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CONTENTS

A NOTE ON AVERAGING IN DIFFERENTIAL EQUATIONS WITH HUKUHARA DERIVATIVE AND DELAY

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Abstract: *The problem of averaging in differential equations with delay and multivalued right-hand side. is studied. Values of all functions are convex compact sets. The derivative is considered in Hukuhara sense. A very general scheme of averaging is investigated.*

Keywords: *differencial equation, averaging, multivalued map (function),Hukuhara derivative, delay, convex set, existence.*

Let $comp\ \ (R^n)\,(conv(R^n))$ be a space, consisting of all nonempty compact convex subsets of R^n having Hausdorff metric $H(.,.)$, defined by the formula

$$H(A,B) = \boldsymbol{min}\{r \geq 0 \mid A \subset S_r(B), B \subset S_r(A)\},$$

where $S_d(C)$ is a closed d - neighbourhood of a compact set $C \subset R^n$.

The sets $comp\ \ (R^n)$ and $conv(R^n)$ are complete and separable but they are not linear [1].

The next series of definitions introduces the basic notion of our work.

Definition 1 *The sum of two sets* $A, B \in comp(R^n)$ *is called the following set:* $C = A + B = \{c = a + b \mid a \in A, b \in B\}$.

Definition 2 $[2]$: *Hukuhara difference of two sets* $A, B \in conv(R^n)$ *(if it exists) is such a set* $C \in conv(R^n)$ *that* $A = B + C$. *We use the notation* $C = A \overset{h}{-} B$.

Definition 3 $[2]$: *A multivalued map* $F(.) : R^1 \to conv(R)$ *is called Hukuhara differentiable in the point* t_0 *if there exists a set* $D_h F(t_0) \in conv(R^n)$ *such that the limits* $\boldsymbol{lim}_{\Delta t \to 0+} \dfrac{F(t_0 + \Delta t) \overset{h}{-} F(t_0)}{\Delta t}$ *and* $\boldsymbol{lim}_{\Delta t \to 0+} \dfrac{F(t_0) \overset{h}{-} F(t_0 - \Delta t)}{\Delta t}$ *exist and are equal to* $D_h F(t_0)$. *The set* $D_h F(t_0)$ *is called Hukuhara derivative of* F *in the point* t_0.

We note that in the definition of Hukuhara derivative it is understood that the differences $F(t_0) \overset{h}{-} F(t_0 - \Delta t)$ and $F(t_0 + \Delta t) \overset{h}{-} F(t_0)$ exist for all Δt sufficiently small.

Differential equations with a multivalued right-hand side and multivalued solutions

$$D_h X(t) = F(t, X(t)),\ X(t_0) = X^0, \qquad (1)$$

where $F : R' \times conv(R^n) \to conv(R^n)$, $X : R' \to conv(R^n)$, $X^0 \in conv(R^n)$ are considered for the first time in [3, 4].

Definition 4 $[3,4]$*: A multivalued map* $X : R^1 \to conv(R^n)$ *is called a solution of the Cauchy problem* (1) *if it is absolutely continuous and satisfies* (1) *almost everywhere.*

In the above cited papers theorems are proved of existence and uniqueness of the solution, as well the equivalency of (1) with the integral equation

$$X(t) = X^0 + \int_{t_0}^{t} F(s, X(s))ds.$$

In [5] it was established that the bunch of the usual solutions of the differential inclusion

$$\dot{x} \in \Phi(t,x), \quad x(t_0) = x_0 \tag{2}$$

is contained in the multivalued solution of the differential equation with Hukuhara derivative

$$D_h X(t) = \Phi(t, X(t)), \quad X(t_0) = x_0,$$

which is generated by the differential inclusion (2).

In [6] the method of averaging was justified for the differential equation

$$D_h X(t) = \varepsilon F(t, X(t)).$$

In [7] differential inclusions with Hukuhara derivative were considered, in [8] the connection is given between differential equations and inclusions with Hukuhara derivative and quasidifferential equations in metric spaces. In [9] the averaging method is justified for differential equations with Hukuhara derivative and assymptotic small delay.

In [10] we consider differential equations with Hukuhara derivative without the assumption that the delay is assympotically small.

In this paper we justify the general scheme of partial averaging indifferential equations with Hukuhara derivative and delay.

