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"MATHEMATICS, INFORMATICS AND PHYSICS"

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A STUDY ON THE INFLUENCE OF INCOMING CALLS FLOW INTENSITY ON THE WAITING TIME CHARACTERISTICS OF AN EMERGENCY MEDICAL AID CENTRE

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Abstract: *The present paper studies the influence of the incoming calls flow on the waiting time characteristics of an emergency aid centre with teams collaboration with each other and a deviation in the incoming calls distribution law and the service time. An instance of "all as one" type of collaboration is considered. The functioning of an emergency aid centre has been simulated with Poisson incoming flow and constant service time, Poisson incoming flow and Erlang service time, as well as with regular incoming flow and exponential service time. Conclusions have been drawn, respectively.*

Keywords: *Mathematical Modelling, Simulation, Queuing Theory, Distribution Laws, Emergency Medical Aid.*

Often, in the case of natural disasters and industrial and road accidents, the patients suffer multiple damages. In view of the timely treatment of the injured patients and saving their lives, they have to be served by more than one emergency aid teams. Thus the question of team collaboration arises. This approach cannot be applied to all types of health conditions.

It is natural to suppose that if several teams *(к)* are working to serve a patient, the intensity of the service $\mu(k)$ should not diminish with the increase of *к,* i.e. this will be a non-decreasing function of the number *к* of the teams working. It could be presented with a graph like the one shown in Fig.1 [10].

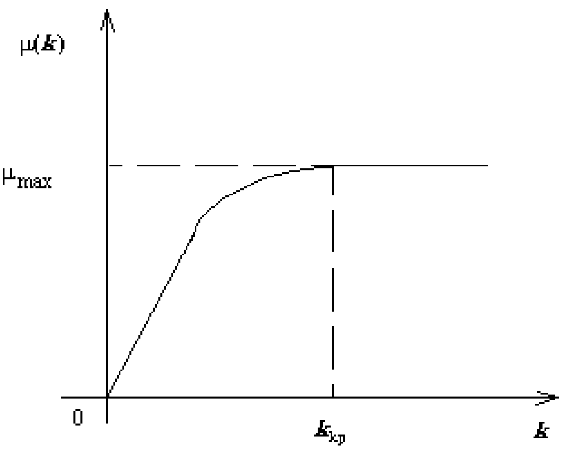

Fig. 1. Dependence of the service intensity on the number of teams

The increase of the number of teams that serve a patient does not always result in proportionate increase of the service intensity. When a certain value $k = k_{kp}$ is reached, a further increase of the number of teams involved does not lead to higher service intensity (Fig.1).

In this case it is important to set the type of the function $\mu(k)$. An example is considered where $\mu(k)$ grows in proportion to *к* with $k \le k_{kp}$, but with $k > k_{kp}$ it remains constant and $\mu_{\max} = k_{kp} * \mu$. If the number of the teams in the system that can cooperate with each other is $n \le k_{kp}$, then one can assume that the patient service intensity with several teams available is proportional to the number of teams.

The simplest example of mutual cooperation is of the “all as one” type. So when a call from a patient is received, all *n* teams start serving it immediately and stay busy until they have completed serving this call. Then all teams move over to provide service to the next call (if there is one) or they wait until they receive one (if there isn’t one at the moment), etc. Therefore, the system in this case would be working as uni-channel with higher service intensity.

Applying team collaboration of the “all as one” type would be best for the patient and he/she would receive timely help but engaging several teams would lead to greater expenses.

The work at an emergency medical aid centre (EMAC) is viewed as a multichannel queuing system with an indefinite number of places in the queue. The influence of the collaboration accomplished among the teams on the waiting time characteristics is studied:

L_0 - number of calls in the system’s queue;

L_c - number of calls in the system;

W_0 - waiting time for the calls in the system’s queue;

W_c - waiting time for the calls in the system.

When no collaboration is applied among the teams, the values of the above parameters are defined from the following dependencies [3]:

(1) $$L_0 = \frac{\rho^{n+1} * P_0}{(n-1)! * (n-\rho)^2},$$

where $P_0 = \dfrac{1}{1 + \dfrac{\rho}{1!} + \dfrac{\rho^2}{2!} + \ldots + \dfrac{\rho^n}{n!} + \dfrac{\rho^{n+1}}{n! * (n-\rho)}}$, $\rho = \dfrac{\lambda}{\mu}$, *n* - number of teams;

