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"MATHEMATICS, INFORMATICS AND PHYSICS"

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AN APPROACH TO DESCRIPTION OF MONOPOLE EXCITATIONS IN NUCLEI

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Abstract: *An extension of the Local-scale transformation method (LSTM) by inclusion of N scalar functions is suggested. An application of LSTM to the Hartree-Fock theory is considered. By means of the suggested approach an investigation of the 'breathing' monopole excitation mode within the adiabatic limit of the Time-dependent Hartree-Fock theory (ATDHF) is carried out. Numeric results in a particular case of pure scale transformation, using the Skyrme forces for the nucleus ^{16}O, are obtained.*

Keywords: *Giant monopole resonance, Local-scale transformation method.*

INTRODUCTION

The Local-scale transformation method (LSTM) [17], [11] affords a natural introduction of collective variables into the wave function of an *A*-particle system with Hamiltonian *H* through the local nucleon density $\rho(\mathbf{r})$ [10]. The many-particle wave functions, obtained in this way, have been successfully used in combination with the Generator coordinate method (GCM) [18], [14] and the adiabatic limit of the Time-dependent Hartree-Fock theory (ATDHF) [10], [6] for the Giant isoscalar and isovector monopole resonance description. In a further development of the LSTM several local-scale transformation functions binding only the one-determinant wave functions, orthogonal with regard to the quantum numbers, remaining unchanged by the implemented LST, are considered. The case of a LST of a pure scale type affords an opportunity for a quantitative analysis of the Giant monopole resonance (GMR) within the ATDHF method. The oscillator frequencies (different for every shell) are taken as collective variables.

ILLUSTRATION OF THE PURE SCALE TRANSFORMATION CASE

A particular case is the one of a pure scale transformation $f(\mathbf{r}) = \alpha\mathbf{r}$. Applied to light nuclei, for which, as it is well known, the oscillator model produces good results, this case provides an opportunity for specifying those results and simultaneously reveals the new features of the suggested method.

For definiteness the nucleus of ^{16}O with its two shells *1s* and *1p* (i.e. the number of shells $N_{sh} = 2$) is considered in approximation of proportional neutron and proton densities (the Coulomb energy not taken into account). The energy functional is minimized within the class of LST functions $f_i(\mathbf{r}) = \alpha_i\mathbf{r} \in F, (i=1,2)$. Using the oscillator model wave function [16] and the Skyrme forces [12], the energy density functional [13] turns into a common algebraic function of the parameters α_1 *and* α_2 (the spin-orbital interaction is neglected). In particular, for even-even spherical nuclei with *N=Z* it has the following form [18]:

$$\mathcal{H}(r) = \left(\frac{\hbar^2}{2m}\right)(A-1)A^{-1}\tau + \frac{3}{8}t_0\rho^2 + \frac{1}{16}t_3\rho^\sigma\rho^2 + \frac{1}{16}(3t_1+5t_2)\rho\tau + \frac{1}{64}(9t_1-5t_2)(\vec{\nabla}\rho)^2.$$

In the above expression A is the mass number, m - the nucleon mass. The spin-orbital and Coulomb interactions are neglected; $\rho_n=\rho_p=\rho/2$, $\tau_n=\tau_p=\tau/2$; σ, t_0, t_1, t_2 and t_3 are parameters of Skyrme forces; ρ and τ are the local and kinetic energy densities, as follows:

$$\rho(r) = 4\pi^{-3/2}\alpha_1^3 e^{-\alpha_1^2 r^2} + 8\pi^{-3/2}\alpha_2^5 r^2 e^{-\alpha_2^2 r^2},$$

$$\tau(r) = 4\pi^{-3/2}\left[\alpha_1^7 r^2 e^{-\alpha_1^2 r^2} + (2\alpha_2^9 r^4 - 4\alpha_2^7 r^2 + 6\alpha_2^5)e^{-\alpha_2^2 r^2}\right],$$

$$\nabla\rho(r) = 4\pi^{-3/2}\left[-2\alpha_1^5 r e^{-\alpha_1^2 r^2} + (4\alpha_2^5 r - 4\alpha_2^7 r^3)e^{-\alpha_2^2 r^2}\right].$$

So, explicitly, the energy functional is expressed through α_1 and α_2.

In particular, at $\sigma=1$ (Skyrme forces modification SIII [3]), the integration over r may be performed analytically, i.e. the energy functional is also an algebraic function of α_1 and α_2. At $\sigma\neq1$ it is necessary to calculate the last term in it by quadratures.

