

# PROCEEDINGS

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of the Union of Scientists - Ruse

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Book 5  
**Mathematics, Informatics and  
Physics**

Volume 10, 2013



RUSE

**The Ruse Branch of the Union of Scientists in Bulgaria**

was founded in 1956. Its first Chairman was Prof. Stoyan Petrov. He was followed by Prof. Trifon Georgiev, Prof. Kolyo Vasilev, Prof. Georgi Popov, Prof. Mityo Kanev, Assoc. Prof. Boris Borisov, Prof. Emil Marinov, Prof. Hristo Beloev. The individual members number nearly 300 recognized scientists from Ruse, organized in 13 scientific sections. There are several collective members too – organizations and companies from Ruse, known for their success in the field of science and higher education, or their applied research activities. The activities of the Union of Scientists – Ruse are numerous: scientific, educational and other humanitarian events directly related to hot issues in the development of Ruse region, including its infrastructure, environment, history and future development; commitment to the development of the scientific organizations in Ruse, the professional development and growth of the scientists and the protection of their individual rights.

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**BOOK 5**

**"MATHEMATICS,  
INFORMATICS AND  
PHYSICS"**

**VOLUME 10**

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This is the jubilee 10-th volume of book 5 Mathematics, Informatics and Physics. The beginning was in Spring, 2001, when the colleagues of the former section Mathematics and Physics decided to start publishing our own book of the Proceedings of the Union of Scientists – Ruse. The first volume included 24 papers. Through the years there have been authors not only from the Angel Kanchev University of Ruse but as well as from universities of Gabrovo, Varna, Veliko Tarnovo and abroad – Russia, Greece and USA.

Since the 6-th volume the preparation and publishing of the papers began to be done in English.

The new 10-th volume of book 5 Mathematics, Informatics and Physics includes papers in Mathematics, Informatics and Information Technologies, Physics and materials from the Scientific Conference ‘Information Technologies in Education’ (ITE), held at the University of Ruse in November 2012 in the frame of Project 2012-FNSE-02.

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## APPLICATION OF MATLAB SOFTWARE FOR DIGITAL IMAGE PROCESSING

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**Abstract:** *In this article an application of software product MATLAB in digital image processing using recursive discrete Kalman filter is presented. The potentiality of the product is analyzed examining the influence of the number of iterations in different disperse of the adaptive Gaussian noise upon the quality of the filtered image.*

**Key words:** *digital image, discrete recursive Kalman filter, Gaussian noise, iterations, MATLAB.*

### INTRODUCTION

The system MATLAB („MATrix LABoratory“) is a software environment for numerical analysis and independent fourth generation programming language. Created by the multinational software corporation for technical applications "The MathWorks", MATLAB allows operations using matrices, functions' drawing, data presenting, algorithm program implementation, development of human-machine interfaces and interfaces with different software products written in different programming languages [6]. One of the most important advantages of the system MATLAB is its ability to expand in purpose to be used as a tool for resolving new scientific tasks. This can be realized by creating a sequence of packages which can serve for expanding the common mathematical abilities of the system (symbolic calculations, optimization, statistics) as well as realization of the directions in computer mathematics like mathematical modeling, building an architecture of neural networks and fuzzy conclusions. MATLAB is used by more than 1 million academic and business users [6]. One possibility for applying the program environment MATLAB in digital image processing using discrete Kalman filter is presented.

### DIGITAL IMAGE PROCESSING USING DISCRETE KALMAN FILTER

No matter the ways of obtaining, a given image is always accompanied with existence of noise and embarrassment of different kind. A main problem in its processing is removing the noise when keeping the important data for further details [7].

In most of the cases the most important goal of the image processing is to obtain a contour. This requires a sequence of procedures (filtration of Gaussian and impulse noise, segmentation aiming to receive black and white image, extraction of the contour line) which should be picked correctly of certain type and in determined sequence. An application of discrete Kalman filter in filtration of Gaussian at radiolocational images of dynamic objects (aircrafts) will be the case under review.

The Kalman filter is consistent recursive algorithm, using a model of dynamic system for receiving marks, which can be edited after every new measurement in temporal sequence [2,9]. This algorithm finds appliance in the process of managing complex dynamic processes, in which it is important to know the stage in each moment of time. The Kalman filter has a possibility to adjust in case of not precisely chosen parameters of the mathematical models. The advantage of the approach is that it does not require huge computational resources and can evaluate the condition vector in real time [5].

