web: suruse.uni-ruse.bg

FINITELY GENERATED GRASSMANN ALGEBRAS - COMMUTATIVITY AND COMPUTER APPROACH

Tsetska Rashkova

Angel Kanchev University of Ruse

Abstract: *In 2015 L. Marki et al. introduced an embedding of the m-generated Grassmann algebra $E^{(m)}$ into a $2^{m-1}\times 2^{m-1}$ matrix algebra over a factor of a commutative polynomial algebra in m variables. Applying this embedding we show examples concerning the degree of the standard identity for the matrix algebra $M_2(E^{(2)})$ and give a negative answer to a question asked by P. Frenkel in the same year for the minimal degree of the standard identity in $M_n(E^{(m)})$.*

Keywords: *Grassmann algebra, standard identity, CT-representation, matrix algebras over m-generated Grassmann algebras for $m=2,3$.*

INTRODUCTION

We consider some matrix algebras over the infinite dimensional Grassmann algebra E and over finite dimensional Grassmann algebras $E^{(m)}$ for different m.

The algebra E is defined as

$$E = E(V) = K\langle e_1, e_2, \ldots \mid e_i e_j + e_j e_i = 0 \quad i, j = 1,2,\ldots\rangle,$$

where the field K has characteristic zero. The elements e_i are called generators of E, while the elements $e_{i_1}e_{i_2}\ldots e_{i_k}$ for $1\le i_1 \le i_2 \ldots \le i_k$ are called basic monomials of E. The element 1 is a generator as well.

The algebra E is in the mainstream of recent research in PI-theory. Its importance is connected with the structure theory for the T- ideals of identities of associative algebras developed by Kemer who proved that any T- prime T- ideal can be obtained as the T-ideal of identities of one of the following algebras: $M_n(K)$, $M_n(E)$ and $M_{n,u}(E)$, the latter being the algebra of $n\times n$ supermatrices over $E = E_0 \oplus E_1$ with two E_0 blocks (with entries of even degree) of sizes $u\times u$ and $(n-u)\times(n-u)$ and with two E_1 blocks (with entries of odd degree) of sizes $u\times(n-u)$ and $(n-u)\times u$.

Another reason for the Grassmann algebra to be one of the fundamental structures in PI-theory is the fact that it generates a minimal variety of exponential growth [4].

Some well known facts concerning the algebra E are the following:

Proposition 1 [5, Corollary, p. 437]. *The T-ideal $Id(E)$ is generated by the identity $[x_1, x_2, x_3] = [x_1, x_2]x_3 - x_3[x_1, x_2] = 0$ (called the Grassmann identity).*

Proposition 2 [1, Lemma 6.1]. *The algebra E satisfies $S_n(x_1,\ldots,x_n)^k = 0$ for all $n,k\ge 2$ and $S_n(x_1,\ldots,x_n) = \sum_{\sigma\in Sym(n)} (-1)^\sigma x_{\sigma(1)}\ldots x_{\sigma(n)}$* being the standard identity.

Proposition 3 [2, Exercise 5.3]. *For* $E^{(m)} = E(V_m)$ *over* m*-dimensional vector space* V_m *all identities follow from the identity* $[x_1, x_2, x_3] = [x_1, x_2]x_3 - x_3[x_1, x_2] = 0$ *and the standard identity* $S_{2p}(x_1, \dots, x_{2p}) = 0$*, where* p *is the minimal integer such that* $2p > m$*.*

The importance of considering the matrix algebra with Grassmann entries $M_n(E)$ is confirmed by the following statement as the trivial isomorphism $E \otimes M_n(K) \cong M_n(E)$ holds:

Proposition 4 [3, Corollary 8.2.4]. *For every PI-algebra* R *there exists a positive* n *such that* $T(R) \supseteq T(M_n(E))$*, i.e.* R *satisfies all polynomial identities of the* $n \times n$ *matrix algebra* $M_n(E)$ *with entries from the Grassmann algebra.*

Proposition 5 [1, Corollary 6.6]. *The algebra* $M_n(E)$ *does not satisfy the identity* $S_m^n(X_1, ..., X_m) = 0$ for any m.

