thorough mathematical knowledge and to be able to verify the results given by the computers [1,7,8,10].

The topic discussed is included in the curriculum of the subject **Linear Algebra and Geometry** for students studying Computer Science, Mathematics & Informatics and Information Technologies in Business. The aim of the study is to show the author's opinion how geometry to be learned easier and to become more interesting by using Maple 13 (Fig. 1).

PRESENTATION

1. Necessary theoretical knowledge ([4, p. 170])

General equation of a plane:

$$\alpha \to Ax+By+Cz+D=0,\ A^2+B^2+C^2 \neq 0, \quad (1)$$

$\overrightarrow{n_\alpha}(A,B,C)$ – the normal vector to the plane.

Equation of a plane through point $A(x_0,y_0,z_0)\in\alpha$ and a normal vector:

$$\alpha \to A(x-x_0)+B(y-y_0)+C(z-z_0)=0 \quad (2)$$

Equation of a plane through three points, $A_1(x_1,y_1,z_1)\in\alpha$, $A_2(x_2,y_2,z_2)\in\alpha$ and $A_3(x_3,y_3,z_3)\in\alpha$:

$$\alpha \to \begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0. \quad (3)$$

Equation of a plane through a point $A(x_0,y_0,z_0)\in\alpha$ and two non-collinear vectors $\overrightarrow{m_1}(a_1,b_1,c_1),\overrightarrow{m_2}(a_2,b_2,c_2)$:

$$\alpha\to\begin{vmatrix} x-x_0 & y-y_0 & z-z_0\\ a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\end{vmatrix}=0. \qquad (4)$$

Oriented distance from a point to a plane:

$$\delta(M,\alpha)=\frac{Ax_0+By_0+Cz_0+D}{-\operatorname{sgn} D\sqrt{A^2+B^2+C^2}},M(x_0,y_0,z_0). \qquad (5)$$

Distance from a point to a plane:

$$d=|\delta| \qquad (6)$$

Table1: Commands in Maple 13 ([6], [9])

Command in MAPLE	Action
plots[implicitplot3d]	calculate the area of a surface in 3D / three dimensional space
matrix(A)	input matrix
det(A)	calculate the determinant of a matrix
eval	most often used to calculate the expressions
solve	solving equations
display	visualization
sqrt(a)	$\sqrt{a}$
abs(a)	absolute value
with(linalg)	Linear algebra package
with(plots)	Plots package

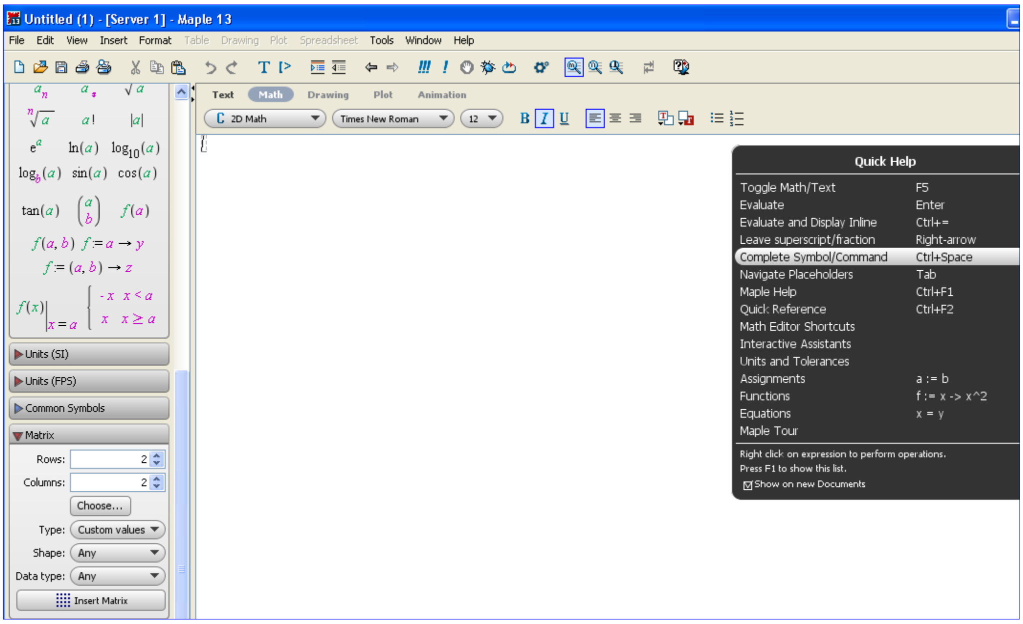

***Figure 1:** Maple workspace*

2. Solved problems

Problem1: Find the general equation of a plane, passing through points $M_1(3,0,4)$, $M_2(5,2,6)$ and $M_3(2,3,-3)$ ([4, p. 172]).

Solution:

We are working with Maple 13 and use formula (3)

```
> with(linalg);

> A:=Matrix([[x,y,z,1],[3,0,4,1],[5,2,6,1],[2,3,-3,1]]);
```

$$A := \begin{bmatrix} x & y & z & 1 \\ 3 & 0 & 4 & 1 \\ 5 & 2 & 6 & 1 \\ 2 & 3 & -3 & 1 \end{bmatrix}$$

```
> det(A)=0;
```

$$-20\,x + 12\,y + 8\,z + 28 = 0$$

```
> det(A)/(-4)=0;
```

$$5\,x - 3\,y - 2\,z - 7 = 0$$

So, we obtain the equation of a plane $\alpha \to 5x\text{-}3y - 2z - 7 = 0$. If we were solving the problem without mathematical software, finding the value of the determinant would have been more difficult.

