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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 11

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 11

Physics

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A STUDY OF THE POSSIBILITIES TO ESTABLISH A STATIONARY MODE IN AN AUTO FLEET

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***Abstract**: The possibilities of establishing a stationary mode in an auto fleet are investigated in the present paper. The probabilities for transition from one state to another have been calculated as well as the probabilities of a system holdup in a given state, and also the probabilities in stationary mode. The graph of the states is shown of a system of four cars in the auto fleet of the Center for emergency medical aid in the town of Ruse. The final probabilities for the functioning of the system have been found and these results have been analyzed.*

***Keywords:** Mathematical modelling, Random processes, Markov's processes, Emergency medical aid*

INTRODUCTION

In a number of practice related problems, we are interested in the so-called **stationary mode** of work of a system, which is established after sufficiently long time has passed since the beginning of the system's operation. This mode comes into effect when certain random processes occur under the above condition. In stationary mode the states of the system change randomly, but their probabilities $p_i(t), \left(i=1,2,...\right)$ remain constant. We will denote these constant probabilities with

$$p_i = \lim_{t\to\infty} p_i\left(t\right) \qquad (1)$$

and if they exist, we will call them final (or marginal) probabilities of the system states. The final probabilities can be interpreted as the average time the system S remains in state s_i.

EXPOSITION

We are going to consider the system of cars in the auto fleet at the Centre for emergency medical aid in the town of Ruse. It is a system with discrete states $s_1, s_2, ..., s_i, ...$. The process taking place in the system is a Markov's process, because the probability for the system to be in one of its states in the future $t > t_0$ depends only on its state at the present moment $t = t_0$ and does not depend on when and how the system has reached this state, i.e. it does not depend on its behaviour in the past (where $t < t_0$).

The process itself is a random walk of the system S through its states. The number of vehicles in the auto fleet of the Centre for emergency medical aid in the town of Ruse is finite, which means that the number of states of the system of cars is also finite. We will denote it by n. We will assume that the random transitions of the system from one state to another can occur only at certain points in time $t_0, t_1, t_2, ...$, which we call "steps" of the process with a starting point t_0.

We set ourselves the task to find the unconditional probabilities for the system S of cars to reside in state s_i at the k-th step, i.e. $p_i(k)$ [5]:

$$p_i(k) = P\{S(k) = s_i\}, \left(i=1,2,...,n;\ k=0,1,...\right).$$

To find these probabilities it is necessary to know the conditional probabilities for the transition of the system S from state s_i at the $(k-1)$-th step to state s_j at the k-th step, i.e.

$$p_{ij}(k)=P\{S(k)=s_j\,|S(k-1)=s_i\},(i,j=1,2,...,n).$$

The probabilities for a holdup $p_{ii}(k)$, which define the likelihood of the system to be held up (or stay) in state s_i at the k-th step, are also introduced.

We will model the system under consideration as a homogeneous Markov chain [2], since the transition probabilities $p_{ij}(k)$ do not depend on the step number k, i.e. $p_{ij}(k)=p_{ij}$.

In the modelled system of cars at the Centre for emergency medical aid, with increasing the step number k a stationary mode is established in which the system continues to walk through its states, but the probabilities for these states no longer depend on the step number.

We find out that the conditions for a stationary mode to exist are present in the system of cars [9]:

1. The set of all states of the system is ergodic because a transition is possible from any state of the system to any other state of this system.

2. The system under consideration, modeled as a Markov chain is homogeneous, since the transition probabilities $p_{ij}(k)$ do not depend on the step number k, i.e. $p_{ij}(k)=p_{ij}$.

3. No cycles are formed in the investigated system (which will be illustrated by plotting the graph of the states of the system).

Since the above mentioned conditions for the existence of stationary mode in the system of cars are present, then the final probabilities for the system's residing in a given state will not depend on what the state of the system was at the initial point $t_0=0$ or what the probability distribution was at the same moment.

Under the above conditions, formula (1) will acquire the form

$$p_i=\lim_{k\to\infty}p_i(k),\quad (i=1,2,...,n). \tag{2}$$

Then the final probabilities will not depend on the initial state of the system.

