PROCEEDINGS

of the Union of Scientists - Ruse

Book 5

Mathematics, Informatics and Physics

Volume 13, 2016

RUSE

PROCEEDINGS
OF THE UNION OF SCIENTISTS - RUSE

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PROCEEDINGS
of the Union of Scientists – Ruse

ISSN 1314-3077

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BOOK 5

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ASSESSMENT OF THE CHARACTERISTICS OF THE SYSTEM 'CENTER FOR EMERGENCY MEDICAL AID' FOR THE PROVISION OF TIMELY SERVICE TO PATIENTS

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***Abstract**: The present paper offers an evaluation of the characteristics of the system 'Center for emergency medical aid' for the provision of timely service to patients. The graphs of the states of the system and of any of the ambulances are shown. The number of functional ambulances is calculated for any point in time, as well as the required number of ambulances in stationary mode.*

***Keywords:** Mathematical modelling, Probability and Statistics, Random processes, Emergency medical aid*

INTRODUCTION

The object of our study is the operation of a Center for emergency medical aid (CEMA). We are using data from a conducted research on the CEMA in the town of Ruse. It is necessary for the Automotive fleet to possess a certain number of vehicles (ambulances or reanomobiles) to ensure the timely service to patients. Some of the ambulances are in working order and are serving patients, others are in working order and expect to service patients, and a third group are being maintained or repaired.

In order to provide a timely service to patients with virtually no waiting in the system queue, it is necessary to carry out some research taking into account the intensity of the incoming requests during each interval of the day, the average time to service a request, the average period during which any of the vehicles is in good working condition, and the time needed for maintenance and repair.

Providing a large number of ambulances as a good prerequisite for the smooth operation of the system would be one possibility to solve the problem under discussion but ambulances are expensive capital goods whose purchase and maintenance require a lot of money. It is therefore necessary to find an optimal variant regarding the number of ambulances needed: on the one hand, to provide patient service with virtually no waiting in the system queue [2], and on the other hand - this should be done with a minimum number of ambulances.

EXPOSITION

Taking into account the need for rational use of CEMA's budget resources, we consider the case where the random process [1] $X(t)$ (number of ambulances ready to serve the process) should not exceed a certain number n ($0 \le X(t) \le n$), i.e. it is not possible to support more ambulances with the available budget.

The graph of the states of the system is shown in Fig.1 . Each state of the system of reanomobiles in CEMA's Auto fleet is shown on it with S_i, $i = 1, 2, 3, ..., n$. The index i indicates the number of ambulances in working order. The intensity of the flow of recovery (repair) of ambulance is given as $\lambda(t)$ [5], and the intensity of their failures (breakdowns) is given as $\mu(t)$ [5].

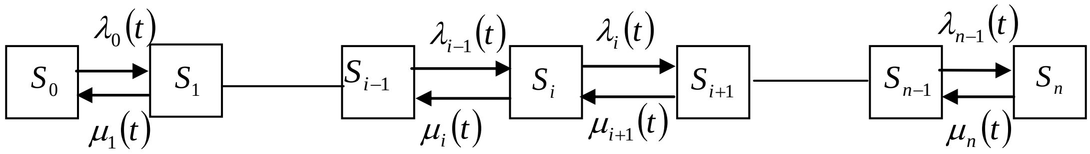

Fig. 1. A graph of the states of a system of n vehicles

The system of differential equations for the probabilistic states of the system of ambulances will have the form (1) :

$$\begin{cases} \frac{dp_0(t)}{dt} = \mu_1(t).p_1(t) - \lambda_0(t).p_0(t) \\ \dots\dots\dots \\ \frac{dp_i(t)}{dt} = \mu_{i+1}(t).p_{i+1}(t) + \lambda_{i-1}(t).p_{i-1}(t) - (\lambda_i(t) + \mu_i(t)).p_i(t), \ (0 < i < n) \\ \dots\dots\dots \\ \frac{dp_n(t)}{dt} = \lambda_{n-1}(t).p_{n-1}(t) - \mu_n(t).p_n(t) \end{cases} \quad (1)$$

By multiplying both sides of the i-th equation in the system (1) with i $(i = 0,1,2,...,n)$ and by adding up the respective sides of these equations, the equation for the mathematical expectation [3] of a random process $X(t)$ is obtained:

$$\frac{dm_x(t)}{dt} = \sum_{i=0}^{n} (\lambda_i(t) - \mu_i(t)).p_i(t) \quad (2)$$

with initial conditions

$$\lambda_n(t) = \mu_0(t) = 0\,. \quad (3)$$

By multiplying both sides of the equations of the system (1) with i^2, and then by adding up the respective sides of these equations and differentiating both sides of the resulting equation , we come to the differential equation for the dispersion [3] of a random process $X(t)$:

$$\frac{dD_x(t)}{dt} = \sum_{i=0}^{n} \left[\lambda_i(t) + \mu_i(t) + 2.(i - m_x(t)).(\lambda_i(t) - \mu_i(t))\right].p_i(t) \quad (4)$$

In solving equation (4), the initial conditions (3) are taken into consideration.