Problem 2: Find the distance from p. $M(3,-1,0)$ to the plane $\alpha \to 3x\text{-}4y+12z=0$ ([4, p. 174]).

Solution:

We are working with Maple 13. After that we use formula (5) to find the oriented distance from a point to a plane:

```
> with(linalg);

> x:=3;y:=-1;z:=0;
```

$$x := 3$$

$$y := -1$$

$$z := 0$$

```
> A:=3;B:=-4;C:=12;G:=-26;
```

$$A := 3$$

$$B := -4$$

$$C := 12$$

$$G := -26$$

```
> d:=abs(A*x+B*y+C*z+G)/sqrt(A^2+B^2+C^2);
```

$$d := 1$$

Problem 3. It's given the equation of a plane $\alpha \to 23x - 25y + 11z + 13 = 0$. Find the intercept of X, Y, Z with axes and calculate the volume of the parallelepiped constructed using vectors $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ [3].

***Solution**:*

To find the intersection with the axis Ox, we have to make the substitutions in the equation the plane $y = 0, z = 0$ and hence to find x.

```
> solve({eval(23*x-25*y+11*z+13,[y=0,z=0])},{x});
```

$$\left\{x = -\frac{13}{23}\right\}$$

```
> solve({eval(23*x-25*y+11*z+13,[x=0,z=0])},{y});
```

$$\left\{y = \frac{13}{25}\right\}$$

```
> solve({eval(23*x-25*y+11*z+13,[x=0,y=0])},{z});
```

$$\left\{z = -\frac{13}{11}\right\}$$

Consequently $X\left(-\frac{13}{23},0,0\right), Y\left(0,\frac{13}{25},0\right), Z\left(0,0,-\frac{13}{11}\right)$.

To find the volume of the parallelepiped, using the application of the mixed product of three vectors, we have $\overrightarrow{OX}\left(-\frac{13}{25},0,0\right) \overrightarrow{OY}\left(0,\frac{13}{25},0\right), \overrightarrow{OZ}\left(0,0,-\frac{13}{11}\right)$.

```
> M:=matrix([[-13/23,0,0 ],[ 0,13/25,0 ],[ 0,0,-13/11 ]]);
```

$$V := \begin{bmatrix} -\frac{13}{23} & 0 & 0 \\ 0 & \frac{13}{25} & 0 \\ 0 & 0 & -\frac{13}{11} \end{bmatrix}$$

```
> V:=abs((-13/23)*(13/25)*(-13/11));
```

$$V := \frac{2197}{6325}$$

Problem 4: It's given the sphere $x^2 + y^2 + z^2 = 9$ and the plane $x + 2y + z = 1$. Determine what their intersection using Maple 13 [2].

Solution:

```
> with(plots);

> p1:=implicitplot3d (x^2+y^2+z^2=9,x=-3..3,y=-3..3, z= 3..3,color=pink):

> p2:=implicitplot3d(x+2*y+z=1,x=-3..3,y=-3..3,z=-3..3,color=brown):

> display([p1,p2]);
```

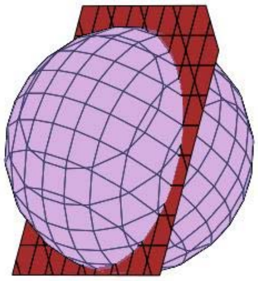

Figure 2: *The intersection of a sphere and a plane*

It can be seen that the plane intersects the sphere, and thus their intersection is a circle (Fig. 2).

Problem 5. Determine the relative positions of the sphere $x^2 + y^2 + z^2 = 1$ and the plane $x + 2y + z + \sqrt{6} = 0$. ([5])

Solution:

After visualization, we can see that the plane does not intersect the sphere (Fig. 3).

```
> with(plots);

> p1:=implicitplot3d(x^2+y^2+z^2=1,x=-1..1,y=-1..1,z=-1..1):

> p2:=implicitplot3d(x+2*y+z+sqrt(6)=0,x=-5..5, y=-5..5,z=5..5, color=red):

> display([p1,p2]);
```

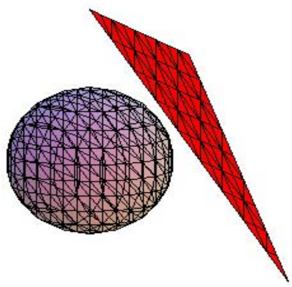

Figure 3: *The intersection of sphere and plane*

CONCLUSION

The usage of mathematical software makes learning mathematics easier and interesting. Also it creates conditions for visualization; motivates and helps students for successful completion of the course **Linear Algebra and Geometry**. The theme is attractive to students, and would be useful for the work of each teacher in mathematics or anyone with an interest in mathematics. In addition, one can develop other themes used in the learning process. Through the use of mathematical software the goals and objectives set in the curriculum are properly achieved.

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„РАВНИНА В ПРОСТРАНСТВОТО" С МАТЕМАТИЧЕСКИ СОФТУЕР

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Резюме: *Статията представя някои подходящи задачи за обучение на студентите по темата "Равнина в пространството", включени в дисциплината Аналитична геометрия. Аналитичните решения са визуализирани с помощта на математически софтуер Maple13. Този метод помага на студентите за по-добро разбиране на преподавания материал в пространствени модели. По този начин успеваемостта на студентите се увеличава значително.*

Ключови думи: *равнина в пространството, преподаване и обучение по математика, математически софтуер, Maple13.*

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