For a homogeneous Markov chain, the probability p_j for the system to be in state s_j at the $(k+1)$-th step is the same as at the k-th step, i.e.

$$p_j(k+1)=\sum_{i=1}^{n}p_i(k).p_{ij}=p_j. \tag{3}$$

or $p_j=\sum_{i=1}^{n}p_i.p_{ij}=p_j$. (4)

The probability for the system to be held up in state s_j will be denoted by p_{jj}.

The product $p_i.p_{ij}$, called "flow of probabilities", converts system S from state s_i into state s_j. The full probability for the transition of system S to state s_j is equal to the sum of all flows of probabilities, converting the system to this state, i.e. the probability for reaching state s_j will be equal to

$$\sum_{i=1}^{n} p_i p_{ij}, (i \neq j). \qquad (5)$$

Similarly, the sum of all flows of probabilities, taking the system out of state s_j and into some other one will be equal to [9]:

$$\sum_{\substack{i=1 \\ (i \neq j)}}^{n} p_j \cdot p_{ji} = p_j \sum_{\substack{i=1 \\ (i \neq j)}}^{n} p_{ji}, \ (j = 1, 2, ..., n) \qquad (6)$$

Under stationary mode, the probability of reaching a certain state must be equal to the probability of leaving the same state, i.e. (5) = (6) or

$$\sum_{\substack{i=1 \\ (i \neq j)}}^{n} p_i p_{ij} = p_j \sum_{\substack{i=1 \\ (i \neq j)}}^{n} p_{ji}, (j = 1, 2, ..., n). \qquad (7)$$

Condition (7) will be called balance condition for state s_j. The states of the system are mutually exclusive. A normalized condition needs to be added to these n equations, too,

$$\sum_{i=1}^{n} p_i = 1 \qquad (8)$$

by removing one of the equations in (6).

Our study [6] on the Centre for emergency medical aid in the town of Ruse has found that when three teams are working (an ambulance and medical staff) the average waiting time of a request in the system queue before it can be handled is 0,019 hours, and when four teams are working it is 0.003 hours, which implies that patients practically receive aid without waiting. This is a favorable situation for them, as delays may be adverse and life-threatening for a patient with certain diseases such as heart attack or stroke [1, 3, 8]. With three teams available, the probability for a queue to occur in the system is 0.055, whereas it is 0.013 when four teams are working.

We are going to consider a system of four cars in the auto fleet at the Center for emergency medical aid in the town of Ruse. At certain points in time, separated from each other by a time interval τ, all cars are checked and as a result every car is either defined as operational and continues to work, or it is considered broken and is withdrawn for repair. The probability for an operational vehicle to break down and be withdrawn within a time interval τ does not depend on how much time it has already worked (the process "prehistory"), and is equal to r. The probability for a car to be restored to working condition for a time interval τ does not depend on how long the repair has lasted or how many car are being repaired and is equal to q. Car failures and their repair are processes that occur independently. We plot a directed graph of the states of the vehicles in the fleet and number them according to the quantity of damaged cars:

s_0 - all four cars are in order;

s_1 - one is damaged, the other three are serviceable;

s_2 - two cars are damaged, two are serviceable;

s_3 - three cars are damaged, one is working;

s_4 - the four cars are damaged.

The directed graph of the states of a system of four vehicles has the form as shown in Figure 1.