(2) $$L_c = L_0 + \rho;$$

(3) $$W_0 = \frac{\rho^n * P_0}{n * \mu * n! * (n-\kappa)^2} = \frac{L_0}{\lambda}, \text{ where } \kappa = \frac{\rho}{n};$$

(4) $$W_c = W_0 + \overline{t_{serv}} = W_0 + \frac{1}{\mu} = \frac{L_c}{\lambda}.$$

In the case of collaboration of the “all as one” type, the system will work as uni-channel with parameters $\mu^* = n * \mu$, $\rho^* = \dfrac{\lambda}{\mu^*} = \dfrac{\lambda}{n * \mu} = \dfrac{\rho}{n} = \kappa$ and its characteristics would be defined in the following way [10] :

(5) $$L_c = \frac{\kappa^2}{1-\kappa},$$

(6) $$W_0 = \frac{\kappa}{n * \mu * (1-\kappa)},$$

(7) $$W_c = \overline{t_{wait}} + \overline{t_{serv}} = W_0 + \overline{t_{serv}} = W_0 + \frac{1}{n * \mu} = \frac{1}{n * \mu * (1-\kappa)}.$$

The study is conducted with service intensity of μ=2.14 c./h. and *n*=4 teams. The following conclusions can be made from the graphs of L_0 and W_0 (W_0 - without collaboration, W_{0ve} - with “all as one” type collaboration) in Fig. 2 and Fig. 3: The acceptable values for λ should not exceed 4, which will allow the call waiting time in the system's queue to be reduced practically to zero. For λ growing to over four calls per hour, the number of calls in the system's queue starts increasing and tends to infinity for λ approaching 8.56 c./h.

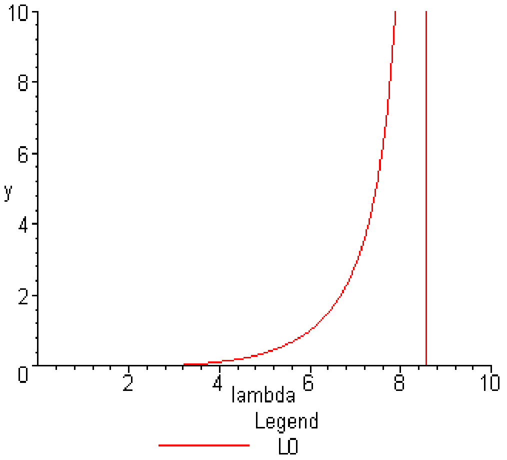

Fig. 2. Dependence of L_0 on the intensity of the incoming calls flow λ

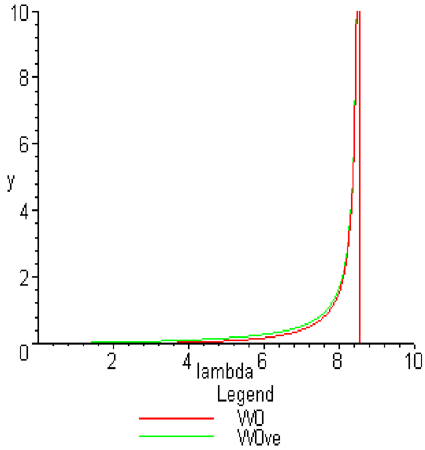

Fig. 3. Dependence of W_0 on the intensity of the incoming calls flow λ

There are also considerable differences in the values of L_c and W_c. The number of calls in the system with no collaboration taking place L_c is larger than the number of those where "all as one" type collaboration is applied L_{cve} (Fig. 4). When the results for the values of W_c are compared (without collaboration among the teams and with "all as one" type collaboration respectively), it turns out that calls waiting time in the system with collaboration applied W_{cve} is almost twice longer than the calls waiting time in a system without collaboration W_c (Fig. 5).

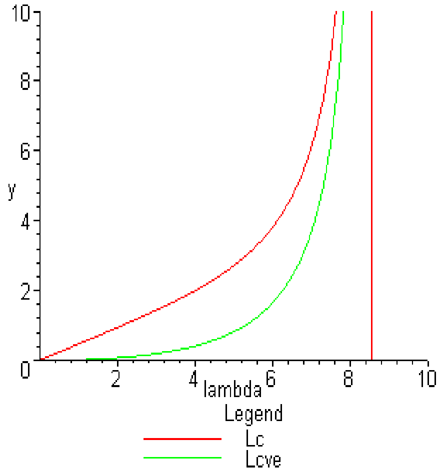

Fig. 4. Dependence of L_c without and with collaboration L_{cve} on the intensity of the incoming calls flow λ