Further on, the variation of the energy functional within the scale class for $\{f\}_{Nsh} \in F$ is equivalent to the solution of the system

$$\frac{\partial E(\alpha_1,\alpha_2)}{\partial \alpha_i} = 0, (i = 1,2).$$

At $\alpha_1 \equiv \alpha_2 = \alpha$ the variation of the energy functional reduces to the solution of the equation

$$\frac{\partial E(\alpha)}{\partial \alpha} = 0.$$

Results concerning the ground state energy for the optimal values of the variational parameters α_{10} , α_{20} and α_0 and root-mean square radii of the ground and first excited states of the nucleus ^{16}O in the two- and one-parameter cases are obtained.

The two equations above are solved independently for the Skyrme forces modifications SIII [3], Ska [8], SkM [9], Sk* M [2]. The obtained results can be summarized as follows:

1. The values of α_{10} and α_{20} are very close. For the Skyrme forces modifications under consideration they form an interval and α_0 belongs to it. The values of α_0 are much closer to these of α_{20}. It shows that the inclusion of the second variational parameter is of greater importance for the *1s* – nucleons. The one-particle wave functions of the *1p* – nucleons change insignificantly.
2. In accordance with the expectations $E(\alpha_0) \geq E(\alpha_{10}, \alpha_{20}) \geq E_{HF}$, i.e., the inclusion of more LST functions practically approximates the results to those of Hartree-Fock.
3. The last conclusion is valid also for the root-mean-square radii.

ISOSCALAR MONOPOLE RESONANCE

This approach is used as a basis of the 'breathing' monopole excitation mode investigation. It reveals some peculiarities, mainly of the monopole vibrations of type 'anti-scaling' [4].

Within the ATDHF theory the classical Hamiltonian of the system with instantaneous density $\rho(r,t)$ may be presented in the form [1]:

$$H = \frac{m}{2}\sum_{i,j} m_{ij}\ \dot{\alpha}_i\dot{\alpha}_j\ + E(\alpha_1, \alpha_2),\ \ (i,j = 1,2),$$

where the first term on the right-hand side is the collective kinetic energy with $m_{i,j}$ (α_1, α_2) - the matrix elements of the mass tensor. The second term is the collective potential energy. Within the harmonic approximation $m_{i,j}$ (α_1, α_2) are taken at the equilibrium point $(\alpha_{10}, \alpha_{20})$ and should be calculated through numerical quadratures.

The potential energy E is developed in series of $\alpha_i - \alpha_{i0}$, $(i=1,2)$ and afterwards is taken with an accuracy to second order terms, taking into account that the first order terms are identically equal to zero, as the energy E has a minimum at the equilibrium point $(\alpha_{10}, \alpha_{20})$.

After the corresponding procedure of diagonalization [5], the collective Hamiltonian $\hat{H}$ is expressed by means of the normal coordinates Q_i , $(i=1,2)$ – linear combinations of α_i, and the eigenvalues λ_i , bound to the energy. After the quantization of $\hat{H}$ [20] and using the phonon operators a^+ and a, the Hamiltonian takes the form [19]:

$$\hat{H} = \sum_{i=1}^{2} \hbar\omega_i(\ a_i^+ a_i + \frac{1}{2}),$$

$$\hat{H} = \ \hbar\omega\ (a^+ a + \frac{1}{2}),$$

respectively, where ω_i and ω denote the phonon frequencies and

$$\hbar\omega_i = \left(\frac{\hbar^2}{m}\right)^{1/2} \lambda_i^{1/2}, (i = 1,2),$$

$$\hbar\omega = \left(\frac{\hbar^2}{m}\right)^{1/2} \lambda^{1/2}.$$

The parameters α_1, α_2 and the local density $\rho(r, \alpha_1, \alpha_2)$ (respectively α, $\rho(r, \alpha)$ in the one-parameter case) have to be expressed in terms of the phonon operators and phonon frequencies. For that purpose $\rho(r, \alpha_1, \alpha_2)$ is developed in Taylor series with an accuracy to second order terms. The matrix elements of $\hat{\rho}$ between the ground and excited nuclear states are calculated: $\langle 0|\hat{\rho}|0\rangle = \rho_{00}(r)$; $\langle 0|\rho_{00} r^2|0\rangle$, which gives r_{00}; $\langle i|\hat{\rho}|0\rangle = \rho_{i0}(r)$, $\langle i|\hat{\rho}|i\rangle = \rho_{ii}(r)$; $\langle i|\rho_{ii} r^2|i\rangle$, which gives r_{ii} *for i=1,2* in the two-parameter case and the corresponding expressions in the one-parameter case. Furthermore, the normalization conditions remain valid:

$$\int \rho_{00}\,(r)d\vec{r} = \int \rho_{ii}\,(r)d\vec{r} = A;\ \int \rho_{i0}\,(r)d\vec{r} = 0.$$

The calculated mean-square radii in the ground state and in the two cases stated above have the following explicit form:

$$r_{rms}^2 = \frac{3}{8}\alpha_{10}^{-2} + \frac{15}{8}\alpha_{20}^{-2},$$

$$r_{rms}^2 = \frac{9}{4}\alpha_0^{-2}.$$

The same quantity is more complicated in the excited states.

