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MODEL OF COUPLED BANDS IN EVEN-EVEN NUCLEI

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Abstract: *A model of coupled rotational bands is considered on the basis of an expansion of the Hamiltonian in terms of elementary transition operators, including direct rotation-vibrational coupling with* β, γ *and* $K^{\pi} = 1^{+}$ *phonons. A method for diagonalization in a suitable constructed multi-phonon basis is discussed.*

Keywords: *Rotation-vibrational coupling, Phonon states, Back-bending.*

The intensive research in the field of heavy-ion reactions allowed high-spin rotational states investigation in a number of nuclei. Various interesting effects (back- and down-bending in the ground state band and even in the β- band, for cranking and band crossing which were found 'in beam' spectroscopy, stimulated intensive theoretical investigations on the possible mechanisms of these phenomena. Besides microscopic descriptions, a plenty of phenomenological models have been proposed.

The band hybridization model was generally accepted by many authors. Almost any physical mechanism might be expressed in its language. The intersecting bands have been experimentally identified. The schematic calculations indicate the necessity of taking a $K^{\pi} = 1^{+}$ rotational band into account. This allows a reproduction of the experimental picture of the back- and down-bending effects.

This paper reveals the main features of the proposed by our group model of coupled rotational bands in even-even nuclei. The model is derived in a natural way from the theory of coupled modes in terms of elementary transition operators. It has the advantage of handling a number of bands, which is difficult in a microscopic theory. A part of it is my M. Sc. Thesis.

MODEL HAMILTONIAN

The description of the nuclear rotation-vibrational motion is based on transition operators $B^{+}_{\alpha IM}$ which are irreducible tensors of rank I with projection M.

$$B^{+}_{\alpha IM} = \sum_{I_1 M_1 I_2 M_2} [(n+1)\,(2I_1+1)\,(2I_2+1)]^{\frac{1}{2}}$$

$$\begin{pmatrix} I & I_1 & I_2 \\ K & K_1 & -K_2 \end{pmatrix} \begin{pmatrix} I & I_1 & I_2 \\ M & M_1 & -M_2 \end{pmatrix} (-1)^{M_2 - K_2} \, |\nu_2 I_2 M_2\rangle \langle M_1 I_1 \nu_1| \,,$$

where α denotes the type of the phonon, created by $B^{+}_{\alpha IM}$, as well as the K- number of the resultant state. The states in the above equation are supposed to have good angular momentum, definite number of phonons of type α: n in the state 1 and $n+1$ in the state 2, K- numbers respectively K_1 and K_2 and they do not need further specification (the index ν includes both K and n numbers). In the case $n=K=0$, $\nu_1=\nu_2$ the operator does not create any phonon and realizes transitions inside the same rotational band. It is denoted by R^{+}_{IM}.

We limit our treatment to three types of phonons: ***β (I=K=0), S (I=K=1)*** and ***γ (I=K=2)***. It can be proved that in this case the basic operators are $B^{+}_{\alpha|K|M}$, $B_{\alpha|K|M}$, R^{+}_{2M} and $\vec{I}$, where $\alpha = \beta, S, \gamma$ and $\vec{I}$ is the angular momentum operator. In order to deal with hermitian irreducible tensor operators having certain time-reversal symmetries, we use combinations of operators, as follows:

$$O^{\beta g(+)}_{00} = B^{+}_{\beta 00} + B_{\beta 00},\ O^{\beta g(-)}_{00} = \frac{1}{i}(B^{+}_{\beta 00} - B_{\beta 00}),$$

$$O^{Sg(+)}_{1M} = \frac{1}{i}(B^{+}_{S1M} - (-)^{M}B_{S1-M}),\ O^{Sg(-)}_{1M} = B^{+}_{S1M} + (-)^{M}B_{S1-M},$$

$$O^{\gamma g(+)}_{2M} = B^{+}_{\gamma 2M} + (-)^{M}B_{\gamma 2-M},\ O^{\gamma g(-)}_{2M} = \frac{1}{i}(B^{+}_{\gamma 2M} - (-)^{M}B_{\gamma 2-M}),$$

$$O^{S\beta(+)}_{1M} = \frac{1}{i}(B^{+}_{S1M}B_{\beta 00} - (-)^{M}B^{+}_{\beta 00}B_{S1-M}),\ O^{S\beta(-)}_{1M} = B^{+}_{S1M}B_{\beta 00} + (-)^{M}B^{+}_{\beta 00}B_{S1-M},$$ etc.