It is obvious that the differential equations in R^n are obtained as a partial case of differential equations with Hukuhara derivative when the function $F(t,x)$ is singlevalued and $X^0 = \{x^0\}$. In connection with this all specific peculiarities of the solutions of differential equations with delay in the space R^n are valid for differential equations with Hukuhara derivative.

However, in differential equations with Hukuhara derivative own peculiarities may appear, connected with the multivaluality. For example, for the solutions $X_1(t)$ and $X_2(t)$, corresponding to the initial functions $\Phi_1(t)$ and $\Phi_2(t)$ the condition $X_1(t) \subset X_2(t)$ may hold in the interval $[t_1, t_2]$. This is not an analogue of the solutions' sticking as $X_1(t) \neq X_2(t)$.

Let us consider differential equations with delay

$$D_h X(t) = F(t, X(t), X(t-\tau_1(t)), \ldots, X(t-\tau_m(t))), \tag{3}$$

$$X(t) = \Phi(t), t \in E_{t_0},$$

where $E_{t_0} = \bigcup_{i=1}^{m} E_{t_0}^{(i)}$, $E_{t_0}^{(i)}$ are sets consisting of the points t_0 and those values $t - \tau_i(t)$ for which $t - \tau_i(t) < t_0$ when $t \geq t_0$, $\Phi : R^1 \to conv(R^n)$.

We proved in [10] that for the equation (3) theorems are valid for existence and continuous dependence of the initial functions which are analogous to the corresponding theorems for the equation in the space R^n [11,12,13].

The theorems are stated as follows:

Theorem 1 *Let* F *be a continuous function in a neighbourhood of the point* $(t_0, \Phi(t_0), \Phi(t_0 - \tau_1(t_0)),...,\Phi(t_0 - \tau_m(t_0)))$, *satisfying the Lipschitz condition in all variables starting with the second one, with a constant* λ*; let the initial function* $\Phi(t)$ *be continuous in* E_{t_0} *and all functions* $\tau_i(t)$ *be continuous for* $t_0 \le t \le t_0 + L$ $(L > 0)$ *and nonnegative. Then there exists a unique solution* $X(t)$ *of the basic initial problem for the equation* (3) *for* $t_0 \le t \le t_0 + \sigma$*, where* σ *is sufficiently small.*

Theorem 2 *Let all the conditions of Theorem* 1 *be fullfilled. Then the solution is continuously dependent in the space* C_0 *of the initial functions and from* $H(\Phi_1(t), \Phi_2(t)) \le \delta, \quad \delta > 0, \quad t \in E_{t_0}$ *it follows that*

$$H(X_1(t), X_2(t)) \le \delta\, e^{\lambda(m+1)(t-t_0)}, \quad t \ge t_0. \tag{4}$$

Let us consider the following differential equation with Hukuhara derivative:

$$\begin{aligned} &D_h X(t,\varepsilon) = \varepsilon F(t, X(t,\varepsilon), X(t - \tau\varepsilon)), \\ &X(s,\varepsilon) = \Phi(s,\varepsilon), \qquad -\tau \le s \le 0, \end{aligned} \tag{5}$$

where $\varepsilon > 0$ is a small parameter, $t \in \left[0, L\varepsilon^{-1}\right]$, L - a constant.

The following partially averaged equation corresponds to (5):

$$\begin{aligned} &D_h Y(t,\varepsilon) = \varepsilon\, F^0(t, Y(t,\varepsilon), Y(t - \tau, \varepsilon)), \\ &Y(s,\varepsilon) = \Phi(s,\varepsilon), \qquad -\tau \le s \le 0, \end{aligned} \tag{6}$$

where

$$\lim_{T \to \infty} H\left(\frac{1}{T}\int_0^T F^0(t, X, Z)\,dt, \frac{1}{T}\int_0^T F(t, X, Z)\,dt \right) = 0. \tag{7}$$

Theorem 3 Let in the domain $Q = \{t \ge 0, X, Z \in D \subset conv(R^n)\}$ the following conditions hold:

1) the functions $F(t, X, Z), F^0(t, X, Z)$ and $\Phi(s,\varepsilon)$ fulfill the conditions of Theorem 1 and there exists a constant M such that

$$|F(t, X, Z)| \le M, \left|F^0(t, X, Z)\right| \le M, \text{ where}$$

$$|A| = H(\{0\}, A), A \in comp(R^n);$$

2) the limit (7) exists for every $X, Z \in D$;

3) there exists a solution of the system (6) for $0 < \varepsilon \le \sigma$ and together with a ρ-neighbourhood it is in the domain D for $t \in \left[0, L^* \varepsilon^{-1}\right]$, where L^* is a constant.