When modeling [6] this particular random process, we assume that :

$$\begin{cases} \lambda_i(t) = (n-i).\lambda(t), & (i = 0,1,...,n) \\ \mu_i(t) = i.\mu(t), & (i = 0,1,...,n) \end{cases}. \quad (5)$$

Taking into account the initial conditions (3), then equations (2) and (4) acquire the form of (6) and (7) respectively:

$$\frac{dm_x(t)}{dt} = n.\lambda(t) - (\mu(t) + \lambda(t)).m_x(t) \quad (6)$$

$$\frac{dD_x(t)}{dt} = n.\lambda(t) + (\mu(t) - \lambda(t)).m_x(t) - 2.(\mu(t) + \lambda(t)).D_x(t) \quad (7)$$

The graph of the states in the case of supporting n vehicles in CEMA Auto fleet, in accordance with conditions (5), is shown in Figure 2.

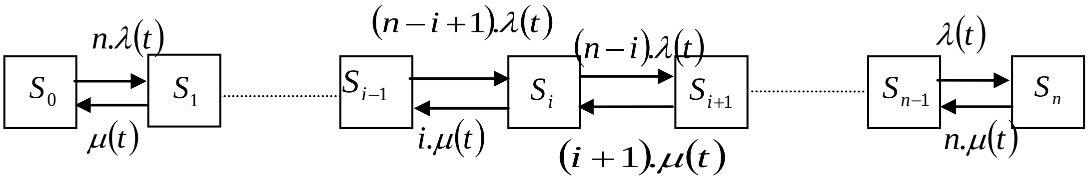

Fig. 2. The graph of the states of the system of vehicles in compliance with conditions (5)

All ambulances (reanomobiles) are identical and can be regarded as interchangeable. Each of them could be in one of the following states:

$S_0^{(i)}$ - when it is damaged and in a state of repair;

$S_1^{(i)}$ - When it is in working order and is capable of operation (servings patients).

The graph of the possible states of any one of the ambulances (or any of the reanomobiles) is shown in Fig. 3 .

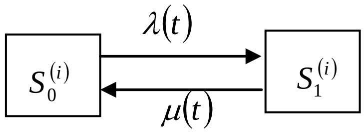

Fig. 3. The graph of the states of any one of ambulances

On Fig. 3 the intensity of the Poisson flow of recovery of a reanomobile is indicated with $\lambda(t)$, and the intensity of the Poisson flow of failures is marked with $\mu(t)$. Moreover, each ambulance passes from one state to another (from those in Figure 3) regardless of what state each of the other ambulances is in. We believe that all reanomobiles have statistically identical parameters $\lambda(t)$ and $\mu(t)$, i.e. the reanomobiles with which the CEMA in the town of Ruse is equipped, are approximately the same age and of about the same mileage.

For this reason we can assume that the graph shown in Figure 3 describes the behavior of an arbitrary i-th reanomobile.

The random process $Z_i(t)$ of roving [9] of the i-th reanomobile between its two states is defined as follows:

$$Z_i(t) = \begin{cases} 1, \textit{if at the point in time t the i-th reanomobile is in state } S_1^{(i)} \\ 0, \textit{if at the point in time t the i-th reanomobile is broken and is in state } S_0^{(i)} \end{cases}$$

Then the total number of reanomobiles which are in the first state (they are intact and are able to serve patients) can be calculated by formula (8):

$$\mathrm{X(t)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{Z_i(t)} \tag{8}$$

We enter the symbols:

$$\begin{cases} \pi_1(t) = P\{Z_i(t) = 1\} \\ \pi_0(t) = P\{Z_i(t) = 0\} = 1 - \pi_1(t) \end{cases} \tag{9}$$

The probabilities (9) do not depend on the consecutive number of the i-th reanomobile in the CEMA Auto fleet (out of n vehicles in total), because it was explained above that each reanomobile behaves statistically uniformly and independently of the others reanomobiles. Then the one-dimensional distribution law of the random process will satisfy the characteristics of a binomial law [8] of distribution with parameters $n, \pi_1(t)$.