▪ **Mathematical model of Discrete Kalman filter**

The equations of the condition and observation are given:

$$x_k = Ax_{k-1} + \omega_{k-1} \quad (1)$$

$$z_k = Hx_k + v_k, \quad (2)$$

where  $x_k \in R^n$  is the state variable in  $k$ -th moment, and  $z_k \in R^m$  is the observation variable. Here  $A$  is  $n \times n$  matrix, connecting the variables of the conditions of the current step  $k$  and the previous step  $k-1$ , and  $H$  is  $m \times n$ -matrix connecting the observation and condition in the  $k$ -th step. The vectors  $w_k$  and  $v_k$  are normally distributed random variables with zero mathematical expectation and covariance matrixes:

$$Q_k = E[w_k w_k^T], \quad R_k = E[v_k v_k^T], \quad (3)$$

where  $E$  is the mathematical expectation [8].

With  $\hat{x}_k^- \in R^n$  is marked the priori mark of the condition variable in the  $k$ -th step, and to  $\hat{x}_k \in R^n$  corresponds a posteriori evaluation in given observation  $z_k$ . Priori and posteriori errors are defined in the following way:

$$e_k^- \equiv x_k - \hat{x}_k^-, e_k \equiv x_k - \hat{x}_k \quad (4)$$

Then the priori covariance matrix of the error will look like:

$$P_k^- = E[e_k^- e_k^{-T}], \quad (5)$$

and the posterior covariance matrix of the error will be respectively:

$$P_k = E[e_k e_k^T]. \quad (6)$$

*Posterior condition* mark is a nonlinear combination of the priori assessment  $\hat{x}_k^-$  and the difference between provided measurement  $H\hat{x}_k^-$  and the real measurement  $z_k$ , multiplied with  $n \times m$  weight matrix  $K_k$  [4].

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H\hat{x}_k^-). \quad (7)$$

The Kalman matrix  $K_k$  in (7) has the look:

$$K_k = P_k^- H^T (HP_k^- H^T + R_k)^{-1}. \quad (8)$$

▪ **Software realization of the model in MATLAB environment**

The function which realizes the Kalman algorithm is the function `s=kalmanf(s)` [9]. Typical for this function is that it is executed for every dot of the image. It uses the primary data from function `start_kal` (Nt). In the beginning a validation check is executed to ensure the main parameters and corresponding assignments. Based upon the choice made concerning the number of filtering, Nt times filtering are executed. This is set as a parameter in function `start_kal` and is chosen from a menu. Appliance of Kalman filter for every row of the image starts, as it starts from the second column. The typical special feature here is that the input data initialized in the beginning are edited for each pixel from the image based on the previous filtered pixel and if the previous pixel is the first it is based on it. The filtration algorithm includes procedure for brightness segmentation (Otsu method) [3] and contour receiving (Roberts operator).

```

function varargout = kfig(varargin)
% Visualization Function
gui_Singleton = 1;
gui_State = struct('gui_Name', mfilename,
if nargin & isstr(varargin{1})
    gui_State.gui_Callback = str2func(varargin{1});
end
if nargin
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
function kfig_OpeningFcn(hObject, eventdata, handles, varargin)
% Choose default command line output for kfig
handles.output = hObject;
% Update handles structure
guidata(hObject, handles);
% Executing actions when visualizing the object
if strcmp(get(hObject,'Visible'),'off')
    I = imread('I1gaus.bmp','bmp'); % Load file (with matrix size 75x100) in I
    mshow(I);
end

% --- Outputs from this function are returned to the command line
function varargout = kfig_OutputFcn(hObject, eventdata, handles)
varargout{1} = handles.output;

% Start (pushbutton1) button Kalman filtering
function pushbutton1_Callback(hObject, eventdata, handles)

    global I; % We create I global matrix
    bw = get(handles.slider1,'Value') % Load from Slider in bw (value for black/white)
    popup_sel_index = get(handles.popupmenu1, 'Value'); % From the drop down menu
(n) times repetitions in k filter.
switch popup_sel_index
case 1 % 1 път
    start_kal(1); % Function Start Kalman
    fig_bw(bw); % Function towards black/white
case 2 % 3 times
    start_kal(3);
    fig_bw(bw);
case 3 % 5 times
    start_kal(5);
    fig_bw(bw);
case 4
    start_kal(10);
    fig_bw(bw);
case 5
    start_kal(20);
    fig_bw(bw);