Then for every $\eta > 0$ and $0 < L \le L^*$ there exists $0 < \varepsilon^0 \le \sigma$ such that for $\varepsilon \in \left[0, \varepsilon^0\right]$ and $t \in \left[0, L\varepsilon^{-1}\right]$

$$H\left(X(t,\varepsilon),Y(t,\varepsilon)\right)\le \eta. \tag{8}$$

Proof: We consider the differential equation

$$D_h Z(t,\varepsilon)=\varepsilon F\left(t,Z(t,\varepsilon),Z(t,\varepsilon)\right),\ Z(0,\varepsilon)=\Phi(0,\varepsilon). \tag{9}$$

Due to $[3,4]$ from the differential equations $(1),(5)$ we come to the corresponding integral equations and get

$$H\left(X(t,\varepsilon),Z(t,\varepsilon)\right)\le \tag{10}$$
$$\varepsilon H\left(\int_0^t F\left(s,X(s,\varepsilon),X(s-\tau,\varepsilon)\right)ds,\int_0^t F\left(s,Z(s,\varepsilon),Z(s,\varepsilon)\right)ds\right).$$

For $t\in[0,\tau]$ taking into account the boundedness of the function $F(s,X,Z)$ we obtain

$$H\left(X(t,\varepsilon),Z(t,\varepsilon)\right)\le \varepsilon 2M\tau. \tag{11}$$

For $t\in\left(\tau,L\varepsilon^{-1}\right)$ we have from (10) the following:

$$H\left(X(t,\varepsilon),Z(t,\varepsilon)\right)\le \tag{12}$$
$$\varepsilon H\left(\int_0^t F\left(s,X(s,\varepsilon),X(s-\tau,\varepsilon)\right)ds,\ \int_0^t F\left(s,X(s,\varepsilon),X(s,\varepsilon)\right)ds\right)+$$
$$\varepsilon H\left(\int_0^t F\left(s,X(s,\varepsilon),X(s,\varepsilon)\right)ds,\ \int_0^t F\left(s,Z(s,\varepsilon),Z(s,\varepsilon)\right)ds\right)\le$$
$$\varepsilon\lambda\int_0^t H\left(X(s-\tau,\varepsilon),X(s,\varepsilon)\right)ds+2\varepsilon\lambda\int_0^t H\left(X(s,\varepsilon),Z(s,\varepsilon)\right)ds.$$

Obviously

$$\int_0^t H\left(X(s-\tau,\varepsilon),X(s,\varepsilon)\right)ds\le M\tau L. \tag{13}$$

Thus, using the Gronwall-Bellman's lemma, we get from (12) that

$$H\left(X(t,\varepsilon),Z(t,\varepsilon)\right)\le \varepsilon M\tau L\,\mathit{exp}\,(\varepsilon 2\lambda\, t). \tag{14}$$

Analogously for equations (6) and the equation

$$D_h W(t,\varepsilon)=\varepsilon F^0\left(t,W(t,\varepsilon),W(t,\varepsilon)\right),W(0,\varepsilon)=\Phi(0,\varepsilon) \tag{15}$$

we have

$$H\left(Y(t,\varepsilon),W(t,\varepsilon)\right)\le \varepsilon M\tau L\,\mathit{exp}\,(\varepsilon 2\lambda\, t). \tag{16}$$

Equation (15) is a partial averaging for equation (9).

According to [14, p.112] for every η and $L\in\left[0,L^*\right]$ there exists $\varepsilon^1\in[0,\tau]$ such that for $\varepsilon\in\left[0,\varepsilon^1\right]$ and $t\in\left[0,L\varepsilon^{-1}\right]$ the following estimate holds:

$$H\left(W(t,\varepsilon),Z(t,\varepsilon)\right)\le \eta/2. \tag{17}$$

From $(14),(16),(17)$ for $\varepsilon^0=\mathit{min}\left\{\varepsilon^1,\eta/\left(4M\tau L\,\mathit{exp}(2\lambda L)\right)\right\}$ we get the estimate (8).