FINITELY GENERATED GRASSMANN ALGEBRAS AND COMMUTATIVITY

In [6] L. Marki et al. introduced an embedding of the m-generated Grassmann algebra $E^{(m)}$ into a $2^{(m-1)} \times 2^{(m-1)}$ matrix algebra over a factor of a commutative polynomial algebra in m variables. Applying it we give examples concerning the degree of the standard identity as stated in [6, Theorem 3.7] for concrete matrix algebras over the Grassmann algebra E. These examples could be related as well to [4, Problem 10], where P.Frenkel asked a question about the degree function $k = k(m, n)$ of the standard identity $S_k = 0$ for $M_n(E^{(m)})$.

Since E does not satisfy any of the standard identities, it follows that E does not embed into any full matrix algebra over a commutative ring.

We give the CT-representation of $E^{(m)}$ according to [6] and the necessary results from [6] as well.

Let ${}_K R$ be an arbitrary and ${}_K \Omega$ be a commutative (associative) algebra over K. For an integer $t \geq 1$ we consider representations of R over Ω which are injective K-algebra homomorphisms (K-embeddings) $\varepsilon : R \to M_t(\Omega)$.

Definition 1. *We call* ε *a constant trace* (CT-) *representation if* $tr(\varepsilon(r)) \in K$ *for all* $r \in R$ *(here* $tr(\varepsilon(r))$ *is the sum of the diagonal entries of the* $t \times t$ *matrix* $\varepsilon(r) \in M_t(\Omega))$*.*

The following representation, namely

$$1 \mapsto \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; v_1 \mapsto m(v_1) = \begin{bmatrix} z_1 & 0 \\ 0 & -z_1 \end{bmatrix}, \; v_2 \mapsto m(v_2) = \begin{bmatrix} 0 & z_2 \\ z_2 & 0 \end{bmatrix}$$

is a CT-representation $\varepsilon^{(2)} : E^{(2)} \to M_2(K[z_1, z_2]/(z_1^2, z_2^2))$ as

$$\varepsilon^{(2)}(c_0 + c_1 v_1 + c_2 v_2 + c_3 v_1 v_2) = \begin{bmatrix} c_0 + c_1 z_1 + (z_1^2, z_2^2) & c_2 z_2 + c_3 z_1 z_2 + (z_1^2, z_2^2) \\ c_2 z_2 - c_3 z_1 z_2 + (z_1^2, z_2^2) & c_0 - c_1 z_1 + (z_1^2, z_2^2) \end{bmatrix},$$

where $c_0, c_1, c_2, c_3 \in K$ and (z_1^2, z_2^2) is the ideal of the commutative polynomial ring $K[z_1, z_2]$ generated by the monomials z_1^2, z_2^2.

Proposition 6 [6, Theorem 3.1]. *For some integers* $m, t \geq 2$, *let* $\varepsilon^{(m)} : E^{(m)} \to M_t(\Omega)$ *be a* CT*-representation of* $E^{(m)}$ *over a commutative* K*-algebra* Ω. *Then the assignments* $1 \mapsto \begin{bmatrix} I_t & 0 \\ 0 & I_t \end{bmatrix}, v_i \mapsto \begin{bmatrix} \varepsilon^{(m)}(v_i) & 0 \\ 0 & -\varepsilon^{(m)}(v_i) \end{bmatrix}$ *for* $1 \leq i \leq m$, *and* $v_{m+1} \mapsto \begin{bmatrix} 0 & \hat{z}I_t \\ \hat{z}I_t & 0 \end{bmatrix}$ *(with* $\hat{z} = z + (z^2)$ *in* $\Omega(z)/(z^2)$*) define a* CT*-representation* $\varepsilon^{(m+1)} : E^{(m+1)} \to M_{2t}(\Omega[z]/(z^2))$.

