PROCEEDINGS

of the Union of Scientists - Ruse

Book 5

Mathematics, Informatics and Physics

Volume 7, 2010

RUSE

ISSN 1311-9184

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SERIES 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 7

CONTENTS

VISHNE IDENTITIES FOR $M_2(G)$ AND THEIR COMPUTER REALIZATION BY *MATHEMATICA*

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Abstract: *Vishne gave in [9] the explicit form of two identities of degree 8 for the matrix algebra $M_2(G)$, where G is the Grassmann algebra. In the paper a programme in Mathematica is used for proving these two identities.*
Keywords: *Grassmann algebra, standard polynomial, Vishne identities.*

PRELIMINARIES

Let G denote the infinite dimensional Grassmann algebra, namely

$$G = G(V) = K\langle v_1, v_2, \ldots \mid v_i v_j + v_j v_i = 0 i, j = 1,2,\ldots\rangle.$$

The field K has a characteristic zero. The algebra G' (without 1) has a basis $v_{i_1} v_{i_2} \ldots v_{i_k}$, where $1 \le i_1 < i_2 \ldots < i_k$. The elements v_i are called generators of G' while the elements $v_{i_1} v_{i_2} \ldots v_{i_k}$ for $1 \le i_1 < i_2 \ldots < i_k$ are called basic monomials of G'. For $G = G' \cup 1$ a generator is 1 as well. The algebras G and G' are PI-equivalent (they satisfy one and the same identities).

The algebra G is in the mainstream of resent research in PI theory. Its importance is connected with the structure theory for the T-ideals of identities of associative algebras developed by Kemer. In [4, Theorem 1.2] he proved that any T-prime T-ideal can be obtained as the T-ideal of identities of one of three algebras, one of which is the algebra $M_n(G)$.

Well known facts concerning the algebra G are the following:

Proposition 1 [5, Corollary, p. 437] *The T-ideal $T(G)$ is generated by the identity* $[x_1, x_2, x_3] = 0$.

Proposition 2 [1, Lemma 6.1] *The algebra G satisfies $S_n(x_1, \ldots, x_n)^k = 0$ for all* $n, k \ge 2$.

Proposition 3 [2, Exercise 5.3] *For $G_k = G(V_k)$ over k-dimensional vector space V_k all identities follow from the identity $[x_1, x_2, x_3] = 0$ and the standard identity*

$$S_{2p}(x_1, \ldots, x_{2p}) = \sum_{\sigma \in Sym(2p)} (-1)^{\sigma} x_{\sigma(1)} \ldots x_{\sigma(2p)} = 0,$$

where p is the minimal integer such that $2p > k$.

Proposition 4 [3, Theorem 3.5] *Let K be an infinite field. A basis of the identities of G_{2k} is given by the polynomials*

$$[x_1, x_2, x_3] = 0, \quad [x_1, x_2] \ldots [x_{2k+1}, x_{2k+2}] = 0.$$

Some facts concerning the identities for the matrix algebra $M_2(G)$ were considered in [7]. There it was proved that the algebra $M_n(G)$ has no identities of degree

$4n-2$. In [8] Vishne described an efficient way to use the $Sym(n)$-module structure of the ideal of multilinear identities in the computation of such dentities of degree n of a given algebra. The method was used to show that $M_2(G)$ has identities of degree 8, but of no smaller degree. Two explicit identities of degree 8 were given. Details on the method used one could see in [8]. Here we give the definition of the identities as done in [8].

The rank and the dimension of the ideal I of 8-degree multilinear identities of $M_2(G)$ were computed and the irreducible representations of the symmetric group $Sym(8)$ were considered. There were found 15 non-zero components of I and in four cases (corresponding to the partitions $8 \vdash (2,2,1,1,1,1)$, $8 \vdash (2,2,2,1,1)$, $8 \vdash (3,1,1,1,1,1)$ and $8 \vdash (4,1,1,1,1)$ the explicit identities could be presented.