The excited states contribution should be estimated, for instance, through the isoscalar energy weighted sum rules (EWSR) [7].

Table 1 presents the normal coordinates, one-phonon excitation energies and the estimates, performed by means of the EWSR, respectively, in the two cases under consideration.

*Table 1. Normal coordinates, one-phonon excitation energies and the **EWSR** estimates in the two- and one-parameter cases under consideration for ^{16}**O** nucleus and for different **Skyrme** type forces*

	SIII	**Ska**	**SkM**	**Sk* M**
Q_1, fm	$2.97\alpha_1+15.62\alpha_2$	$2.77\alpha_1+16.06\alpha_2$	$2.85\alpha_1+15.87\alpha_2$	$2.88\alpha_1+16.20\alpha_2$
Q_2, fm	$1.87\alpha_1-1.21\alpha_2$	$1.98\alpha_1-1.63\alpha_2$	$1.91\alpha_1-1.64\alpha_2$	$1.96\alpha_1-1.69\alpha_2$
$\lvert E_0 - E_1 \rvert$, MeV	34.253	29.658	27.747	27.181
$EWSR_1$, %	92.689	92.124	90.994	93.652
$\lvert E_0 - E_2 \rvert$, MeV	133.350	121.348	115.056	112.177
$EWSR_2$, %	0.144	0.057	0.037	0.057
Q, fm	18.58 α	18.80 α	18.71 α	19.06 α
$\lvert E_0 - E_1 \rvert$, MeV	34.586	29.780	27.826	27.256
$EWSR_1$, %	99.242	97.443	96.640	96.388

These results can be summarized as follows:

1. According to Table 1, the excited state with a greater contribution to the EWSR in the two-parameter case is close to the excited state in the one-parameter case. The contribution to the EWSR of the last is also great. This state is known to have vibrations of type ‘scaling’ [4], i.e., with the increasing of α, the nuclear half-radius and diffuseness increase too.
2. The excited state, that nearly exhausts the isoscalar EWSR in the two-parameter case, is lower than the corresponding one-parameter case state. It proves that the two-parameter form is reasonable.
3. In the two-parameter case exists one more high excited state with a small contribution to the EWSR. The comparison of the transition densities in the two cases reveals one node more of the corresponding to this state transition density. This is typical for vibrations of type ‘anti-scaling’ [4].
4. Table 2 displays the excitation energies $E_1(\alpha_1, \alpha_2)$, $E_1(\alpha)$ of ^{16}O compared to those obtained in the GCM with one generator coordinate [7, 9, 6]. The results are in good agreement.

5. From the comparison of the normal coordinates, corresponding to 'scaling' and 'anti-scaling' type of vibrations, it could be seen that the vibrations are, respectively, in phase and opposite in phase with regard to the normal coordinates α_1 and α_2. Hence, within the considered monopole resonance dynamics, in particular for the ^{16}O nucleus, the two shells *1s* and *1p* of this nucleus perform mutual vibrations in phase and opposite in phase.

Table 2. Excitation energies $E_1(\alpha_1, \alpha_2)$ *and* $E_1(\alpha)$ *of* ^{16}O *in comparison with those in the GCM with one generator coordinate [7], Nuclear fluid dynamics (NFD) [9] and ATDHF [6] (**r** - the half-radius, **b** - the surface thickness of the nucleus*)

E_1, MeV	SIII	Sk* M
$E_1(\alpha_1,\alpha_2)$	34.253	27.181
$E_1(\alpha)$	34.586	27.256
E_1 (NFD)	32.8	25.3
E_1 (GCM(α))	31.7	26.1
E_1 (GCM(b))	31.5	25.8
E_1 (GCM(R))	42.4	32.5
E_1(ATDHF(R,b))	28.4	23.3