In some of them we have to take into account the coupling to angular momentum ***L*** by summation with Clebsch-Gordan coefficients.

Any physical quantity, i.e., a multipole operator acting between the states of our model space, which includes states with any number of three types of phonons - ***β, S*** and ***γ***, can be expanded in a power series in the basic operators. In particular, the model Hamiltonian is obtained as an irreducible tensor operator of rank zero:

$\hat{h} = \hat{h}_R + \hat{h}_v + \hat{h}_c$ with rotational part $\hat{h}_R = \frac{\vec{I}^2}{2\mathcal{J}}$, where $\mathcal{J}$ is the moment of inertia, and $\hat{h}_v = \omega_0\hat{n}_\beta + \omega_1\hat{n}_S + \omega_2\hat{n}_\gamma$ is the vibrational part. The phonon number operators $\hat{n}_\beta$, $\hat{n}_S$ and $\hat{n}_\gamma$ generate the vibrational part $\hat{h}_v$ of $\hat{h}$. For example, the ***β***–phonon number operator is $\hat{n}_\beta = B^{+}_{\beta 00}B_{\beta 00}$. $\hat{h}_c$ is of more complicated form, as it takes into account the rotation-vibrational coupling. It includes ***β–g***, ***S–g***, ***γ–g***, ***S–β***, ***S–γ*** and ***γ–β*** coupling terms with respective coupling strengths.

We make the following modification of the rotational part:

$$\hat{h}_R = \sum_{\alpha=g,\beta,S,\gamma} \frac{\vec{I}^2\hat{P}_\alpha}{2\mathcal{J}_\alpha}$$

with an intra-band projection operator
$\hat{P}_g = 1, \hat{P}_\alpha = P(n_\alpha), P(n_\alpha) = 0\ for\ n_\alpha = 0, P(n_\alpha) = 1\ for\ n_\alpha \neq 0$, where n_α is the number of phonons of type α and $\mathcal{J}_g$ is the moment of inertia of the ground state band, and $[\mathcal{J}_g^{-1} + \mathcal{J}_\alpha^{-1}]^{-1}$ - of any band with α phonons only. So we take into account the difference between the moments of inertia of our four bands in a simple way.

BASIC STATES AND MODEL HAMILTONIAN MATRIX

The basis of operators $B^{+}_{\alpha|K|M}$, $B_{\alpha|K|M}$, R^{+}_{2M} and $\vec{I}$ acting on the ground state is difficult to be handled since the operators have to be coupled by means of Clebsch-Gordan technique to definite angular momenta. To calculate the Hamiltonian matrix elements between such states is a complicated procedure. For this reason we have introduced the zero rank operators:

$$b^{+}_0 = B^{+}_{\beta 0}\vec{I}^2, b^{+}_1 = B^{+}_{S1}T_1, b^{+}_2 = B^{+}_{\gamma 2}T_2.$$

In the right hand side of b_1^+ and b_2^+ the coupling of the two operators is taken into account. Together with their hermitian conjugates these three operators obey definite commutation relations.

The above zero rank operators can not bring any angular momentum into the state they create, so we make them acting not only on the ground state, but also on each state of the ground-state rotational band with appropriate values of ***I*** and ***M***: $|0IM\rangle$. Such procedure is very convenient since the model Hamiltonian $\hat{h}$ is invariant with respect to rotations and thus it does not mix different values of ***I*** and ***M***. Therefore the problem can be solved for each ***IM*** separately. Thus our basic states are:

$$|\alpha IM\rangle = |n_0 n_1 n_2 IM\rangle = N_\alpha (b_0^+)^{n_0} (b_1^+)^{n_1} (b_2^+)^{n_2} |0IM\rangle,$$

where n_0, n_1, n_2 are the numbers of β, S and γ phonons (the eigenvalues of $\hat{n}_\beta$, $\hat{n}_S$ and $\hat{n}_\gamma$), respectively.