Let us consider the differential equation

$$D_h X(t,\varepsilon)=\varepsilon F\left(t,X(t,\varepsilon),X(t-\tau_1,\varepsilon)\right),X(t-\tau_2/\varepsilon,\varepsilon) \tag{18}$$

$$X(s,\varepsilon)=\Phi(s,\varepsilon), \qquad -\tau_2/\varepsilon \le s \le (0).$$

In correspondence to equation (18) we put the following partially averaged equation

$$D_h Y(t,\varepsilon)=\varepsilon F^0(t, Y(t,\varepsilon), Y(t-\tau_1,\varepsilon)), Y(t-\tau_2/\varepsilon,\varepsilon), \tag{19}$$
$$Y(s,\varepsilon)=\Phi(s,\varepsilon), \qquad -\tau_2/\varepsilon \le s \le 0,$$

where

$$\lim_{T\to\infty} H\left(\frac{1}{T}\int_0^T F(t,X,Y,Z)\,dt, \frac{1}{T}\int_0^T F^0(t,X,Y,Z)\,dt\right)=0. \tag{20}$$

Theorem 4 Let in the domain $Q=\{t\ge 0, X,Y,Z\in conv(R^n)\}$ the following conditions hold:

1) the functions $F(t,X,Y,Z), F^0(t,X,Y,Z)$ are continuous, uniformly bounded by the constant M and fulfill the Lipschitz condition in all variables starting with the second one with a constant λ;

2) the initial function $\Phi(s,\varepsilon)$ is continuous and

$$H\left(\Phi\left(s',\varepsilon\right),\Phi\left(s'',\varepsilon\right)\right)\le \varepsilon\lambda\left(s'-s''\right),$$
$$\Phi(s,\varepsilon)\in D, \qquad -\tau_2/\varepsilon \le s \le 0;$$

3) there exists the limit (20) for every $X,Y,Z\in D$;

4) the solution of equation (19) exists for $\varepsilon\in[0,\tau]$ and together with a δ-neighbourhood it is in D for $t\in\left[0, L^*\varepsilon^{-1}\right]$, L^* is a constant.

Then for every $\eta>0$ and $0<L\le L^*$ there exists $0<\varepsilon^0\le\tau$ such that for $\varepsilon\in\left(0,\varepsilon^0\right)$ and $t\in\left[0, L\varepsilon^{-1}\right]$ the following unequality holds:

$$H(X(t,\varepsilon), Y(t,\varepsilon))\le\eta. \tag{21}$$

Proof. We consider a solution of the systems (18), (19) on the interval $[0,\tau_2/\varepsilon]$:

$$D_h X^1(t,\varepsilon)=\varepsilon F\left(t, X^1(t,\varepsilon), X^1(t-\tau_1,\varepsilon), \Phi(t-\tau_2/\varepsilon,\varepsilon)\right), \tag{22}$$
$$X^1(0,\varepsilon)=\Phi(0,\varepsilon);$$

$$D_h Y^1(t,\varepsilon)=\varepsilon\, F^0\left(t, Y^1(t,\varepsilon), Y^1(t-\tau_1,\varepsilon), \Phi(t-\tau_2/\varepsilon,\varepsilon)\right), \tag{23}$$
$$Y^1(0,\varepsilon)=\Phi\,(0,\varepsilon).$$

According to Theorem 3 for every $\eta_1>0$ there exists $\varepsilon_1\in(0,\sigma]$ such that for $\varepsilon\in(0,\varepsilon_1)$ and $t\in(0,\tau_2/\varepsilon]$ the following estimate is valid:

$$H\left(X^1(t,\varepsilon), Y^1(t,\varepsilon)\right)\le\eta_1. \tag{24}$$

Now we consider on $[\tau_2/\varepsilon, 2\tau_2/\varepsilon]$ the solutions $Y^2(s,\varepsilon)$ and $Z^2(s,\varepsilon)$ of the system (19) with initial conditions

$$Y^2(s-\tau_2/2,\varepsilon)=Y^1(s,\varepsilon),$$
$$Z^2(s-\tau_2/2,\varepsilon)=X^1(s,\varepsilon).$$

Due to Theorem 2 for every $\eta_2>0$ there exists $\delta>0$ such that for $H\left(Y^1(s,\varepsilon),X^1(s,\varepsilon)\right)\le\delta$ we have

$$H\left(Y^2(s,\varepsilon),Z^2(s,\varepsilon)\right)\le\eta_2/2,\qquad s\in[\tau_2/\varepsilon,\,2\tau_2/\varepsilon]. \tag{25}$$