Our task is to find the mathematical expectation of a random process $X(t)$ defined by equation (8). Taking into account the symbols (9), we obtain:

$$M[X(t)] = m_x(t) = M\left[\sum_{i=1}^{n} Z_i(t)\right] = \sum_{i=1}^{n} M[Z_i(t)] = n.\pi_1(t). \tag{10}$$

In accordance with the graph from Figure 3, we come to the result:

$$\begin{aligned}\frac{d\pi_1(t)}{dt} &= \lambda(t).\pi_0(t) - \mu(t).\pi_1(t) = \lambda(t).(1-\pi_1(t)) - \mu(t).\pi_1(t) = \\ &= \lambda(t) - (\mu(t)+\lambda(t)).\pi_1(t)\end{aligned}. \tag{11}$$

By multiplying the left and the right side of equation (11) with n and taking into account equation (10), the following result is obtained:

$$\begin{aligned}&\frac{n.d\pi_1(t)}{dt} = \frac{d(n\pi_1(t))}{dt} = \frac{d(m_x(t))}{dt} = n.\lambda(t) - (\mu(t)+\lambda(t)).n.\pi_1(t) = \\ &= n.\lambda(t) - (\mu(t)+\lambda(t)).m_x(t)\end{aligned}, \tag{12}$$

We pointed out above that in terms of its characteristics, the one-dimensional law of distribution of a random process $X(t)$ is binomial with parameters $n,\ \pi_1(t)$, and from the literature [8] it is known that with such a law of distribution the mathematical expectation and variance are calculated by formulas (13)

$$m_x(t) = n.\pi_1(t) \text{ и } D_x(t) = n.\pi_1(t).(1-\pi_1(t)). \tag{13}$$

Therefore:

$$D_x(t) = n.\pi_1(t).(1-\pi_1(t)) = n.\pi_1(t) - n.\pi_1^2(t) = m_x(t) - \frac{m_x^2(t)}{n} = m_x(t).\left[1 - \frac{m_x(t)}{n}\right]. \tag{14}$$

In accordance with the graph of the states of a reanomobile (or an ambulance) shown in Figure 3, the probability for a reanomobile to be in state $\pi_1(t)$, i.e. intact and working, will be determined after solving the linear differential equation (11) :

$$\pi_1(t) = e^{-\int_0^t (\mu(x)+\lambda(x)).dx}\left[\int_0^t \lambda(x).e^{\int_0^x (\mu(\tau)+\lambda(\tau))d\tau} dx + \pi_{i0}\right], \text{ where } \pi_{i0} = \pi_1(0). \tag{15}$$

In searching for a solution to equation (11) by formula (15), we assume that the intensities of the flows of failures and recovery (repair) do not depend on time, i.e. $\mu = const$ and $\lambda = const$. At the initial moment $t = 0$ all ambulances are operational $(m_x(0) = n,\ D_x(0) = 0,\ \pi_1(0) = 1)$. Under these conditions the characteristics of the random process $X(t)$ will be determined – the number of ambulances in working order. Then equation (15) will acquire the form (16)

$$\pi_1(t) = e^{-\int_0^t (\mu+\lambda).dx}\left[\int_0^t \lambda.e^{\int_0^x (\mu+\lambda)d\tau} dx + 1\right] = \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}.e^{-(\lambda+\mu).t}, \quad t > 0. \tag{16}$$

The system has ergodic property [9], and since $\lambda_i(t) = (n-i).\lambda = const.$, $\mu_i(t) = i.\mu = const.$, the number of states is finite. Therefore, a stationary mode will exist for this system for which:

$$m_x = \lim_{t \to \infty} m_x(t) = \frac{n.\lambda}{\lambda + \mu} = n.\pi_1, \text{ where } \pi_1 = \frac{\lambda}{\lambda + \mu}. \quad (17)$$

$$D_x = \lim_{t \to \infty} D_x(t) = \frac{n.\lambda.\mu}{(\lambda + \mu)^2} = n.\pi_1.(1 - \pi_1). \quad (18)$$

Since the CEMA aims to provide around the clock service to needy patients, it must have at its disposal the necessary number of teams. Let us suppose the number of ambulances (or reanomobiles) is six [7] and the average uptime of each ambulance is 10 days. The average repair time is half a day. Our goal is to determine the characteristics of the random process $X(t)$ - number of ambulances in working order in stationary mode, assuming that the failures and recovery (repair) of each ambulance are simple flows and all ambulances operate independently from one another.

For the process under consideration

$$\mu = 0{,}1\,(1/day); \quad \lambda = \frac{1}{0{,}5} = 2\,(1/day)$$

$$\pi_1 = \frac{\lambda}{\lambda + \mu} = \frac{2}{2 + 0{,}1} \approx 0{,}95; \; \pi_0 = 1 - \pi_1 = 1 - 0{,}95 = 0{,}05.$$

Then the probability for a random ambulance to be functional is 0.95, and the probability to be faulty is 0.05.