The calculation of the normalization factor N_α requires all the commutators of the operators in b_0^+, b_1^+, b_2^+ and it is useful to introduce a ***K***-number operator which gives the ***K***-number of the state when acting on $|\alpha IM\rangle$. Its ***K***-number eigenvalue is $K=n_1+2n_2$. The states $|\alpha IM\rangle$ are orthonormal. Before calculating the matrix elements of $\hat{h}$ it is convenient to express $\hat{h}_c$ in terms of the operators b_0^+, b_1^+ and b_2^+:

$$\hat{h}_c = \chi_0 (b_0^+ + b_0) + \chi_1 \sqrt{\frac{3}{2}} (b_1^+ + b_1) + \chi_2 \sqrt{5} (b_2^+ + b_2) + \chi_{10} \sqrt{\frac{3}{2}} \frac{1}{\hat{I}^2} (b_0^+ b_1 + b_1^+ b_0) +$$
$$\chi_{12} \sqrt{\frac{5}{2}} 2\sqrt{3} \left(\frac{1}{\hat{K}^2 - \hat{K} - \hat{I}^2} b_1^+ b_2 + b_2^+ b_1 \frac{1}{\hat{K}^2 - \hat{K} - \hat{I}^2} \right) + \chi_{20} \sqrt{5} \frac{1}{\hat{I}^2} (b_0^+ b_2 + b_2^+ b_0)$$

.

Finally we obtain a complicated expression for $\hat{h}|n_0 n_1 n_2\rangle$ which is ready for the calculation of the Hamiltonian matrix elements between the orthonormal states $|\alpha IM\rangle$:

$$\begin{aligned}
\hat{h}|n_0 n_1 n_2\rangle = & \left[\left(\frac{1}{2\mathcal{J}_g} + \sum_{\alpha=\beta,S,\gamma} \frac{P(n_\alpha)}{2\mathcal{J}_\alpha} \right) D(I,0) + \sum_{\alpha=\beta,S,\gamma} \omega_\alpha n_\alpha \right] |n_0 n_1 n_2\rangle \\
& + \chi_0 D(I,0) \left[n_0^{1/2} |n_0 - 1 n_1 n_2\rangle + (n_0+1)^{1/2} |n_0 + 1 n_1 n_2\rangle \right] \\
& + \frac{\chi_1}{2} \left\{ [n_1 D(I,K-1)]^{1/2} |n_0 n_1 - 1 n_2\rangle \right. \\
& \left. + [(n_1+1) D(I,K)]^{1/2} |n_0 n_1 + 1 n_2\rangle \right\} + \frac{\chi_2}{2} \left\{ [n_2 D(I,K-1) D(I,K \right. \\
& \left. - 2)]^{1/2} |n_0 n_1 n_2 - 1\rangle + [(n_2+1) D(I,K+1) D(I,K)]^{1/2} |n_0 n_1 n_2 + 1\rangle \right\} \\
& + \frac{\chi_{10}}{2} \left\{ [(n_0+1) n_1 D(I,K-1)]^{1/2} |n_0 + 1 n_1 - 1 n_2\rangle \right. \\
& \left. + [n_0 (n_1+1) D(I,K)]^{1/2} |n_0 - 1 n_1 + 1 n_2\rangle \right\} \\
& - \frac{\chi_{12}}{2} \left\{ [(n_1+1) n_2 D(I,K-1)]^{1/2} |n_0 n_1 + 1 n_2 - 1\rangle \right. \\
& \left. + [n_1 (n_2+1) D(I,K)]^{1/2} |n_0 n_1 - 1 n_2 + 1\rangle \right\} \\
& + \frac{\chi_{20}}{2} \left\{ [(n_0+1) n_2 D(I,K-1) D(I,K-2)]^{1/2} |n_0 + 1 n_1 n_2 - 1\rangle \right. \\
& \left. + [n_0 (n_2+1) D(I,K+1) D(I,K)]^{1/2} |n_0 - 1 n_1 n_2 + 1\rangle \right\}.
\end{aligned}$$

After truncating the basis by inclusion of a reasonable number of low lying phonon states $|\alpha IM\rangle$ one may evaluate the matrix elements of the model Hamiltonian $\hat{h}$ and diagonalize it by standard numerical methods to fit the experimental spectrum with an optimal choice of the corresponding parameters $\mathcal{J}_\alpha, \omega_\alpha \; and \; \chi_\mu$.