Now we consider the solutions $X^2(s,\varepsilon)$ and $Z^2(s,\varepsilon)$ of the systems (18), (19) on the interval $[\tau_2/\varepsilon,2\tau_2/\varepsilon]$ with initial conditions

$$X^2(s-\tau_2/2,\varepsilon)=X^1(s,\varepsilon),$$
$$Z^2(s-\tau_2/2,\varepsilon)=X^1(s,\varepsilon).$$

According to Theorem 3 for every $\eta_2>0$ there exists for $\varepsilon\in(0,\varepsilon_2']$ and $t\in[\tau_2/\varepsilon,\,2\tau_2/\varepsilon]$ the following estimate holds

$$H\left(X^2(t,\varepsilon),Z^2(t,\varepsilon)\right)\le\eta_2/2. \tag{26}$$

We choose $\varepsilon_2''\in(0,\sigma]$ such that $\eta_1\le\delta$ for $\varepsilon\in(0,\varepsilon_2'']$. Then for $\varepsilon_2=\min\{\varepsilon_2',\varepsilon_2''\}$ from (25),(26) we get

$$H\left(Y^2(s,\varepsilon),X^2(s,\varepsilon)\right)\le\eta_2,\qquad t\in[\tau_2/\varepsilon,2\tau_2/\varepsilon].$$

Let $k<\dfrac{L}{\tau_2}\le k+1$. Then after k steps for $\varepsilon^0=\min\limits_{1\le i\le k}\varepsilon_i$ we come to the conclusion of the theorem.

Remark 1 If the function $F^0(t,X,Y,Z)$ does not depend explicitly on t, relation (20) means the existence of an average, i.e.

$$F^0(X,Y,Z)=\lim_{T\to\infty}\frac{1}{T}\int_0^T F^0(t,X,Y,Z)\,dt.$$

In this case the full averaging scheme justification follows from Theorem 4.

We consider the system of differential equations with Hukuhara derivative

$$D_hX_i(t,\varepsilon)=\varepsilon F_i\left(t,X(t,\varepsilon),X(t-\tau_1,\varepsilon),X^1(t-\tau_2/\varepsilon,\varepsilon)\right), \tag{27}$$
$$X_i(s,\varepsilon)=\Phi(s,\varepsilon)\qquad -\tau_2/\varepsilon\le s\le 0, i=1,2,\dots,m,$$

where $X=(X_1,X_2,\dots,X_m)$,

$$X\in D\subset W=\left(conv\left(R^{n_1}\right)\text{x}\,conv\left(R^{n_2}\right)\text{x}\dots\text{x}\,conv\left(R^{n_m}\right)\right),$$
$$F=(F_1,F_2,\dots,F_m),\quad F(t,X,Y,Z)=F^1(t,X,Y,Z)+F^2(t,X,Y,Z),$$
$$\Phi=(\Phi_1,\Phi_2,\dots,\Phi_m).$$

We put the following partially averaged system in correspondence to system (27):

$$D_h Y_i(t,\varepsilon) = \varepsilon F_i^0\left(t, Y(t,\varepsilon), Y(t-\tau_1,\varepsilon), Y(t-\tau_2/\varepsilon,\varepsilon)\right), \tag{28}$$

$$Y_i(s,\varepsilon) = \Phi(s,\varepsilon) \qquad -\tau_2/\varepsilon \le s \le 0,$$

where

$$F^0(t,X,Y,Z) = F^1(t,X,Y,Z) + \overline{F^2}(t,X,Y,Z),$$

$$\overline{F^2}(X,Z) = \lim_{T\to\infty} \frac{1}{T}\int_0^T F^2(t,X,X,Z)dt. \tag{29}$$

Theorem 5 Let in the domain $Q = \{\, t \ge 0, X, Y, Z \in D \subset W \,\}$ the following conditions hold:

1) the function $F(t,X,Y,Z)$ is continous, uniformly bounded by a constant M and fulfills the Lipschitz condition with a constant λ in all variables starting with the second one;

2) the initial function $\Phi(s,\varepsilon)$ is continous and

$$H(\Phi(s',\varepsilon), \Phi(s'',\varepsilon)) \le \varepsilon\lambda|s'-s''|,$$

$$\Phi(s,\varepsilon) \in D, \qquad -\tau_2/\varepsilon \le s \le 0;$$

3) there exists the limit (29) for all $X,Y,Z \in D$;

4) the solution of the system (27) for $\varepsilon \in (0,\sigma]$ exists and together with a g-neighbourhood is in the domain D for $t \in \left[0, L\varepsilon^{-1}\right]$.