```

```

case 6
    start_kal(30);
    fig_bw(bw);
end
function start_kal(Nt)
figure(1);
global I
I=I
clear s
s.x = 1;
s.A = 1;
s.Q = 2^2;           % Measurement
s.H = 1;
s.R = 2^2;
s.B = 0;
s.u = 0;
s.x = nan;
s.P = nan;

for t2=1:Nt
for t1=1:346
for t=2:572
    tr(end+1)=I(t1,t);
    s(end).z=tr(end) ;
    s(end+1)=kalmanf(s(end));   % Filtration
    I(m(t1,t)=s(end).x;
end
    clf;
    J = imresize(I,m,4);
    imshow(J);
end
I=I;
end
global I;

function fig_bw(bw)
figure(2);
global I;
bw_210 = I > bw;
bw_210 = imresize(bw_210,4);
imshow(bw_210);title(['Prag B/W # ',int2str(bw)]);
global bw_210;
end
return

```

**AN APPROACH FOR DETERMINING THE OPTIMAL NUMBER OF ITERATIONS FOR DIFFERENT DISPERSE OF GAUSSIAN NOISE IN FILTRATION WITH DISCRETE KALMAN FILTER**

The optimal number of iterations in different disperse of Gaussian noise is determined by basing on quantity characteristics of two geometrical characteristics of the object (aircraft) – fuselage axis length and wingspan of the aircraft. The fuselage axis length and the wingspan are measured in number of pixels, respectively between the “nose” and “tail” of the aircraft and the end points of the two wings. The optimal number of iterations is determined by using the following criteria: the values of the chosen geometrical characteristics, received after image filtering to be most similar to the corresponding etalon aircraft before filtration. The change of the geometrical characteristics is examined in different number of iterations with discrete Kalman filter. Digital images from 10 types of aircrafts, excepted as etalon, have been used. They have been chosen by random pick of data bases [1]. They are: F16, An 124, Mc Donnell, B 52, Buccaneer, F117, Jaguar, Mig 29, Miraj 2000, Su34. A random image is chosen out of the choice. It is noised with adaptive Gaussian noise with normal distribution at zero mathematical expectation and disperse relatively 0,03; 0,05; 0,07. The results of the filtration are examined after 5, 10, 20 and 30 iterations. The results after filtration of etalon object are given respectively in Table 1, Table 2 and Table 3.

Table 1. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,03

Geometrical characteristics	Number of iterations			
	5	10	20	30
Fuselage length	60	62	63	58
Wingspan	39	40	41	37

Table. 2. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,05

Geometrical characteristics.	Number of iterations			
	5	10	20	30
Fuselage length	55	57	60	56
Wingspan	35	36	38	33

Table.3. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,07

Geometrical characteristics	Number of iterations			
	5	10	20	30
Fuselage length	40	42	42	38
Wingspan	28	29	30	27

The results are given graphically on *Fig. 1*. They show that the filter gives best results at 20 iterations and Gaussian noise disperse 0,03

For etalon object, presented in black color, the wingspan is 69 pixels and the fuselage axis length – 45 pixels.



