PROCEEDINGS

of the Union of Scientists - Ruse

Book 5

Mathematics, Informatics and Physics

Volume 8, 2011

RUSE

ISSN 1311-9974

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

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ANALYSIS OF THE IMPACT OF THE INCOMING CALLS FLOW INTENSITY ON SOME BASIC CHARACTERISTICS OF AN EMERGENCY AID CENTRE

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Abstract: *The emergency aid centre in Ruse is modelled as a queuing system with an unlimited number of places in the system queue. The influence of the incoming calls flow intensity on some major characteristics of the system is examined. The paper reaches some conclusions about the number of emergency teams needed in order to provide prompt emergency service for the patients at a definite intensity of the incoming calls flow. The results of the simulation of priority service of incoming calls are analysed and the respective conclusions are made.*

Keywords: *Mathematical Modelling, Queuing Theory, Emergency Medical Care, Priority Service.*

The development of technically feasible and socially effective systems of medical service, which form the basis of public health care, is a very suitable medium for the application of the investigation of operations methods. A great number of problems which come up in these cases can be solved with the help of analytical methods; the efficiency of these methods depends on the researcher's ability to form a good enough mathematical model which takes into account as many factors as possible and describes the system as comprehensively as possible.

The paper examines an emergency aid centre (EAC) where the intensity of incoming calls is λ calls per hour. The medical team and the ambulance car accept a new call when they get back to the coordination centre or immediately after they have attended to the previous call [7]. The observations and the analysis of the performance of the EAC in Ruse show that the average time needed to provide the medical service for each call is 28 minutes, i.e. the intensity of the service flow is $\mu = 2.14$ calls per hour and the EAC uses n=8 ambulance cars (and the same number of medical teams).

EAC is examined as a queuing system (QS) with an unlimited number of places in the system queue [2]. The transport-medical teams are interchangeable. In order to establish a steady working mode which can guarantee that all patients (incoming calls) in the system will get proper service, the value $\frac{\rho}{n}$, where $\rho = \frac{\lambda}{\mu}$, should be less than 1 [10]. This is a necessary requirement to limit the queue of waiting patients. When $\frac{\rho}{n}$>1 such a mode does not exist.

The graph of the states of the system is shown in fig.1 [1].

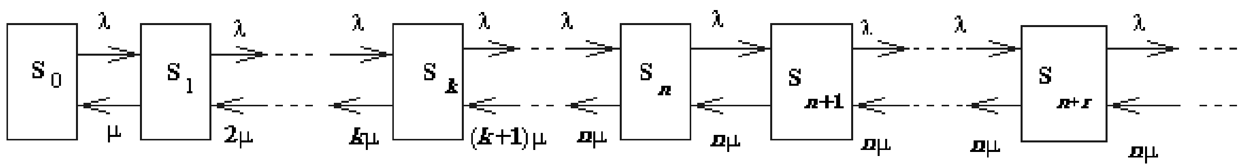

Fig.1 Graph of the states of the system

The separate states correspond to the following meanings:

S_0 - all teams are free;

S_1 - one team is busy, the rest are free;

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S_n - all *n* teams are busy, but there is no queue;

S_{n+1} - all *n* teams are busy and one incoming call is waiting in the queue;

…………………………………………………….

S_{n+r} - all *n* teams are busy and *r* incoming calls are waiting in the queue.

In [10] it has been proved that the condition for the final probabilities to exist is expressed with $\frac{\rho}{n} < 1$. Then the final probabilities are determined according to the Erlang formulae [3]:

(1) $$P_0 = \left[1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + ... + \frac{\rho^n}{n!} + \frac{\rho^{n+1}}{n! * (n-\rho)}\right]^{-1}$$

………………………………………..

(2) $$P_k = \frac{\rho^k}{k!} * P_0,\ k=1,2,\dots,n;$$

……………………………………….