This could be done using the relationship of the $Sym(n)$-module and the GL_m-module structures of the considered ideal for a partition $\lambda \vdash (\lambda_1, \ldots, \lambda_m)$ of n [9].

Here we give only the definition of the highest weight vector of the irreducible GL_m-module. It is a non-zero element

$$f_\lambda(x_1,\ldots,x_m) = \left(\prod_{i=1}^{r} S_{q_i}\left(x_1,\ldots,x_{q_i}\right)\right) \sum_{\sigma\in Sym(n)} \alpha_\sigma \sigma$$

for some $\alpha_\sigma \in K$, where $q_1,\ldots,q_r$ are the lengths of the columns of the Young diagram related to λ.

For example for a partition $8 \vdash (2,2,1,1,1,1)$ the lengths of the columns of the corresponding Young diagram are 6 and 2. This explains the construction of the multilinear polynomials $T_1(x_1,\ldots,x_6;y_1,y_2)$ and $T_2(x_1,\ldots,x_5;y_1,y_2,y_3)$ done by Vishne and given in the forthcoming exposition.

A *pattern* is a finite sequence of the letter A,B. If π is a pattern with a appearances of A and b of B, we denote by $\pi(x_1,\ldots,x_a;y_1,\ldots,y_b)$ the product of variables where the $x's$ and $y's$ are combined according to π. For example $ABBA(x_1,x_2;y_1,y_2) = x_1y_1y_2x_2$. A coefficient in front of a pattern π means that the monomial should be multiplied by that coefficient.

Now let

$$P_\pi^+ = \sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)\pi(x_{\sigma(1)},\ldots,x_{\sigma(a)};y_{\tau(1)},\ldots,y_{\tau(b)}),$$

$$P_\pi^- = \sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)sign(\tau)\pi(x_{\sigma(1)},\ldots,x_{\sigma(a)};y_{\tau(1)},\ldots,y_{\tau(b)}).$$

Let

$$\mathrm{P} = \begin{pmatrix} +AAAABAAB, & +AABBAAAA, & -AABAAAB, \\ -AAAABBAA, & -BAABAAAA, & +BAAAABAA \end{pmatrix}.$$

The component of I in the representation $8 \vdash (2,2,1,1,1,1)$ contains (and is thus generated by) $\sum_{\pi\in\mathrm{P}} P_\pi^-(x_1,\ldots,x_6;y_1,y_2)$. Similarly, the component of I in the representation $8 \vdash (3,1,1,1,1,1)$ contains $\sum_{\pi\in\mathrm{P}} P_\pi^+(x_1,\ldots,x_6;y_1,y_2)$. In the sum

$$T_1(x_1,\ldots,x_6;y_1,y_2)=\sum_{\pi\in\mathsf{P}}(P_\pi^- + P_\pi^+) \qquad (1)$$

only the monomials with y_1 preceding y_2 appear. On the other hand the original two identities are given back by $T_1(\ldots;y_1,y_2)\pm T_1(\ldots;y_2,y_1)$.

The same phenomenon happens for another couple of representations.

Let

$$\mathsf{PP}=\begin{pmatrix} -AAABAABB, & -AABBAABA, & +ABBAABAA, \\ +AAABBAAB, & +AABAABBA, & -ABAABBAA \\ -ABBAAAAB, & +BAABBAAA, & -BAAAABBA \\ +ABAAAABB, & -BBAABAAA, & +BBAAAABA \end{pmatrix}.$$

The component in $8\vdash(2,2,2,1,1)$ contains $\sum_{\pi\in\mathsf{PP}}P_\pi^-(x_1,\ldots,x_5;y_1,y_2,y_3)$ and the component in $8\vdash(4,1,1,1,1)$ contains $\sum_{\pi\in\mathsf{PP}}P_\pi^+(x_1,\ldots,x_5;y_1,y_2,y_3)$. Again

$$T_2(x_1,\ldots,x_5;y_1,y_2,y_3)=\sum_{\pi\in\mathsf{PP}}(P_\pi^- + P_\pi^+) \qquad (2)$$

has only the monomials in which the order of y_1,y_2,y_3 is even and $T_2(\ldots;y_1,y_2,y_3)\pm T_2(\ldots;y_3,y_2,y_1)$ gives the original identities.