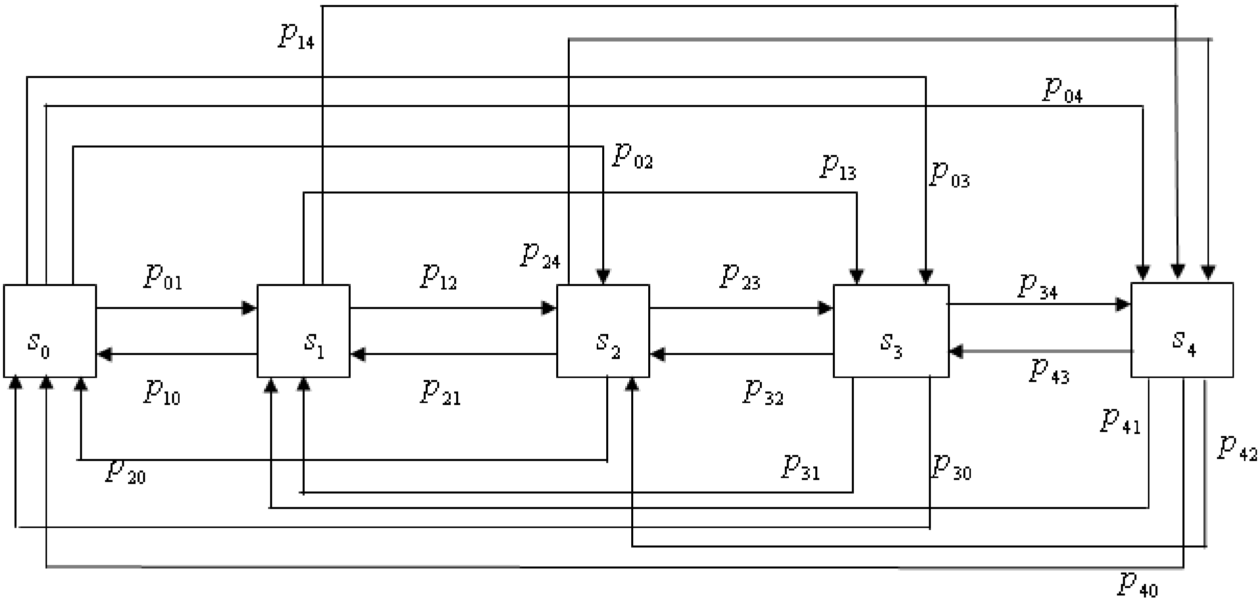

Fig. 1 Graph of the states of a system of four cars

For the system to go from state s_0 to state s_1, one of the four cars needs to break down within a time interval τ. This probability can be found in accordance with the binomial distribution [4] and $p_{01} = C_4^1 r(1-r)^3$. Similarly, the other transition probabilities are found – $p_{02} = C_4^2 r^2 (1-r)^2$, $p_{03} = C_4^3 r^3 (1-r)$, $p_{04} = r^4$, $p_{00} = (1-r)^4$.

The following equation is satisfied: $\sum_{i=0}^{4} C_4^i r^i (1-r)^{4-i} = \sum_{i=0}^{4} p_{0i} = 1.$

For the system to pass from state s_1 to state s_0, the faulty car needs to be repaired within a time interval τ, and the other three cars must remain serviceable: $p_{10} = q(1-r)^3$.

By similar reasoning, the other transition probabilities are defined:

$$p_{20} = q^2.(1-r)^2,\ p_{30} = q^3.(1-r),\ p_{40} = q^4,$$

$$p_{11} = C_3^2.q.r.(1-r)^2 + (1-q).(1-r)^3,\ p_{12} = C_3^2.q.r^2.(1-r) + C_3^2(1-q).r.(1-r)^2,$$

$$p_{13} = C_3^2(1-q).r^2.(1-r) + q.r^3,\ p_{14} = (1-q).r^3,\ \sum_{i=0}^{4} p_{1i} = 1;$$

$$p_{21} = 2.q^2.r.(1-r) + 2.(1-r)^2.q.(1-q),\ p_{22} = (1-q)^2(1-r)^2 + 4.r.q.(1-q).(1-r),$$

$$p_{23} = 2.(1-q)^2.r.(1-r) + 2.(1-q).q.r^2,\ p_{24} = (1-q)^2.r^2,\ \sum_{i=0}^{4} p_{2i} = 1; \qquad (9)$$

$$p_{31} = q^3.r + 3.q^2.(1-q).(1-r),\ p_{32} = C_3^2.q.(1-q)^2.(1-r) + C_3^2.(1-q).r.q^2,$$

$$p_{33} = (1-q)^3(1-r) + 3.r.q.(1-q)^2,\ p_{34} = (1-q)^3.r,\ \sum_{i=0}^{4} p_{3i} = 1;$$