The notation I_t stands for the unit matrix of order t.

Applying Proposition 6 we form the CT-representation $\varepsilon^{(3)} : E^{(3)} \to M_4(\Omega[z]/(z^2))$, namely,

$$v_1 \mapsto M(v_1) = \begin{bmatrix} z_1 & 0 & 0 & 0 \\ 0 & -z_1 & 0 & 0 \\ 0 & 0 & -z_1 & 0 \\ 0 & 0 & 0 & z_1 \end{bmatrix}, v_2 \mapsto M(v_2) = \begin{bmatrix} 0 & z_2 & 0 & 0 \\ z_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -z_2 \\ 0 & 0 & -z_2 & 0 \end{bmatrix},$$

$$v_3 \mapsto M(v_3) = \begin{bmatrix} 0 & 0 & z_3 & 0 \\ 0 & 0 & 0 & z_3 \\ z_3 & 0 & 0 & 0 \\ 0 & z_3 & 0 & 0 \end{bmatrix}.$$

We formulate the corresponding propositions from [6, 4]:

Proposition 7 [6, Theorem 3.7]. *The standard identity* $S_{2^m n} = 0$ *of degree* $2^m n$ *is a polynomial identity on* $M_n(E^{(m)})$.

Proposition 8 [4, Theorem 7]. *The standard identity of degree* $k = 2n([m/2]+1)$ *holds in* $M_n(E^{(m)})$.

Proposition 9 [4, Proposition 8]. *The standard identity of degree* 6 *holds in* $M_2(E^{(2)})$.

Proposition 10 [4, Proposition 9]. *The standard identity of degree* $k = 2(n+[m/2])-1$ *does not hold in* $M_n(E^{(m)})$ *if the base ring is a field of characteristic either zero or a prime* $p > 2[m/2]$.

Problem 1 [4, Problem 10]. *Does the standard identity of degree* $2(n+[m/2])$ *hold in* $M_n(E^{(m)})$?

In the cases $m = 0$, $m = 1$, $n = 1$ and $m = n = 2$ the answer is an affirmative one.

Now we expose results in relation to the above statements in the partial cases of $M_2(E^{(2)})$, $M_2(E^{(3)})$, $M_3(E^{(2)})$ and $M_3(E^{(3)})$.

COMPUTER APPROACH FOR THE CASES $m=2,n=2$ AND $m=2,n=3$

Using a program written in the system for computer algebra *Mathematica* [7] for working in the second order matrix algebra over a finite Grassmann algebra we give an illustration (in a computer way) of the validity of Proposition 9.

We start with the case $m=n=2$ applying Proposition 6:

Using the CT-representation of $E^{(2)}$ we form the following 4×4 matrices (instead of z_1, z_2 we use the letters a,b):

A1={{a,0,0,0},{0,-a,0,0},{0,0,0,0},{0,0,0,0}};
A2={{0,0,0,0},{0,0,0,0},{0,0,0,b},{0,0,b,0}};
A3={{1,0,a,0},{0,1,0,-a},{0,0,0,0},{0,0,0,0}};
A4={{0,0,0,a b},{0,0,-ab,0},{0,0,0,0},{0,0,0,0}};
A5={{0,b,0,0},{b,0,0,0},{0,0,1,0},{0,0,0,1}};
A6={{1,0,1,0},{0,1,0,1},{1,0,1,0},{0,1,0,1}}.

Here we give the matrices in a way suitable for the system *Mathematica*. Regarding Proposition 6 for example $A3=\begin{bmatrix} I_2 & m(v_1) \\ 0 & 0 \end{bmatrix}$ in a block way form.

Proposition 11. $S_6(A1,A2,A3,A4,A5,A6)=0$ *in the algebra* $M_2(E^{(2)})$.