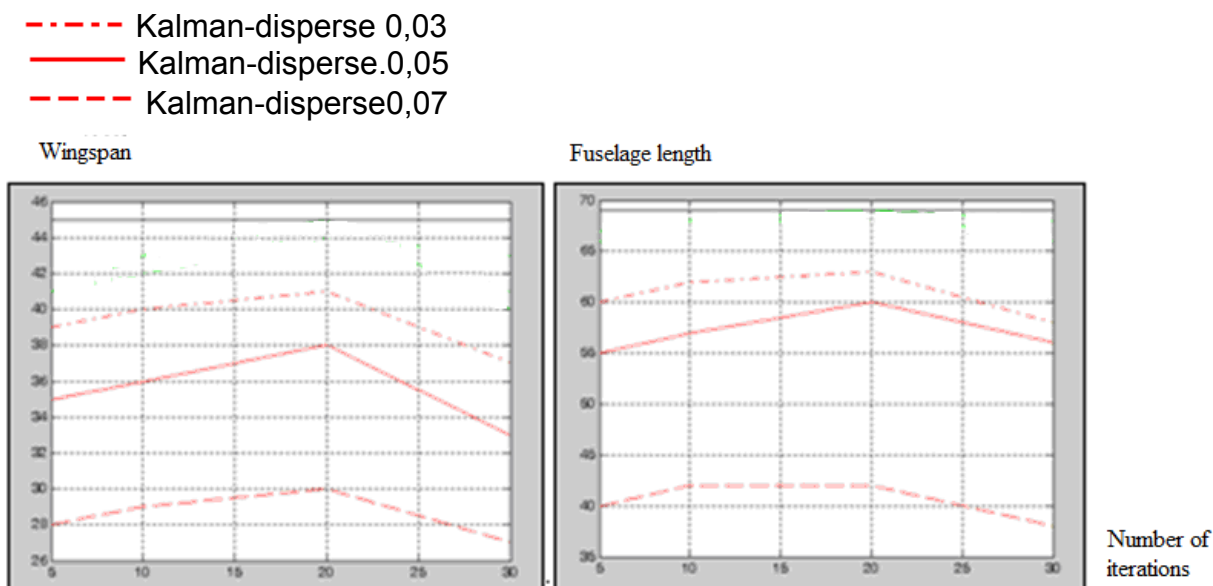


Fig. 1. Dependence of the values of the geometrical characteristics upon the number of iterations in Discrete Kalman filter

Stages of the image processing with Discrete Kalman filter at 0,03 and 20 iterations are visualized on Fig. 2. In Table 4 are given the geometrical characteristics of all the aircrafts, noised with adaptive Gaussian noise (disperse 0, 03) after filtering with Kalman filter.

Table. 4 Aircrafts geometrical characteristics from the sample noised with adaptive Gaussian noise with adaptive Gaussian noise (disperse 0, 03) after filtration with Kalman filter (20 iterations)

Type of aircraft		Etalon object		After filtration of Kalman Filter	
		Fuselage length	Wingspan	Fuselage length	Wingspan
1	<b>F16</b>	50	33	47	29
2	<b>AN 124</b>	91	100	89	90
3	<b>Mc Donnell</b>	69	59	66	57
4	<b>Buccaneer</b>	50	36	47	33
5	<b>F117</b>	51	38	48	35
6	<b>Jaguar</b>	80	45	75	41
7	<b>Mig29</b>	43	38	39	32
8	<b>Miraj 2000</b>	50	31	47	26
9	<b>Su34</b>	87	61	84	55
10	<b>B52</b>	69	45	68	44

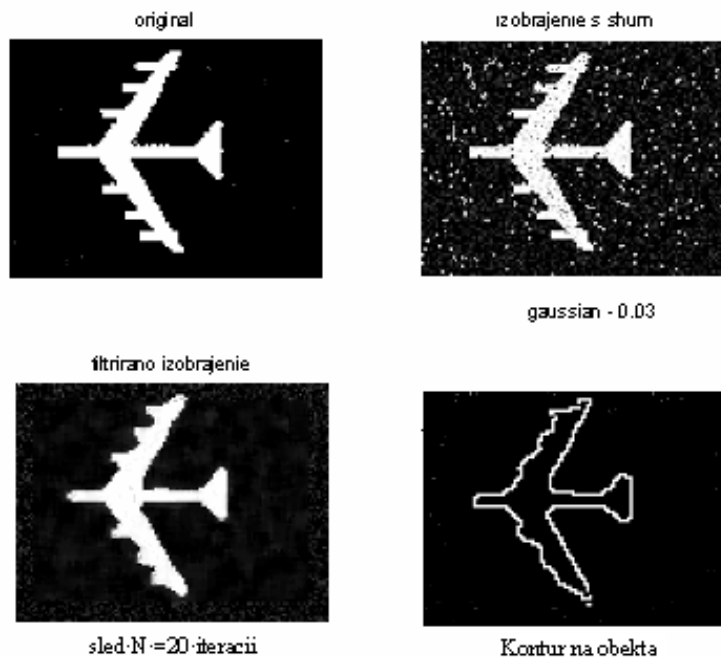


Fig. 2. Stages of image processing with Discrete Kalman filter in disperse 0,03 and 20 iterations