CONCLUSION

The increase of the number *N* of the LST functions affords an opportunity for the wave functions and ground state energy improvement. This approach, in combination with GCM or ATDHF, provides further possibilities for nuclear collective motion description and different collective modes analysis. The fact, that at *N=1* the suggested method leads to the previously considered LSTM, allows a generalization of a number of statements and results, such as those, presented in [15,16,17]. As an illustration, the case of $N=N_{sh}$ (the shell-number) of the nucleus ^{16}O is considered with Hamiltonian, built on the basis of Skyrme forces modifications SIII, Ska, SkM, Sk* M. The model basis consists of oscillator wave functions. Through a scale transformation $f_i(r) = \alpha_i r$, $(i=1,2,...,N_{sh})$, the equilibrium values of the parameters α_i and the corresponding full nuclear energy and root-mean-square radius have been obtained. The LST functions number increase leads to better results for these nuclear quantities.

The suggested approach has been used for the 'breathing' monopole excitation mode investigation within the ATDHF method. The obtained results for the first excited state energies, root-mean-square radii of the nucleus in these states and in the ground state, the local and transition densities and the EWSR estimates in the case of the nucleus ^{16}O reveal the two-parameter approach advantage. The results are in good agreement with those, obtained within the GCM and NFD. The two normal modes for the nucleus ^{16}O under consideration correspond to 'scaling' and 'anti-scaling' vibrations. These two types

of vibrations in ^{16}O are connected with the mutual oscillations of the *1s*- and *1p*- nucleons in phase ('scaling') and opposite in phase ('anti-scaling'), respectively.

REFERENCES

[1] Baranger, M., M. Véneroni. Ann. Phys.114, 1975, p. 123.
[2] Bartel, J., P. Quentin, M. Brack, C. Guet, H. B. Hakansson. Nucl. Phys. A386, 1982, p. 79.
[3] Beiner, M., H. Flockard, M. V. Giai, P. Quentin. Nucl. Phys. A238, 1975, p. 29.
[4] Brack, M., W. Stocker. Nucl. Phys. A388, 1982, p. 230.
[5] Brack, M., W. Stocker. Nucl. Phys. A406, 1983, p. 413.
[6] Dimitrova, S. S., I. Zh. Petkov, M. V. Stoitsov. Z. Phys. A325, 1986, p. 15.
[7] Flocard, H., D. Vautherin. Nucl. Phys. A264, 1976, p. 197.
[8] Köhler, S. Nucl. Phys. A258, 1976, p. 301.
[9] Krivine, H., J. Treiner, O. Bohigas. Nucl. Phys. A366, 1980, p. 155.
[10] Petkov, I. Zh., M. V. Stoitsov. ICTP, IC (85), 45, Trieste, 1985.
[11] Petkov, I. Zh., M. V. Stoitsov, E. S. Kryacko. Int. J. Quantum Chem., 29, 1986, p. 149.
[12] Skyrme, T. H. R. Nucl. Phys. 9, 1959, p. 615.
[13] Vautherin, D., D. M. Brink. Phys. Rev. C5, 1972, p. 626.
[14] Маринова, Л. П., И. Ж. Петков, М. В. Стоицов. Препринт ОИЯИ Р4-86-42, Дубна,1986.
[15] Петков, И. Ж., М. В. Стоицов. Доклады БАН, 34, 1981, с. 1657.
[16] Петков, И. Ж., М. В. Стоицов. ЯФ, 37, 1983, с. 1167.
[17] Петков, И. Ж., М. В. Стоицов. ТМФ, 55, 1983, с. 407.
[18] Петков, И. Ж., М. В. Стоицов. Препринт ОИЯИ Р4-85-785, Дубна, 1985.
[19] Фейнман, Р. Статистическая механика, Москва, „Мир", 1978.
[20] Эллиот, Дж., П. Добер. Симметрия в физике, т. 1, Москва, „Мир", 1983.

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ПОДХОД ЗА ОПИСАНИЕ НА МОНОПОЛНИ ВЪЗБУДЕНИ СЪСТОЯНИЯ В ЯДРАТА

Галина Крумова

Русенски университет „Ангел Кънчев"

Резюме: *Предложено е разширение на Метода на локално-мащабното преобразование (МЛМП) посредством включване на N скаларни функции. Разгледано е приложение на МЛМП в теорията на Хартри-Фок. С помощта на предложения метод е проведено изследване на 'дихателния' монополен мод на възбуждане в рамките на адиабатичната зависеща от времето теория на Хартри-Фок. Получени са числени резултати за ядрото ^{16}O в частен случай на чисто мащабно преобразование с използване на силите на Скирм.*

Ключови думи: *Гигантски монополен резонанс, Метод на локално-мащабното преобразование.*

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