Proof: We give a part of the program in *Mathematica* for evaluating that $T6=S_6(A1,A2,A3,A4,A5,A6)=0$. We define the standard polynomial recurrently and give the last two steps:

$$T5[x\_,y\_,z\_,t\_,u\_]:=x.T4[y,z,t,u]+y.T4[z,t,u,x]+z.T4[t,u,x,y]$$
$$+t.T4[u,x,y,z]+u.T4[x,y,z,t];$$
$$T6[x\_,y\_,z\_,t\_,u\_,v\_]:=x.T5[y,z,t,u,v]-y.T5[z,t,u,v,x]$$
$$+z.T5[t,u,v,x,y]-t.T5[u,v,x,y,z]$$
$$+u.T5[v,x,y,z,t]-v.T5[x,y,z,t,u]$$

T6[A1,A2,A3,A4,A5,A6]={{0,0, $4a^3b^2$, $4a^3b^2$},{0,0,$-4a^3b^2$,$-4a^3b^2$}, {0,0,0,0}, {0,0,0,0}}.

As we are working in the T-ideal, generated by a^2 and b^2, we get the desired result.

Now we consider the case $m=2,n=3$:

Proposition 10 gives that for $m=2,n=3$ the standard polynomial S_7 is not an identity in $M_3(E^{(2)})$ while Problem 1 asks is $S_8=0$ an identity in $M_3(E^{(2)})$. Using again the above CT-representation of $E^{(2)}$ we give an example when Proposition 10 is true and a negative answer to Problem 1.

We form the following 12 matrices of type 6×6:

B1={{1,0,0,0,1,0},{0,1,0,0,0,1},{0,0,a,0,0,0},
0,0,0,-a,0,0},{0,b,0,0,0,b},{b,0,0,0,b,0}};
B2={{a,0,0,0,a,0},{0,-a,0,0,0,-a},{0,0,1,0,0,0},
{0,0,0,1,0,0},{0,b,0,0,0,b},{b,0,0,0,b,0}};
B3={{1,0,0,0,1,0},{0,1, 0,0,0,1},{0,0,a,0,0,0},
{0,0,0,-a,0,0},{1,0,0,0,1,0},{0,1,0,0,0,1}};
B4={{a,0,0,0,a,0},{0,-a,0,0,0,-a},{0,0,1,0,0,0},
{0,0,0,1,0,0},{a,0,0,0,a,0},{0,-a,0,0,0,-a}};

B5={{1,0,0,0,0,0},{0,1,0,0,0,0},{a,0,0,b,0,ab},
{0,-a,b,0,-ab,0},{0,0,0,0,1,0},{0,0,0,0,0,1}};
B6={{0,0,1,0,1,0},{0,0,0,1,0,0},{1,0,0,0,1,0},
{0,1,0,0,0,1},{0,0,0,0,1,0},{0,0,0,0,0,1}};
B7={{a,0,0,0,a,0},{0,-a,0,0,0,-a},{0,0,1,0,1,0},
{0,0,0,1,0,1},{0,0,1,0,1,0},{0,0,0,1,0,1}};
B8={{1,0,0,0,0,0},{0,1,0,0,0,0},{1,0,a,0,a,0},
{0,1,0,-a,0,-a},{1,0,0,0,0,0},{0,1,0,0,0,0}};
B9={{1,0,0,0,0,0},{0,1,0,0,0,0},{1,0,a,0,a,0},
{0,1,0,-a,0,-a},{1,0,a,0,a,0},{0,1,0,-a,0,-a}};
B10={{1,0,1,0,1,0},{0,1,0,1,0,1},{1,0,1,0,1,0},
{0,1,0,1,0,1},{1,0,1,0,0,0},{0,1,0,1,0,0}};
B11={{0,0,1,0,1,0},{0,0,0,1,0,1},{1,0,1,0,1,0},
{0,1,0,1,0,1},{1,0,1,0,0,0},{0,1,0,1,0,0}};
B12={{1,0,1,0,1,0},{0,1,0,1,0,1},{1,0,1,0,1,0},
{0,1,0,1,0,1},{1,0,1,0,1,0},{0,1,0,1,0,1}}.