Theorem 1 [8, Corollary 4.2] *T_1 and T_2 are multilinear identities of degree 8 of $M_2(G)$.*

COMPUTER REALIZATIONS OF THE IDENTITIES

Calculations in the Grassmann algebra are not done easily. We considered the problem of finding a computer realization of the multiplication in it. Using the 1-1 correspondence between the integer numbers from 0 to 2^n and the basic elements of a Grassmann algebra over a n-dimensional vector space a programme in *Mathematica* was written [6] using which we could prove in a computer way Vishne identities. For a guide book in the system *Mathematica* we use [9]. We point that the programme considers a finite dimensional Grassmann algebra. But this is not a limitation. Knowing the degree of the polynomial, say n, it is enough to work in the algebra G_n.

Firstly we introduce the polynomial $T_1=T1(x1,x2,x3,x4,x5,x6,y1,y2)$ according to (1) done by *Mathematica*. We use the notation $\otimes$ for the Grassmann multiplication.

It is easier to present the polynomial in parts corresponding to the parts of the pattern P.

Let we consider $AAAABAAB$. The corresponding part of the polynomial $T_1=T1(x1,x2,x3,x4,x5,x6,y1,y2)$ is denoted as $A[x1,x2,x3,x4,x5,x6,y1,y2]$. Then we have:

$S2[x1,x2] := x1\otimes x2 - x2\otimes x1$;

$S3[x1,x2,x3] := S2[x1,x2]\otimes x3 + S2[x2,x3]\otimes x1 + S2[x3,x1]\otimes x2$;

$S4[x1, x2, x3, x4] := S3[x1, x2, x3] \otimes x4 - S3[x2, x3, x4] \otimes x1 +$
$S3[x3, x4, x1] \otimes x2 - S3[x4, x1, x2] \otimes x3;$

$A1[x1, x2, x3, x4, y1] := S4[x1, x2, x3, x4] \otimes y1;$

$A2[x1, x2, x3, x4, x5, y1] := A1[x1, x2, x3, x4, y1] \otimes x5 +$
$A1[x2, x3, x4, x5, y1] \otimes x1 + A1[x3, x4, x5, x1, y1] \otimes x2 +$
$A1[x4, x5, x1, x2, y1] \otimes x3 + A1[x5, x1, x2, x3, y1] \otimes x4;$

$A3[x1, x2, x3, x4, x5, x6, y1] := A2[x1, x2, x3, x4, x5, y1] \otimes x6 -$
$A2[x2, x3, x4, x5, x6, y1] \otimes x1 + A2[x3, x4, x5, x6, x1, y1] \otimes x2 -$
$A2[x4, x5, x6, x1, x2, y1] \otimes x3 + A2[x5, x6, x1, x2, x3, y1] \otimes x4 -$
$A2[x6, x1, x2, x3, x4, y1] \otimes x5;$

$A[x1, x2, x3, x4, x5, x6, y1, y2] := A3[x1, x2, x3, x4, x5, x6, y1] \otimes y2;$

For $AABBAAAA$ the corresponding polynomial is $B[x1, x2, x3, x4, x5, x6, y1, y2]$.