In our previous studies [7] it was found that for patients to receive service without having to wait in the system queue of the Center for emergency medical aid, in accordance with the intensity of incoming requests and the intensity of service (defined mainly by the average service time for one patient), it is required that the patients of the investigated CEMA are served by at least four ambulances at any point of time. Taking into consideration the fact that some of the ambulances break down, and others are taken out for maintenance, we have reached the conclusion that CEMA should have at least six ambulances at their disposal. Then we obtain the results (19)

$$m_x = \frac{n.\lambda}{\lambda + \mu} = \frac{6 * 2}{(2 + 0{,}1)} \approx 5{,}71 \text{ ambulances.}$$

$$D_x = \frac{n.\lambda.\mu}{(\lambda + \mu)^2} = \frac{6 * 2 * 0{,}1}{(2 + 0{,}1)^2} \approx 0{,}27. \quad (19)$$

$$\sigma_x = \sqrt{D_x} \approx 0{,}52 \text{ ambulances.}$$

The interpretation of the results (19) is as follows:

The standard deviation is 0.52 ambulances, which shows that more than five ambulances are in working order at any point in time and this will be a prerequisite for timely patient service without waiting in the system queue.

We consider the utilization of n ambulances by CEMA which work around the clock to provide service to patients. Each ambulance is involved in serving patients λ times a day on average, regardless of the involvement of other ambulances. Each instance of providing service to a patient with an ambulance lasts for a random period of time depending on the condition of the individual patient (this is the time of service). The time is distributed by exponential law with a parameter μ, regardless of how many ambulances from the fleet of CEMA are busy serving patients. We believe that the inclusion of each ambulance is a simple flow with a parameter λ. Our task is to determine the characteristics of the random process $X(t)$ - number of ambulances in operation at point in time t, if at point in time $t = 0$ none of the ambulances serves patients.

The likelihood for whichever ambulance (out of all n ambulances) to be engaged in the service of patients is determined from (15) by formula (16) :

$$\pi_1(t) = e^{-\int_0^t (\mu+\lambda)dx} \int_0^t \lambda . e^{\int_0^x (\mu+\lambda)d\tau} dx = \frac{\lambda}{\mu+\lambda}.\left(1-e^{-(\mu+\lambda).t}\right), \qquad \text{where } \pi_1(0)=0. \qquad (20)$$

Therefore

$$\mathrm{M}[\mathrm{X}(t)] = \mathrm{m_x}(t) = \mathrm{M}\left[\sum_{i=1}^{n} \mathrm{Z_i}(t)\right] = \sum_{i=1}^{n} \mathrm{M}[\mathrm{Z_i}(t)] = n.\pi_1(t) \qquad (21)$$

Then:

$$\mathrm{m_x}(t) = \frac{n.\lambda.\left(1-e^{-(\mu+\lambda).t}\right)}{\mu+\lambda}, \ (t>0), \qquad (22)$$

$$D_x(t) = \frac{n.\lambda.\mu.\left(1-e^{-(\mu+\lambda).t}\right)}{(\mu+\lambda)^2}.\left(1+\frac{\lambda}{\mu}.e^{-(\mu+\lambda).t}\right). \qquad (23)$$

Under these conditions, there is a stationary mode for which

$$m_x(t) = m_x = \frac{n.\lambda}{\mu+\lambda}; \qquad (24)$$

$$D_x(t) = D_x = \frac{n.\lambda.\mu}{(\mu+\lambda)^2}. \qquad (25)$$

In accordance with previous research carried out by us [4], it was found that the average number of requests from patients received by CEMA in Ruse for one day is 40 and the average service time for a request is 29 minutes. Four ambulances (or reanomobiles) at least are needed for the timely service of patients. Then the average number of calls that one ambulance will respond to in one day is $\lambda = 10$, and the service intensity of an ambulance is $\mu = 50$.

If $n = 6$, $\lambda = 10$ (1/day), $\mu = 50$ (1/day), then

$$m_x = \frac{6*10}{50+10} = 1, \qquad D_x = \frac{6*10*50}{(50+10)^2} \approx 0{,}83.$$

CONCLUSION

Therefore, in stationary mode of patient service, it is sufficient to involve one ambulance.

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ОЦЕНКА НА ХАРАКТЕРИСТИКИТЕ НА СИСТЕМАТА ЦЕНТЪР ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ ЗА ОСИГУРЯВАНЕ СВОЕВРЕМЕННОТО ОБСЛУЖВАНЕ НА ПАЦИЕНТИТЕ

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Русенски университет „Ангел Кънчев“

Резюме: *В настоящата работа е направена оценка на характеристиките на системата Център за спешна медицинска помощ за осигуряване своевременното обслужване на пациентите. Показани са графите на състоянията на системата и за коя да е от линейките. Пресметнат е броят на изправните линейки във всеки момент от време, както и необходимият брой при стационарен режим.*

Ключови думи: *Математическо моделиране, Вероятности и статистика, Случайни процеси, Спешна медицинска помощ.*

ISSN 1314-3077

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