(3) $$P_{n+r} = \frac{\rho^{n+r}}{n^r * n!} * P_0,\ r=1,2,\dots.$$

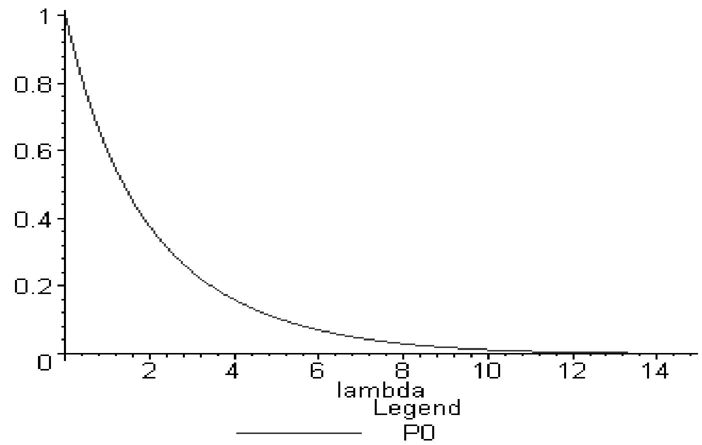

Fig. 2. Dependence of P_0 on λ

Fig. 2 illustrates that when the intensity of the incoming calls flow goes up the probability for the system to have no waiting calls goes down. When the value of λ reaches ten, this probability approaches zero.

The parameters expressing the system's load can be calculated according to the following formulae [8]:

- average length of the queue

(4) $$L_0 = \frac{\rho^{n+1}}{n * n! * \left(1 - \frac{\rho}{n}\right)^2};$$

- average number of patients in the system

(5) $$L_c = L_0 + \rho;$$

- average time the incoming call spends in the system's queue

(6) $W_0 = \dfrac{L_0}{\lambda}$;

- average time the incoming call spends in the system

(7) $W_c = \dfrac{L_c}{\lambda}$.

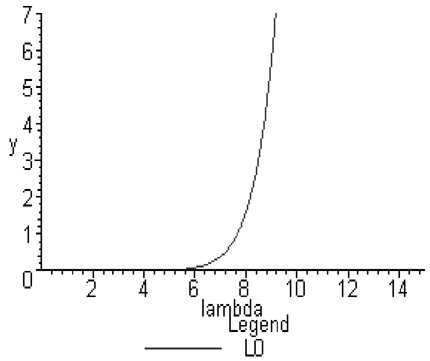

Fig. 3. Dependence of L_0 on λ

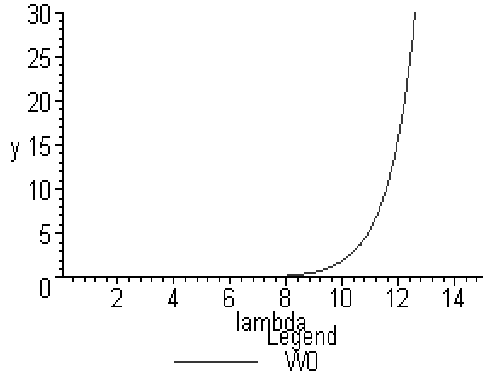

Fig. 4. Dependence of W_0 on λ

Fig.3 demonstrates that when the incoming calls flow intensity reaches the value of 8, the length of the waiting patients queue begins to grow unrestrictedly; this means that the available number of 8 teams is not enough to respond to the incoming calls of this level of intensity and therefore, it is necessary to ask other emergency medical centres for assistance. Such uncontrollably great number of incoming calls for the existing EAC may become a fact at times of traffic accidents, natural adversities or industrial disasters [4].

The dependence of the time each call spends in the system queue as a function of the incoming calls intensity is analogical (fig. 4).

So, the conclusion is that the ambulance cars available at the EAC in Ruse are not enough to provide adequate emergency medical service when the number of incoming calls reaches 8 or more per hour. It is necessary to buy more vehicles or to ask the nearest emergency centres for help. If there is a chance increase in the incoming calls flow intensity, assistance from the nearest emergency aid centres is necessary. However, if the increase in the incoming calls flow intensity is a permanent tendency, new ambulance cars should be purchased.

The present paper deals with the influence of the incoming calls flow intensity on some characteristics of the system when it accepts priority emergency medical service.

In some countries in the world, health services are provided according to the rank or income of the citizens – those with higher rank or higher income are provided with different services from the citizens with lower rank or lower income. The paper examines a case with two levels of priority, and when the priority service does not interrupt the realization of the current emergency medical call.

It is assumed that the incoming calls flow is a Poisson stream, and the service time for each of the *m*-number of queues of waiting patients (depending on the chosen number of priorities) is distributed according to a random law. $M_k[t]$ and $D_k[t]$ respectively, denote the mathematical expectation and the variance for the *k*-team, and λ_k denotes the intensity of the incoming calls flow for the same team.