Using the CT-representation of $E^{(2)}$ $B1=\begin{bmatrix} I_2 & 0 & I_2 \\ 0 & m(v_1) & 0 \\ m(v_2) & 0 & m(v_2) \end{bmatrix}$ and

$B5=\begin{bmatrix} I_2 & 0 & 0 \\ m(v_1) & m(v_2) & m(v_1v_2) \\ 0 & 0 & I_2 \end{bmatrix}$ in a block way form.

Proposition 12. *$S_7(x_1,...,x_7)=0$ and $S_8(x_1,...,x_8)=0$ are not identities in the algebra $M_3(E^{(2)})$.*

Proof: Evaluating $S_7(B2,B3,B4,B5,B6,B7,B10)=(a_{ij})$ and $S_8(B1,B4,B6,B7,B8,B9,B10,B11)=(b_{ij})$ in the system *Mathematica* we get that modulo the ideal generated by a^2 and b^2 these are not zero matrices as

$$a_{12}=-a_{21}=-66ab \quad a_{32}=-a_{41}=-16ab \quad a_{52}=-a_{61}=-12ab$$
$$a_{14}=-a_{23}=-10ab \quad a_{34}=-a_{43}=-22ab \quad a_{54}=-a_{63}=4ab \quad .$$
$$a_{16}=-a_{25}=-28ab \quad a_{36}=-a_{45}=-42ab \quad a_{56}=-a_{65}=-70ab$$

and $b_{52}=-b_{61}=4ab$.

The above result means that if the standard identity $S_k(x_1,...,x_k)=0$ holds in $M_n(E^{(m)})$ for $n>2$, then $k>2(n+[m/2])$.

We want to find a better estimation of the degree k. Using again the system *Mathematica* we get that $S_9(B2,B3,B4,B5,B6,B7,B10,B11,B12)=(c_{ij})$ is not zero as

$$\begin{aligned} c_{14} &= -c_{16}=-c_{23}=c_{25}=c_{34}=c_{36}=-c_{43}=-c_{45} \\ &= c_{52}=-c_{54}=-c_{56}=-c_{61}=c_{63}=c_{65}=-4ab \end{aligned}.$$

We could formulate the following proposition:

Proposition 13. *If the standard identity* $S_k(x_1,...,x_k)=0$ *holds in* $M_n(E^{(m)})$ *for* $n>2$*, then* $k>2(n+[m/2])+1$.

Remark 1. *The memory of 8.00 GB of the computer used does not allow to increase the degree of the standard polynomial in direct calculations made by Mathematica. In order to use the system to calculate for example* $S_{10}(B1,B7,B2,B3,B4,B10,B5,B6,B11,B12)$ *(as from above* $S_9(B2,B3,B4,B5,B6,B7,B10,B11,B12)\neq 0$ *and* $B1.S_9(B2,B3,B4,B5,B6,B7,B10,B11,B12)\neq 0$*) we calculate the polynomial in parts (the ten-th ones in the recurrent formula, analogous to the formulas from the proof of Proposition 11). Firstly we find the values of any of the standard polynomials of degree* 9 *and then simplify them modulo the ideal generated by* a^2, b^2. *Then we multiply any of the obtained polynomials with the corresponding matrix and at last we find the value of the standard polynomial of degree* 10. *But in this case the standard polynomial*

$S_{10}(B1,B7,B2,B3,B4,B10,B5,B6,B11,B12)$ *appeared to be zero.*

The method is not fruitful if applying several choices for the ten-th matrices. Thus at the moment we can't give a better estimation than the one in Proposition 13.

On the other hand it is logical to suppose that if for some n the value of S_n is a symmetrical in some way matrix (like the form $\begin{bmatrix} 0 & 0 & 0 & x & 0 & -x \\ 0 & 0 & -x & 0 & x & 0 \\ 0 & 0 & 0 & x & 0 & x \\ 0 & 0 & -x & 0 & -x & 0 \\ 0 & x & 0 & -x & 0 & -x \\ -x & 0 & x & 0 & x & 0 \end{bmatrix}$ of $S_9(B2,B3,B4,B5,B6,B7,B10,B11,B12)$), then $S_{n+1}=0$ is maybe an identity.