As a criteria for accuracy is used the relative error [8]:

$$\hat{\delta}_x = \frac{\delta_x}{x_{u3M}} \cdot 100, \quad \% , \quad (9)$$

where  $\delta_x = |x_{em} - x_{u3M}|$  - absolute error;

$x_{em}$  is the value of the parameter of the etalon object:

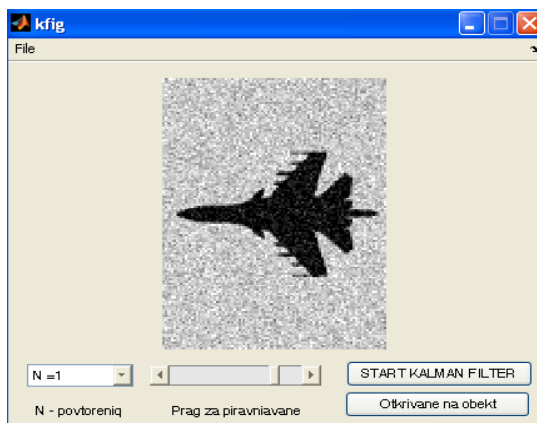
$x_{u3M}$  - value of the parameter's object after Kalman filter applying

Values of the relative error for the examined types of aircrafts, calculated on (9), are shown in Table 5.

Table.5 Values of the relative error for the studied types of aircraft

Type of aircraft	$\hat{\delta}_x, \%$	
	Length of the fuselage axis	Wingspan
<b>F16</b>	6.38	13.79
<b>AN 124</b>	1.12	11.11
<b>Mc Donnell</b>	4.55	3.5
<b>Buccaneer</b>	6.38	9.09
<b>F117</b>	6.25	8.57
<b>Jaguar</b>	6.67	9.75
<b>Mig29</b>	10.25	18.75
<b>Miraj 2000</b>	6.66	19.23
<b>Su34</b>	3.57	10.9
<b>B52</b>	3.7	6.25

For image filter using the Kalman method is created graphical user interface (GUI). The overall appearance is presented on *Fig. 3*.



*Fig. 3. Overall appearance of GUI after filtration for digital image using the Kalman method*

The interface gives the following possibility for choice:

- Number of repetitions for filtration;
- Grey threshold – for receiving the binary image;
- Object detecting in the image.

The input and the filtered image are visualized.

## CONCLUSION

The presented development shows one application of program product MATLAB in software realization for created mathematical model, which gives possibility for visualizing the stages of image processing as well as examining the influence of iteration numbers of Discrete Kalman filter upon the quality of the filtered image.

For reasonable error usually is accepted the value  $\hat{\delta}_x = 5\%$ . From the results we can see that the relative error when using Kalman filter reaches up to 10% for length of the fuselage axis and up to 19% for wingspan. The result gives rise for looking to another filter (eventually based on neural network), which would give better results (minimization the error), connected with the inputted geometrical characteristics.

This work is appropriate for classes, related to Digital image processing. It could be an interesting subject for graduate students studying Computer systems, Mathematics, Informatics, Radiolocation and navigations and so on.

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## ПРИЛОЖЕНИЕ НА ПРОГРАМЕН ПРОДУКТ МАТЛАВ ПРИ ОБРАБОТКА НА ЦИФРОВА ИНФОРМАЦИЯ

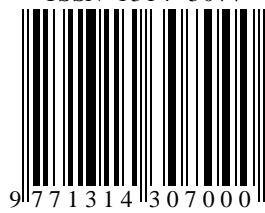
Милена Костова, Иван Георгиев

*Русенски университет „Ангел Кънчев“*

**Абстракт:** В статията е представено приложение на програмен продукт MATLAB при обработка на цифрово изображение чрез дискретен рекурсивен Калманов филтър. Анализирани са възможностите на продукта при изследване влиянието на броя итерации, при различна дисперсията на адитивния Гаусов шум, върху качеството на отфилтрираното изображение.

**Ключови думи:** цифрово изображение, дискретен рекурсивен Калманов филтър, Гаусов шум, итерации, MATLAB.

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