The influence of the intensity of the incoming calls flow on the basic characteristics of the system is studied. With one team providing the emergency service, their values are determined by the following dependencies [9]:

$W_0^{(k)}$ - average time the patient spends waiting in the *κ*-team queue

(8) $$W_0^{(k)} = \frac{\sum_{i=1}^{m} \lambda_i * (M_k^2[t] + D_k[t])}{2*(1-S_{k-1})*(1-S_k)};$$

$W_c^{(k)}$ - average time the patient spends with the system's *κ*-team

(9) $$W_c^{(k)} = W_0^{(k)} + M_k[t];$$

$L_0^{(k)}$ - average number of patients in the *κ*-team service queue

(10) $$L_0^{(k)} = \lambda_k * W_0^{(k)};$$

$L_c^{(k)}$ - average number of patients with the *κ*-team;

(11) $L_c^{(k)} = L_0^{(k)} + \rho_k$, where $\rho_k = \lambda_k * M_k[t]$;

(12) $S_k = \sum_{i=1}^{k} \rho_i < 1$, *κ=1,2,…,m.*

(13) $S_0 \equiv 0$.

The average time the patient spends in the queue does not depend on the kind of priority they have and is equal to

(14) $W_0 = \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} W_0^{(k)}$, where $\lambda = \sum_{i=1}^{m} \lambda_i$.

The average time the incoming calls stay in the system, is determined by the dependency

(15) $$W_c = \sum_{k=1}^{m} \frac{\lambda_k}{\lambda} W_c^{(k)}.$$

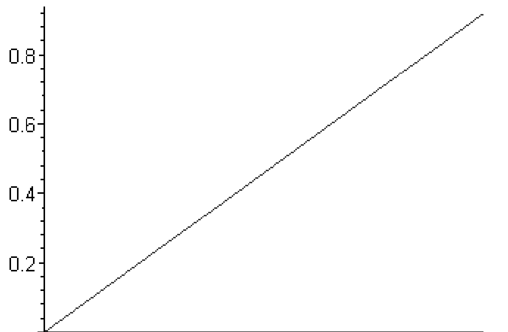

Fig.5. Dependency of $W_0^{(1)}$ on λ with one team providing the service

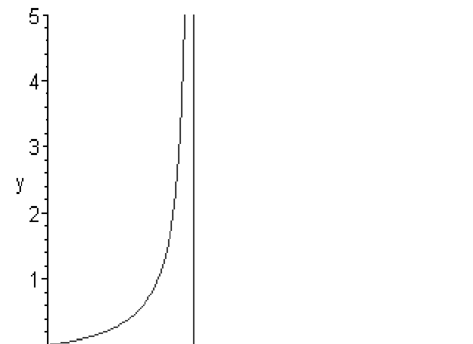

Fig.6. Dependency of $W_0^{(2)}$ on λ with one team providing the service

The results of the examined case with one team are as follows:

$W_0^{(1)}$ - the average time the call stays in the system's queue increases linearly with the increase in the incoming calls flow intensity λ (fig.5). The average time $W_0^{(2)}$ the calls of second (lower) priority stay in the queue increases unrestrictedly when the calls intensity increases to 2.07 incoming calls per hour (fig.6); so with the introduction of priority service by one team the lower priority calls cannot get the service needed in due time, which can be fatal to the patient. It is possible to provide adequate emergency medical service without threatening the patient's life if the intensity of the incoming calls of second (lower) priority does not exceed 1 call per hour [5].

In the simulation of the emergency aid centre as a multichannel queuing system with a priority channel, in the case which is examined (with two teams), it is supposed that that the time needed to provide the service will be the same for the calls from the two priority categories and the distribution of the time spent providing the service by all the teams (*m* is their number), is according to an exponential law with average service intensity μ. Incoming calls with *k*-priority is distributed in time according to Poisson's law and is characterized by average frequency λ_k *(κ=1,2,…,m)*. So for the k-queue of waiting patients the average time the patient (the call) stays in the queue is [9]:

(16) $$W_0^{(k)} = \frac{M[\xi]}{(1-S_{k-1})*(1-S_k)}, \quad k=1,2,\ldots,m,$$

where $S_0 \equiv 0,\ S_k \frac{\lambda_i}{m*\mu} < 1$ for *κ=1,2,…m*,

(17) $$M\left[\xi\right] = \frac{1}{\mu*m*\left[\rho^{-m}*(m-\rho)*(m-1)!*\sum_{j=0}^{m-1}\frac{\rho^j}{j!}+1\right]}, \ \rho = \frac{\lambda}{\mu}.$$

For the examined case it is supposed that there are two queues, the first one being of higher priority and with intensity $\lambda_1 = 0.5,$ and the second one – with intensity $\lambda_2 = \lambda - \lambda_1$, where λ is the intensity of the incoming calls flow for the whole system. The average service time for the incoming calls is $\overline{t_{serv}}$ =28 minutes. It can be accepted that it is the same for the incoming calls from the two groups and therefore, the service intensity is $\mu_1 = \mu_2 = \frac{1}{\overline{t_{serv}}} = 2.14$ calls per hour.