COMPUTER APPROACH FOR THE CASES $m=3,n=2$ AND $m=3,n=3$

Using the CT-representation of $E^{(3)}$ as shown in Proposition 6 we illustrate the validity of a theorem of Uzi Vishne [9, Corollary 4.2] for $m=3$ and a special choice of the variables:

In [9] Vishne gave the explicit form of 2 multilinear polynomials being identities in the matrix algebra $M_2(E)$. Their definition is not a simple one. Here we only sketch it.

A pattern is a finite sequence of the letters A,B. If π is a pattern with a appearances of A and b of B, we denote by $\pi(x_1,...,x_a;y_1,...,y_b)$ the product of variables where the x' s and y' s are combined according to π. For example $ABBA(x_1,x_2;y_1,y_2)=x_1y_1y_2x_2$.

We construct the polynomials

$$P_\pi^+=\sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)\pi(x_{\sigma(1)},\dots,x_{\sigma(a)};y_{\tau(1)},\dots,y_{\tau(b)});$$

$$P_\pi^- = \sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)sign(\tau)\pi(x_{\sigma(1)},\ldots,x_{\sigma(a)};y_{\tau(1)},\ldots,y_{\tau(b)}).$$

Let

$$\text{P} = \begin{pmatrix} +AAAABAAB, & +AABBAAAA, & -AABAAAAB, \\ -AAAABBAA, & -BAABAAAA, & +BAAAABAA \end{pmatrix} \text{ and}$$

$$\text{PP} = \begin{pmatrix} -AAABAABB, & -AABBAABA, & +ABBAABAA, \\ +AAABBAAB, & +AABAABBA, & -ABAABBAA \\ -ABBAAAAB, & +BAABBAAA, & -BAAAABBA \\ +ABAAAABB, & -BBAABAAA, & +BBAAAABA \end{pmatrix}.$$

The two polynomials introduced in [9] are the following ones:

$$T_1(x_1,\ldots,x_6;y_1,y_2) = \sum_{\pi\in \text{P}} (P_\pi^- + P_\pi^+);$$

$$T_2(x_1,\ldots,x_5;y_1,y_2,y_3) = \sum_{\pi\in \text{PP}} (P_\pi^- + P_\pi^+).$$

Theorem 1 [9, Corollary 4.2]. $T_1(x_1,\ldots,x_6;y_1,y_2)$ *and* $T_2(x_1,\ldots,x_5;y_1,y_2,y_3)$ *are multilinear identities of degree* 8 *of* $M_2(E)$*.*

Now we consider the case $m=3, n=2$**:** Using the embedding $\varepsilon^{(3)}: E^{(3)} \to M_4(\Omega[z]/(z^2))$, shown after Proposition 6 and the corresponding embedding $M_2(E^{(3)}) \to M_8(\Omega[z]/(z^2))$ we confirm the validity of Theorem 1 in the following partial case, namely evaluating the above two polynomials on the 8×8 matrices

$$M1 = \begin{bmatrix} I_4 & M(v_1) \\ I_4 & I_4 \end{bmatrix}, \quad M2 = \begin{bmatrix} I_4 & I_4 \\ M(v_1) & I_4 \end{bmatrix}, \quad M3 = \begin{bmatrix} M(v_2) & I_4 \\ I_4 & 0 \end{bmatrix},$$

$$M4 = \begin{bmatrix} I_4 & I_4 \\ I_4 & M(v_3) \end{bmatrix}, \quad M5 = \begin{bmatrix} I_4 & I_4 \\ I_4 & I_4 \end{bmatrix}, \quad M6 = \begin{bmatrix} M(v_1) & I_4 \\ I_4 & M(v_2) \end{bmatrix},$$

$$M7 = \begin{bmatrix} M(v_3) & M(v_2) \\ I_4 & 0 \end{bmatrix}, \quad M8 = \begin{bmatrix} I_4 & I_4 \\ 0 & M(v_3) \end{bmatrix}.$$

Here $M(v_i)$ are the matrices representing $E^{(3)}$, while I_4 is the 4×4 unit matrix, 0 is the 4×4 zero matrix.