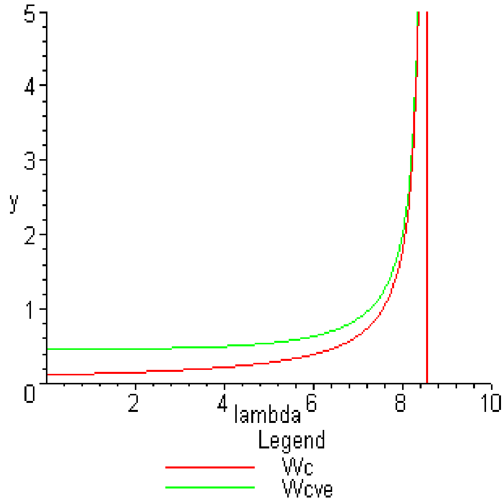

Fig. 5. Dependence of W_c without and with collaboration W_{cve} on the intensity of the incoming calls flow λ

It is evident from the graphs (Fig. 4 and Fig. 5) that the system can work with these parameters with a maximum intensity of incoming calls flow of not more than four calls per hour because the calls waiting time in the system increases and this could be fatal for the health and the life of the patient. It is important for a patient with a heart attack or a stroke to receive the necessary medical aid within the so called "golden hour" [4, 5].

On the basis of the research on the functioning of the emergency medical aid centre in the town of Ruse, the laws regulating the distribution of incoming calls flow and the service time have been defined. It has been established that the incoming calls flow is distributed in accordance with Poisson's law and the service time - in accordance with an exponential law [7].

Sometimes deviations from these laws are likely to occur. In some of the affiliate branches of the emergency medical aid centre in the town of Ruse [2] the distance travelled for one call is about twice longer than the distance travelled by the teams from the Ruse emergency medical aid centre. The increased time for reaching a patient increases the service time [8, 9]. The share of the transportation time as a portion of the service time goes up, respectively. This also causes deviations in the laws for the distribution of the service time.

When the incoming flow is not of Poisson's type, and the service time is not distributed in accordance with an exponential law, the mathematical apparatus of the research becomes too complicated and systems characteristics are found only for the simplest cases [6]. A simulation has been developed of the functioning of the emergency medical aid centre in the town of Ruse with Poisson's incoming flow and constant service time, Poisson's incoming flow and Erlang service time, as well as with regular incoming flow and exponential service time.

Coffman and Kryuon [1] have shown that for a uni-channel queuing system with Poisson's flow and random distribution of service time, the average number of calls located in the queue is defined by the following dependency:

(8) $$L_0 = \frac{\rho^2 * (1+\nu^2)}{2*(1-\rho)},$$

where $\rho = \dfrac{\lambda}{\mu}$; λ is the incoming flow intensity, μ is the service intensity, ν is the variation coefficient of service time.

The average waiting time in the system's queue is calculated according to the formula (9):

(9) $$W_0 = \frac{\rho^2 * (1+\nu^2)}{2*\lambda*(1-\rho)}.$$

When the service time for all queries is a constant value equal to its mathematical expectation and $\overline{t_{serv}} = \dfrac{1}{\mu}$, it follows that $\sigma_{\overline{t_{serv}}} = 0$, $\nu = 0$. Then [6]:

(10) $$L_0 = \frac{\rho^2}{2*(1-\rho)}, \qquad W_0 = \frac{\rho^2}{2*\lambda*(1-\rho)}.$$

For a uni-channel queuing system with Poisson's incoming flow and exponential service time, the waiting time characteristics are defines by the following dependencies (11), see [10]:

(11) $$L_0 = \frac{\rho^2}{(1-\rho)}, \quad L_c = \frac{\rho}{1-\rho}, \quad W_0 = \frac{\rho^2}{\lambda*(1-\rho)}, \quad W_c = \frac{1}{\mu*(1-\rho)}.$$

The following dependencies are obtained (12) for the values characterizing the system efficiency for Poisson's incoming flow and constant service time, see [6]:

(12) $L_c = \dfrac{\rho^2 - 2*\rho}{2*(\rho-1)}$, $L_0 = L_c - \rho = \dfrac{\rho^2}{2*(1-\rho)}$, $W_c = \dfrac{L_c}{\lambda} = \dfrac{\rho-2}{2*\mu*(\rho-1)}$,

$W_0 = \dfrac{L_0}{\lambda} = \dfrac{\rho}{2*\mu*(1-\rho)}$.