SEPARABLE SOLUTION APPROXIMATION

In order to simplify the problem we can consider only two types of phonons - β and γ, or β and S phonons. The K-number of the state is now $K_{n_1 0} = n_1$, or $K_{0 n_2} = 2n_2$. Then we have

$$\hat{h}|n_0 n_\mu\rangle = [E_0(n_0) + E_\mu(n_\mu)]\,|n_0 n_\mu\rangle + h_0(n_0)|n_0 - 1 n_\mu\rangle + h_0(n_0+1)|n_0 + 1 n_\mu\rangle + h_\mu(n_\mu)|n_0 n_\mu - 1\rangle + h_\mu(n_\mu+1)|n_0 n_\mu + 1\rangle$$

with the notations:

$$E_0(n_0) = \left[\frac{1}{2\mathcal{J}_g} + \frac{P(n_0)}{2\mathcal{J}_0}\right] D(I,0) + \omega_0 n_0 ,$$

$$E_\mu(n_\mu) = \frac{P(n_\mu)}{2\mathcal{J}_\mu} D(I,0) + \omega_\mu n_\mu ,$$

$$h_0(n_0) = \chi_0 n_0^{1/2} D(I,0),$$

$$h_1(n_1) = \frac{\chi_1}{2}[n_1 D(I, n_1 - 1)]^{1/2} ,$$

$$h_2(n_2) = \frac{\chi_2}{2}[n_2 D(I, 2n_2 - 1) D(I, 2n_2 - 2)]^{1/2} ,$$

where $\mu = 1$ or 2 refers to the case of S or γ phonons.

For a given spin value I, the basis is composed of the states $|n_0 n_\mu\rangle$ with all possible combinations of $n_0 = 0, 1, 2, \ldots \; and \; 2n_2 \le I$. The Schrödinger equation is: $\hat{h}|\Psi\rangle = E|\Psi\rangle$, where $|\Psi\rangle = \sum_{n_0 n_\mu} a(n_0 n_\mu)|n_0 n_\mu\rangle$. It is useful to write this matrix equation as an infinite system of equations, since $\hat{h}$ has many zero matrix elements. Then the row, labelled by n_0, n_μ, is:

$$[E_0(n_0) + E_\mu(n_\mu) - E]a(n_0, n_\mu) = h_0(n_0)a(n_0 - 1, n_\mu) + h_0(n_0+1)a(n_0 + 1, n_\mu) + h_\mu(n_\mu)a(n_0, n_\mu - 1) + h_\mu(n_\mu+1)a(n_0, n_\mu + 1).$$

By inserting $a(n_0, n_\mu) = a_0(n_0)a_\mu(n_\mu)$, $E = E_0 + E_\mu$, the above equations can be separated in two parts: an infinite subsystem for the β phonons and a finite one (remember that $D(I,K)$ vanishes for $K = I$) for the μ phonons ($\mu = 1 \; or \; 2$). So we get for the K^{th} equation:

$$[E_\nu - E_\nu(K)]a_\nu(K) = h_\nu(K)a_\nu(K-1) + h_\nu(K+1)a_\nu(K+1),$$

where $\nu = 0, \mu$, i.e. $\nu = 0, 1$ or $\nu = 0, 2$. The g.s.b. rotational energy is

$$E_0(0) = \frac{I(I+1)}{2\mathcal{J}_g}.$$

Defining

$$\Psi_\nu(K) = \frac{a_\nu(K)}{a_\nu(K-1)}$$

one gets a recurrence relation for $\Psi_\nu(K)$:

$$h_\nu(K)\Psi_\nu(K) + h_\nu(K+1)\Psi_\nu(K+1) = E_\nu - E_\nu(K)$$

and an equation for $\Psi_\nu(1)$:

$$h_\nu(1)\Psi_\nu(1) = E_\nu - E_\nu(0),$$

which allows to obtain the following eigenvalue infinite fraction equation:

$$E_\nu - E_\nu(0) = \frac{h_\nu^2(1)}{E_\nu - E_\nu(1) - \frac{h_\nu^2(2)}{E_\nu - E_\nu(2) - \cdots}}.$$