By the system *Mathematica* and working modulo the corresponding ideal we get

Proposition 14. $T_1(M1,\ldots,M6;M7,M8)=0$ *and* $T_2(M1,\ldots,M5;M6,M7,M8)=0$ *in the algebra* $M_2(E^{(3)})$.

This proposition gives a good evidence that the approach of working in a matrix algebra over a factor of a commutative polynomial algebra in m indeterminates is an effective one in spite of the fact that the order of the corresponding matrix algebra increases with the increase of the number m of the generators of $E^{(m)}$.

We consider Problem 1 in the case of $m=n=3$ in relation to some concrete matrix subalgebras of $M_3(E)$ trying to find a lower degree of the standard identity for them.

The first algebra is the algebra $M1A_3(E)$ of matrices of type $\begin{bmatrix} x_1 & x_2 & x_3 \\ \alpha x_1 & \alpha x_2 & \alpha x_3 \\ \beta x_1 & \beta x_2 & \beta x_3 \end{bmatrix}$, where $x_j \in E$ and $\alpha, \beta \in K^+$.

According to [8,Theorem 1] the algebra $M1A_3(E)$ satisfies the identities $[X_1, X_2, X_3]X_4 = 0$ and $[X_1, X_2][X_1, X_3]X_4 = 0$.

Let $M2A_3(E)$ be the algebra of the matrices of type $\begin{bmatrix} x & 0 & x \\ 0 & y & 0 \\ z & 0 & z \end{bmatrix}$ for $x, y, z \in E$. Its T-ideal is generated by the identity $X_4[X_1, X_2, X_3] = 0$ [8].

Considering the algebra $M3A_3(E)$ of the matrices of type $\begin{bmatrix} x & 0 & 0 \\ y & z & t \\ u & 0 & 0 \end{bmatrix}$ for $x, y, z, t, u \in E$ we could prove that $[X_1, X_2, X_3]X_4[X_5, X_6, X_7] = 0$ is an identity in it [8, Theorem 3].

Using the possibilities of the system *Mathematica* we could show that the best estimation for the degree of the standard identity is in the case of the third algebra. For the corresponding values of m and n we have $k > 2(n + [m/2]) - 2$, namely

Proposition 15. *If the standard identity* $S_k(x_1,..., x_k) = 0$ *holds in* $M3A_3(E^{(3)})$, *then* $k > 6$.

Proof: Considering the CT-presentation of $E^{(3)}$ we find concrete 12×12 matrices for which $S_6(x_1,..., x_6) \neq 0$. These are the matrices

B01={{M(v$_1$),0,0)},{M(v$_1$),M(v$_1$),0},{0,0,0}};
B02={{M(v$_2$),0,0},{M(v$_1$),0,M(v$_1$v$_2$)},{M(v$_1$),0,0}};
B03={{I$_4$,0,0},{0,M(v$_2$),M(v$_2$)},{M(v$_1$),0,0}};
B04={{M(v$_2$),0,0},{0,M(v$_1$),0},{I$_4$,0,0}};
B05={{I$_4$,0,0},{I$_4$,I$_4$,I$_4$},{I$_4$,0,0}};
B06={{I$_4$,0,0},{0,I$_4$,0},{0,0,0}}.

By the system *Mathematica* we get $S_6(B01, B02, B03, B04, B05, B06) = (a_{ij}) \neq 0$, namely $a_{52} = -a_{61} = a_{74} = -a_{83} = 2ab$ modulo the ideal, generated by a^2 and b^2.