Then for $\eta > 0$ there exists $\varepsilon^0 \in (0,\sigma)$ such that for $\varepsilon \in (0,\varepsilon^0]$ and $t \in (0,\varepsilon^{-1}]$ the following estimate holds:

$$H(X(t,\varepsilon), Y(t,\varepsilon)) \le \eta.$$

The validity of Theorem 5 follows from Theorem 4 as from the fulfillness of conditions (29) the fulfillness of conditions (20) follows.

REFERENCES

[1] Borisovicz U.G., R.D.Gelman , A.D.Myshkis, V.V.Obuchovskii, Multivalued maps, VINITY, Itogi Nauki i Techniki, 19 (1982), 127--231 (in Russian).

[2] Hukuhara M., Sur l'application mesurables dont la valeur est un compact convexe, Func. Ekvacioj, 10 (1967), 43--66.

[3] Brandao Lopes Pinto A.J., F.S.De Blasi, F.Iervolino, Uniqueness and existence theorems for differential equations with compact convex valued solutions, Boll. U.M.I., 4 (1970), 534--538.

[4] De Blasi F.S., F.Iervolino, Equazioni differentiali con soluzioni a valore compatto convesso, Boll. U.M.I., 2:4-5 (1969), 491--501.

[5] Tolstonogov A.A., Differential Inclusions in Banach Space, Novosibirsk, Nauka, 1986 (in Russian).

[6] Kisielewicz M., Method of averaging for differential equations with compact convex valued solutions. Rend. math., 9:3 (1976), 397--408.
[7] Plotnikov A.V., Averaging of differential inclusions with Hukuhara derivative, Ukr. Mat. Zurn. 41:1 (1989), 121--125 (in Russian).
[8] Plotnikov V.A., L.I.Plotnikova, Averaging of quasidifferential equations in a metric space, Differenzialnie Uravnenija, 31:10 (1995), 1678--1683 (in Russian).
[9] Janiak T., E.Luczak-Kumorek, Averaging of neutral differential inclusions with unbounded right-hand side, Discuss. Math. 12 (1992), 67--74 (1993).
[10] Plotnikov V.A., P.I.Rashkov, Existence, Continuous Dependence and Averaging in Differential Equations with Hukuhara Derivative and Delay, Mathematics and Education in Mathematics, Proceedings of 26-th Spring Conf. of UBM, Plovdiv, April 22--25 1997, UBM, Sofia, 179--184.
[11] Myshkis A.D., General Theory of Differential Equations with Retarded Argument, A.M.S. Translations, series I, 4, American Mathematical Society, Providence, RI, 1962 (Russian original 1949).
[12] Hale, J.K., S.M.Verduyn Lunel, Introduction to Functional Differential Equations, Springer-Verlag, New York, 1993.
[13] El'sgol'tz, L.E., S.B.Norkin, Intorduction to the Theory of Differential Equations with Deviating Argument (revised edition), Academic Press, New York, 1973 (Russian edition 1971).
[14] Plotnikov V.A., A.V.Plotnikov, A.N.Vitjuk, Differential Equations with Multivalued Right-Hand Side, Asymptotic Methods (in Russian), Astro-Print, Odessa, 1999.

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ВЪРХУ УСРЕДНЯВАНЕТО В ДИФЕРЕНЦИАЛНИ УРАВНЕНИЯ С ПРОИЗВОДНА НА ХУКУХАРА И ЗАКЪСНЕНИЕ

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Резюме: *В работата се разглеждат диференциални уравнения с многозначни десни страни.и закъснение. Стойностите на всички разглеждани многозначни функции са изпъкнали компактни множества. Производните на решенията са в смисъл на Хукухара. Обоснована е една обща схема за усредняване в три варианта.*

Ключови думи: *диференциално уравнение, усредняване, многозначни изображения (функции), закъснение, изпъкнало множество, съществуване.*

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