Both systems $(\nu = 0, \mu)$ can be solved separately. Let E_ν be a solution of the above equation. Then one can get $\Psi_\nu(K)$ for any K, using $\Psi_\nu(1)$. In calculations one may cut off the infinite chain fraction for β phonons $(\nu = 0)$ at a fixed number, say 30 phonons, and include in it, in the case of γ phonons $(\nu = \mu)$, finite fraction, all the allowed states. The above equation can be solved only numerically by evaluating its right-hand side for each value of E_ν. Then one gets $\Psi_\nu(1)$ and may find each $\Psi_\nu(K)$ afterwards. They give the eigenvector components $a_\nu(K), K = 0, 1, 2, \ldots$ within a constant factor, which may be used to normalize the vector.

An exact solution can be found in the case $\nu = 0$ (β phonons) if we take

$$\Psi_0(K) = -\frac{\chi_0 D(I,0)}{\omega_0\sqrt{K}}, \mathcal{J}^{-1} = 0.$$

The ground state-band energies become

$$E_I = \frac{I(I+1)}{2\mathcal{J}_g} - \frac{{\chi_0}^2}{\omega_0}[I(I+1)]^2.$$

If we consider the ground state and β phonon states, we can get a simple solution not only for the energies, but also for the eigenvectors. After inserting $a_0 = 1$ we obtain the correlated ground state band:

$$|IM\rangle = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}}\left[-\frac{\chi_0 I(I+1)}{\omega_0}\right]^n |nIM\rangle,$$

where the n^{th} term represents the admixture of the band built on the state with n β phonons.

The physical meaning of the solution can be easily seen in a simplified treatment when one considers the ground state band and a second band built on a state with one phonon of type ν $(\nu = \beta \text{ or } S, \text{ or } \gamma)$. Then $h_\nu(1) \neq 0, h_\nu(K) = 0 \text{ for } K \geq 2$ and solving a quadratic equation we obtain the following solutions:

$$E_\nu^{I,II} = \frac{E_\nu(0) + E_\nu(1)}{2} \pm \left\{\left[\frac{E_\nu(0) - E_\nu(1)}{2}\right]^2 + {h_\nu}^2(1)\right\}^{1/2},$$

the second one corresponding to the yrast band. If the two bands intersect at spin $I = I_0$, for low spins $I \ll I_0$ the energy is $E^{II} \approx E_\nu(0)$, i.e., near the g.s.b. energy and for $I \gg I_0$ $E^{II} \approx E_\nu(1)$, i.e., the well known picture of band hybridization as a special case.

CONCLUSION

Calculations after fitting to experimental level energies must show in the first place how far in the high spin region the experiment can be reproduced phenomenologically. Secondly, they can give some rude estimates of the role of many phonon states in the band hybridization picture. The last can be viewed upon as a special case of the model, taking into account only the ground state and one-phonon bands. Thirdly, they can give experimental values of the model Hamiltonian parameters, in particular-of the different type coupling strengths. And finally, we hope to be able to reproduce the model Hamiltonian parameters microscopically and thus - to obtain a simple microscopic description of band crossing and in particular - of the back-bending.

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МОДЕЛ НА СВЪРЗАНИТЕ ИВИЦИ В ЧЕТНО-ЧЕТНИ ЯДРА

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Резюме: *Разгледан е модел на свързаните ротационни ивици на базата на хамилтониан, развит по елементарните оператори на преходи, включващ директна ротационно-вибрационна връзка с β, γ и $K^\pi = 1^+$ фонони. Обсъден е метод за диагонализация в подходящо конструиран многофононен базис.*

Ключови думи: *Ротационно-вибрационно свързване, Фононни състояния, Бек-бендинг.*

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