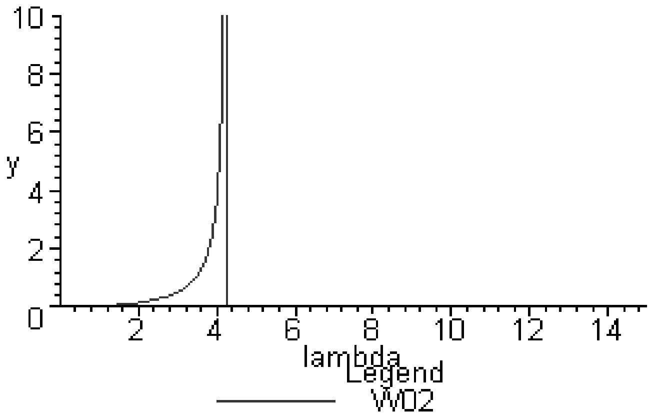

Fig.7. Dependence of $W_0^{(2)}$ on λ with two service teams

The MAPLE program product is used to draw the graphs.

For two service teams the results are shown in fig.7: the intensity of the incoming calls of second priority λ_2 should not exceed 2, as according to this graph the time the calls stay waiting in the queue is practically zero, which means that the patients will get the medical service they need without having to wait in the system's queue. When the intensity of the incoming calls is larger than 3, the situation becomes intolerable considering the service provided by the emergency aid system [6], as the time spent waiting in the queue of the system increases unrestrictedly. For λ_2 approaching 4.28 calls per hour, $W_0^{(2)}$ approaches infinity.

REFERENCES

[1] Chinnaswamy, M., M. Kamath, On queuing network models of service systems, in: IE, Research Conference, Atlanta, Georgia, May 2005.

[2] Edlich R.F., My Revolutionary Adventures in the Development of Modern Emergency Medical Systems in Our Country, Journal of Emergency Medicine, Volume 34, Issue 4, May 2008, pp. 359-365.

[3] Hillier F., G. Lieberman, Introduction to Operation Research (eighth ed.), NY, USA, McGraw Hill, 2005.

[4] Hirokazu M., Advancement of postwar system of emergency medical services in Japan (in Japanese), J Jpn Assoc Acute Med 11, 2000, pp. 311–322.

[5] Hladki W., J. Andres, M. Trybus, R. Drwila, Emergency medicine in Poland, Resuscitation, Volume 75, Issue 2, November 2007, pp. 213-218.

[6] Lambert T.E., P. B. Meyer, New and Fringe Residential Development and Emergency Medical Services Response Times in the United States, State and Local Government Review, Volume 40, Issue 2, August 1, 2008, pp. 115 - 124.

[7] Lewin M.R., Sh. Hori, N. Aikawa, Emergency medical services in Japan: An opportunity for the rational development of pre-hospital care and research, Journal of Emergency Medicine, Volume 28, Issue 2, February 2005, pp. 237-241.

[8] Mitkov A., D. Minkov, Mathematical Methods of Engineering Research, Ruse, 1985.

[9] Taha H., Introduction to Operation Research, Volume 2, Moscow, „Mir”, 1985.

[10] Ventcel E.S., Operation Research, Moscow, “Sovetskoe radio”, 1972.

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ИЗСЛЕДВАНЕ НА ВЛИЯНИЕТО НА ИНТЕНЗИВНОСТТА НА ВХОДЯЩИЯ ПОТОК ОТ ЗАЯВКИ ВЪРХУ НЯКОИ ОСНОВНИ ХАРАКТЕРИСТИКИ НА ЦЕНТЪР ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ

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Резюме: *Центърът за спешна медицинска помощ в град Русе е моделиран като система за масово обслужване с неограничен брой места в опашката на системата. Изследвано е влиянието на интензивността на входящия поток заявки върху някои основни характеристики на системата. Направени са изводи за необходимия брой екипи, за да може да се осигури своевременно обслужване на пациентите при определена интензивност на входящия поток от заявки. Анализирани са и резултатите от симулацията на въвеждането на приоритетно обслужване на заявки и са направени съответните изводи.*

Ключови думи: *Математическо моделиране, Теория на масовото обслужване, Спешна медицинска помощ, Приоритетно обслужване.*

ISSN 1311-9974

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