$$p_{41} = C_4^3.q^3.(1-q),\ p_{42} = C_4^2.q^2.(1-q)^2,\ p_{43} = C_4^3.q.(1-q)^3,\ p_{44} = (1-q)^4,$$

$$\sum_{i=0}^{4} p_{4i} = 1.$$

The car park of Center for emergency medical aid in the town of Ruse is relatively new, therefore $r = 0{,}1$ and $q = 0{,}4$ Hence, the values found for the transition probabilities are as follows:

$$\begin{array}{lllll} p_{00} = 0{,}6561, & p_{01} = 0{,}2916, & p_{02} = 0{,}0486, & p_{03} = 0{,}0036, & p_{04} = 0{,}0001, \\ p_{10} = 0{,}2916, & p_{11} = 0{,}5346, & p_{12} = 0{,}1566, & p_{13} = 0{,}0166, & p_{14} = 0{,}0006, \\ p_{20} = 0{,}1296, & p_{21} = 0{,}4176, & p_{22} = 0{,}3796, & p_{23} = 0{,}0696, & p_{24} = 0{,}0036, \\ p_{30} = 0{,}0576, & p_{31} = 0{,}2656, & p_{32} = 0{,}4176, & p_{33} = 0{,}2376, & p_{34} = 0{,}0216, \\ p_{40} = 0{,}0256, & p_{41} = 0{,}1536, & p_{42} = 0{,}3456, & p_{43} = 0{,}3456, & p_{44} = 0{,}1296. \end{array} \quad (10)$$

Based on the values calculated for the transition probabilities (10), system (11) is solved

$$\begin{cases} (p_{00}-1).p_0 + p_{10}.p_1 + p_{20}.p_2 + p_{30}.p_3 + p_{40}.p_4 = 0 \\ p_{01}.p_0 + (p_{11}-1).p_1 + p_{21}.p_2 + p_{31}.p_3 + p_{41}.p_4 = 0 \\ p_{02}.p_0 + p_{12}.p_1 + (p_{22}-1).p_2 + p_{32}.p_3 + p_{42}.p_4 = 0 \\ p_{03}.p_0 + p_{13}.p_1 + p_{23}.p_2 + (p_{33}-1).p_3 + p_{43}.p_4 = 0 \\ p_0 + p_1 + p_2 + p_3 + p_4 = 1 \end{cases} \quad (11)$$

and the values of the final probabilities are obtained:

$$p_0 = 0{,}4096, \; p_1 = 0{,}4096, \; p_2 = 0{,}1536, \; p_3 = 0{,}0256, \; p_4 = 0{,}0016. \quad (12)$$

The calculations have been made using the MAPLE software product [7].

CONCLUSIONS

The analysis of the results (12) shows that the probability for at least 3 cars in the investigated system to be operational ($p_1 + p_0$) is 0.8192, which means that the system will be able to perform its functions flawlessly and the patients of the Centre for emergency medical aid in Ruse will receive the necessary help practically without having to wait in the system queue. Their lives will not be endangered due to a delay of the team caused by servicing a previous request.

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ИЗСЛЕДВАНЕ НА ВЪЗМОЖНОСТИТЕ ЗА УСТАНОВЯВАНЕ НА СТАЦИОНАРЕН РЕЖИМ В СИСТЕМА ОТ АВТОМОБИЛИ

Веселина Евтимова

Русенски университет „Ангел Кънчев”

***Резюме**: В настоящата работа са изследвани възможностите за установяване на стационарен режим в система от автомобили. Пресметнати са преходните вероятности от състояние в състояние, вероятностите на задръжка на системата в дадено състояние, както и вероятностите в стационарен режим. Показан е графът на състоянията на система от четири автомобила в автостопанството на Центъра за спешна медицинска помощ в град Русе. Намерени са финалните вероятности на работата на системата и е направен анализ на тези резултати.*

***Ключови думи:** Математическо моделиране, Случайни процеси, Марковски процеси, Спешна медицинска помощ*

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