We construct

$B1[x1, x2, y1, y2] := (S2[x1, x2] \otimes y1) \otimes y2;$

$B2[x1, x2, x3, y1, y2] := B1[x1, x2, y1, y2] \otimes x3 +$
$B1[x2, x3, y1, y2] \otimes x1 + B1[x3, x1, y1, y2] \otimes x2;$

$B3[x1, x2, x3, x4, y1, y2] := B2[x1, x2, x3, y1, y2] \otimes x4 -$
$B2[x2, x3, x4, y1, y2] \otimes x1 + B2[x3, x4, x1, y1, y2] \otimes x2 -$
$B2[x4, x1, x2, y1, y2] \otimes x3;$

$B4[x1, x2, x3, x4, x5, y1, y2] := B3[x1, x2, x3, x4, y1, y2] \otimes x5 +$
$B3[x2, x3, x4, x5, y1, y2] \otimes x1 + B3[x3, x4, x5, x1, y1, y2] \otimes x2 +$
$B3[x4, x5, x1, x2, y1, y2] \otimes x3 + B3[x5, x1, x2, x3, y1, y2] \otimes x4;$

$B[x1, x2, x3, x4, x5, x6, y1, y2] := B4[x1, x2, x3, x4, x5, y1, y2] \otimes x6 -$
$B4[x2, x3, x4, x5, x6, y1, y2] \otimes x1 + B4[x3, x4, x5, x6, x1, y1, y2] \otimes x2 -$
$B4[x4, x5, x6, x1, x2, y1, y2] \otimes x3 + B4[x5, x6, x1, x2, x3, y1, y2] \otimes x4 -$
$B4[x6, x1, x2, x3, x4, y1, y2] \otimes x5;$

For $AABAAAB$ the corresponding polynomial is $CH[x1, x2, x3, x4, x5, x6, y1, y2]$.

We have

$CH1[x1,x2,y1] := S2[x1,x2] \otimes y1$;
$CH2[x1,x2,x3,y1] := CH1[x1,x2,y1] \otimes x3 + CH1[x2,x3,y1] \otimes x1 +$
$CH1[x3,x1,y1] \otimes x2$;

$CH3[x1,x2,x3,x4,y1] := CH2[x1,x2,x3,y1] \otimes x4 -$
$CH2[x2,x3,x4,y1] \otimes x1 + CH2[x3,x4,x1,y1] \otimes x2 -$
$CH2[x4,x1,x2,y1] \otimes x3$;

$CH4[x1,x2,x3,x4,x5,y1] := CH3[x1,x2,x3,x4,y1] \otimes x5 +$
$CH3[x2,x3,x4,x5,y1] \otimes x1 + CH3[x3,x4,x5,x1,y1] \otimes x2 +$
$CH3[x4,x5,x1,x2,y1] \otimes x3 + CH3[x5,x1,x2,x3,y1] \otimes x4$;

$CH5[x1,x2,x3,x4,x5,x6,y1] := CH4[x1,x2,x3,x4,x5,y1] \otimes x6 -$
$CH4[x2,x3,x4,x5,x6,y1] \otimes x1 + CH4[x3,x4,x5,x6,x1,y1] \otimes x2 -$
$CH4[x4,x5,x6,x1,x2,y1] \otimes x3 + CH4[x5,x6,x1,x2,x3,y1] \otimes x4 -$
$CH4[x6,x1,x2,x3,x4,y1] \otimes x5$;

$CH[x1,x2,x3,x4,x5,x6,y1,y2] := CH5[x1,x2,x3,x4,x5,x6,y1] \otimes y2$;

The corresponding part to $AAAABBAA$ is denoted as $DH[x1,x2,x3,x4,x5,x6,y1,y2]$. We get

$DH1[x1,x2,x3,x4,y1,y2] := (S4[x1,x2,x3,x4] \otimes y1) \otimes y2$;

$DH2[x1,x2,x3,x4,x5,y1,y2] := DH1[x1,x2,x3,x4,y1,y2] \otimes x5 +$
$DH1[x2,x3,x4,x5,y1,y2] \otimes x1 + DH1[x3,x4,x5,x1,y1,y2] \otimes x2 +$
$DH1[x4,x5,x1,x2,y1,y2] \otimes x3 + DH1[x5,x1,x2,x3,y1,y2] \otimes x4$;