With regular incoming flow and exponential service time, the average calls waiting time in the system's queue is obtained from the dependency

(13) $W_0 = \int_0^\infty t.dP(T<t) = \dfrac{\rho}{2*\mu*(1-\rho)}$,

where $P(T<t)$ is the probability for the waiting time to be *<t.*

The system parameters are defined according to the following formulas:

(14) $W_0 = \dfrac{\rho}{2*\mu*(1-\rho)}$; $L_0 = W_0 * \lambda$; $L_c = L_0 + \rho$; $W_c = \dfrac{L_c}{\lambda}$.

If the incoming flow is of Poisson's type and the service time is distributed according to Erlang's law, then changes in the system parameters occur.

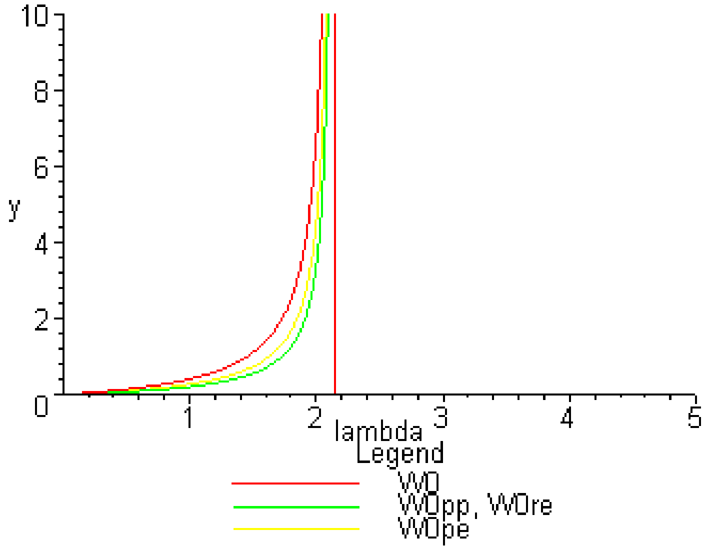

Fig. 6. Dependence of W_0 on the incoming calls flow intensity λ

For Erlang's distribution of service time E_k, it can be assumed that the call coming into the system passes through *κ* service phases, which have the same exponential distribution with a $\mu*k$ parameter. The density of the distribution of the sum of *κ* mutually independent random values which have identical exponential distribution with a $\mu*k$ parameter, is Erlang's flow of *κ*-th order E_k. In this case the incoming flow will have a mathematical expectation λ/k. This means that only $\dfrac{1}{k}$ part of the calls received in accordance with Poisson's law are used.

In the examined case the system characteristics will be defined in accordance with the formulas (15), see [6]:

(15) $L_c = \dfrac{\rho*(k+1)}{2*(1-k*\rho)}$; $W_c = \dfrac{L_c}{\lambda}$; $W_0 = \dfrac{\rho*(k+1)}{2*\mu*(1-k*\rho)}$; $\rho = \dfrac{\lambda}{k*\mu}$.

The study of Erlang's distribution of service time is done for *κ=3*.

The following can be inferred from the research on the waiting time characteristics with different variants of the distribution laws related to the incoming calls flow and the service time: When increasing the incoming calls flow intensity, the system achieves best waiting time characteristics with Poisson's incoming flow and constant service time as well as with regular incoming flow and exponential service time because in these cases the calls waiting time in the system's queue remains always smaller than the values W_o in the other cases (Fig. 6). Thus the most favourable situation for the serviced patient could be accomplished – treatment with the minimum waiting time in the queue of the system.

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НЯКОИ ИЗСЛЕДВАНИЯ НА ВЛИЯНИЕТО НА ИНТЕНЗИВНОСТТА НА ВХОДЯЩИЯ ПОТОК ОТ ЗАЯВКИ ВЪРХУ ХАРАКТЕРИСТИКИТЕ НА ИЗЧАКВАНЕ НА ЦЕНТЪР ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ

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Резюме: *В настоящата работа се изследва влиянието на интензивността на входящия поток от заявки върху характеристиките на изчакване на център за спешна медицинска помощ при прилагане на взаимопомощ между екипите и при отклонение в законите на разпределение на входящия поток от заявки и на времето на обслужване. Разглежда се случай на взаимопомощ от вида „всички като един". Симулирано е функционирането на център за спешна медицинска помощ при поасонов входящ поток и постоянно време на обслужване, поасонов входящ поток и ерлангово време на обслужване, както и на регулярен входящ поток и експоненциално време на обслужване. Направени са и съответните изводи.*

Ключови думи: *Математическо моделиране, Симулация, Теория на масовото обслужване, Закони на разпределение, Спешна медицинска помощ.*

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