$DH[x1,x2,x3,x4,x5,x6,y1,y2] := DH2[x1,x2,x3,x4,x5,y1,y2] \otimes x6 -$
$DH2[x2,x3,x4,x5,x6,y1,y2] \otimes x1 + DH2[x3,x4,x5,x6,x1,y1,y2] \otimes x2 -$
$DH2[x4,x5,x6,x1,x2,y1,y2] \otimes x3 + DH2[x5,x6,x1,x2,x3,y1,y2] \otimes x4 -$
$DH2[x6,x1,x2,x3,x4,y1,y2] \otimes x5$;

The part $BAABAAAA$ gives rise to $EH[x1,x2,x3,x4,x5,x6,y1,y2]$. Thus

$EH1[x1,x2,y1,y2] := ((y1 \otimes x1) \otimes x2) \otimes y2 - ((y1 \otimes x2) \otimes x1) \otimes y2$;
$EH2[x1,x2,x3,y1,y2] := EH1[x1,x2,y1,y2] \otimes x3 +$
$EH1[x2,x3,y1,y2] \otimes x1 + EH1[x3,x1,y1,y2] \otimes x2$;

$EH3[x1, x2, x3, x4, y1, y2] := EH2[x1, x2, x3, y1, y2] \otimes x4 -$
$EH2[x2, x3, x4, y1, y2] \otimes x1 + EH2[x3, x4, x1, y1, y2] \otimes x2 -$
$EH2[x4, x1, x2, y1, y2] \otimes x3;$

$EH4[x1, x2, x3, x4, x5, y1, y2] := EH3[x1, x2, x3, x4, y1, y2] \otimes x5 +$
$EH3[x2, x3, x4, x5, y1, y2] \otimes x1 + EH3[x3, x4, x5, x1, y1, y2] \otimes x2 +$
$EH3[x4, x5, x1, x2, y1, y2] \otimes x3 + EH3[x5, x1, x2, x3, y1, y2] \otimes x4;$

$EH[x1, x2, x3, x4, x5, x6, y1, y2] := EH4[x1, x2, x3, x4, x5, y1, y2] \otimes x6 -$
$EH4[x2, x3, x4, x5, x6, y1, y2] \otimes x1 + EH4[x3, x4, x5, x6, x1, y1, y2] \otimes x2 -$
$EH4[x4, x5, x6, x1, x2, y1, y2] \otimes x3 + EH4[x5, x6, x1, x2, x3, y1, y2] \otimes x4 -$
$EH4[x6, x1, x2, x3, x4, y1, y2] \otimes x5;$

The polynomial $FH[x1, x2, x3, x4, x5, x6, y1, y2]$ is related to $BAAAABAA$ and is constructed in a similar way, namely

$FH1[x1, x2, y1] := (y1 \otimes x1) \otimes x2 - (y1 \otimes x2) \otimes x1;$

$FH2[x1, x2, x3, y1] := FH1[x1, x2, y1] \otimes x3 + FH1[x2, x3, y1] \otimes x1 +$
$FH1[x3, x1, y1] \otimes x2;$

$FH3[x1, x2, x3, x4, y1] := FH2[x1, x2, x3, y1] \otimes x4 -$
$FH2[x2, x3, x4, y1] \otimes x1 + FH2[x3, x4, x1, y1] \otimes x2 -$
$FH2[x4, x1, x2, y1] \otimes x3;$

$FH4[x1, x2, x3, x4, y1, y2] := FH3[x1, x2, x3, x4, y1] \otimes y2;$

$FH5[x1, x2, x3, x4, x5, y1, y2] := FH4[x1, x2, x3, x4, y1, y2] \otimes x5 +$
$FH4[x2, x3, x4, x5, y1, y2] \otimes x1 + FH4[x3, x4, x5, x1, y1, y2] \otimes x2 +$
$FH4[x4, x5, x1, x2, y1, y2] \otimes x3 + FH4[x5, x1, x2, x3, y1, y2] \otimes x4;$

$FH[x1, x2, x3, x4, x5, x6, y1, y2] := FH5[x1, x2, x3, x4, x5, y1, y2] \otimes x6 -$
$FH5[x2, x3, x4, x5, x6, y1, y2] \otimes x1 + FH5[x3, x4, x5, x6, x1, y1, y2] \otimes x2 -$
$FH5[x4, x5, x6, x1, x2, y1, y2] \otimes x3 + FH5[x5, x6, x1, x2, x3, y1, y2] \otimes x4 -$
$FH5[x6, x1, x2, x3, x4, y1, y2] \otimes x5;$

At the end we form the entire polynomial

$$T1[x1,x2,x3,x4,x5,x6,y1,y2] := A[x1,x2,x3,x4,x5,x6,y1,y2] +$$
$$B[x1,x2,x3,x4,x5,x6,y1,y2] - CH[x1,x2,x3,x4,x5,x6,y1,y2] -$$
$$DH[x1,x2,x3,x4,x5,x6,y1,y2] - EH[x1,x2,x3,x4,x5,x6,y1,y2] +$$
$$FH[x1,x2,x3,x4,x5,x6,y1,y2];$$

In a similar way the recurrent construction of $T_2 = T2(x1,x2,x3,x4,x5,y1,y2,y3)$ is realized due to (2).

Then we set $n = 8$ in the *Mathematica* programme realized in [6]. The evaluation on $M_2(G)$ was done in two stages.

We use the operator **Random[type,{min,max}]** bringing out an arbitrary number of type Integer, Real or Complex. At the beginning we specify the type as **Real**. Due to the rounding done in calculations the needed result was not obtained.

Then for exact calculations we specify the type as **Integer**. For max=500 for example we have

$$For[i=1, i \le 2^8, i_{++}, \{a[i] = \mathrm{R}andom[\mathrm{I}nteger,\{0,500\}], b[i] = \mathrm{R}andom[\mathrm{I}nteger,\{0,500\}],$$
$$c[i] = \mathrm{R}andom[\mathrm{I}nteger,\{0,500\}], d[i] = \mathrm{R}andom[\mathrm{I}nteger,\{0,500\}]\}]$$

for a 2×2 matrix

$$x = \{\{Array[a,256], Array[b,256]\}, \{Array[c,256], Array[d,256]\}\}.$$

For the evaluation of the polynomial $T_1(x_1,\ldots,x_6,y_1,y_2)$ by Intel Pentium computer with 2GB RAM 40 minutes were needed.

The calculation of $T_2(x_1,\ldots,x_5,y_1,y_2,y_3)$ with random matrices took 70 minutes time.

Then we consider the general case with arbitrary matrices introducing a matrix variable x only as

$$x = \{\{Array[a,256], Array[b,256]\}, \{Array[c,256], Array[d,256]\}\}.$$

The possibilities of our Pentium computer were not enough for calculating the polynomials $T_1(x_1,\ldots,x_6,y_1,y_2)$ and $T_2(x_1,\ldots,x_5,y_1,y_2,y_3)$ in the general case.

We point that the identity $[x_1,x_2,x_3]^2 = 0$ for the upper triangular two by two matrices with entries from G_n for $n = 12$ was confirmed for 3 hours by Intel Celeron computer with 2GB RAM.

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ТЪЖДЕСТВА НА ВИШНЕ ЗА $M_2(G)$ И ТЯХНАТА КОМПЮТЪРНА РЕАЛИЗАЦИЯ ЧРЕЗ *MATHEMATICA*

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Резюме: *В своя работа [8] Вишне дава явната форма на две тъждества от степен* 8 *за матричната алгебра* $M_2(G)$*, където* G *е Грасмановата алгебра. В статията се използва програма на Mathematica за доказване на тези две тъждества.*

Ключови думи*: Грасманова алгебра, стандартен полином, тъждества на Вишне*

ISSN 1311-9184