For now we are not able to find a better lower bound of the degree of a standard polynomial to be an identity in $M3A_3(E^{(3)})$. As it was pointed in Remark 1 the symmetry in the matrix $S_6(B01, B02, B03, B04, B05, B06) = (a_{ij})$ is not an impulse to continue the computer trials.

The paper is partially supported by Grant I 02/08 “Computer and Combinatorial Methods in Algebra and Applications” of the Bulgarian National Science Fund and under Project 17-FNSE-03 “Investigations of Mathematical Models with Analytic and

Computer Methods and Generating Mathematical Problems with a Specialized Software" by the Fund "Science Research" at the University of Ruse.

REFERENCES

[1] Berele A., A. Regev. Exponential growth for codimensions of some P.I. algebras, J. Algebra, vol. 241, 2001, 118-145.

[2] Drensky V. Free Algebras and PI-Algebras, Springer-Verlag, 1999.

[3] Drensky V. E. Formanek. Polynomial Identity Rings, Birkhauser Verlag, 2004.

[4] Frenkel P. Polynomial identities for matrices over the Grassmann algebra, http://xxx.laul.gov/pdf/1511.05549.pdf.

[5] Krakowski D., A. Regev. The polynomial identities of the Grassmann algebra, Trans. Amer. Math. Soc., vol. 181, 1973, 429-438.

[6] Marki L., J. Meyer, J. Szigeti and L. van Wyk. Matrix representations of finitely generated Grassmann algebras and some consequences, Israel J. Math., vol. 208 (1), 2015, 373-384.

[7] Mihova A. Mathematica for calculations in the finite dimensional Grassmann algebra, Acta Universitatis Apulensis, Special Issue, Alba Iulia, Romania 2009, 279-285.

[8] Rashkova Ts. On some varieties of algebras defined by low degree identities, University of Ruse "A. Kanchev", Proceedings, vol. 54, book 6.1 Mathematics, Informatics and Physics, 2015, 39-43.

[9] Vishne U. Polynomial identities of $M_2(G)$, Commun. Algebra, vol. 30 (1), 2002, 443-454.

CONTACT ADDRESS

Assoc. Prof. Tsetska Rashkova, PhD,
Department of Mathematics,
Faculty of Natural Sciences and Education,
Angel Kanchev University of Ruse,
8, Studentska Str., 7017 Ruse, BULGARIA,
Phone: (++359 82) 888 489,
E-mail: tsrashkova@uni-ruse.bg

КРАЙНОПОРОДЕНИ ГРАСМАНОВИ АЛГЕБРИ – КОМУТАТИВНОСТ И КОМПЮТЪРЕН ПОДХОД

Цецка Рашкова

Русенски университет "Ангел Кънчев"

***Резюме**: През 2015г. Л. Марки и други въведоха влагане на m-породенатa Грасманова алгебра $E^{(m)}$ в $2^{m-1} \times 2^{m-1}$ матрична алгебра над фактор на комутативна полиномна алгебра на m променливи. Използвайки това влагане, ние посочваме примери, свързани със степента на стандартното тъждество в матричната алгебра $M_2(E^{(2)})$ и даваме отрицателен отговор на въпрос, зададен от П.Френкел през същата година за минималната степен на стандартно тъждество в $M_n(E^{(m)})$.*

***Ключови думи**: Грасманова алгебра, стандартно тъждество, СТ-представяне, матрични алгебри над m-породени Грасманови алгебри при $m = 2,3$.*

Requirements and guidelines for the authors - "Proceedings of the Union of Scientists - Ruse"

The Editorial Board of "Proceedings of the Union of Scientists - Ruse" accepts for publication annually both scientific, applied research and methodology papers, as well as announcements, reviews, information materials, adds. No honoraria are paid.

The paper scripts submitted to the Board should answer the following requirements:

1. Papers submitted in Bulgarian, Russian and English are accepted. Their volume should not exceed 8 pages, formatted following the requirements, including reference, tables, figures and abstract.

2. The text should be computer generated (MS Word 97 for Windows or higher versions up to Word 2003) and printed in one copy, possibly on laser printer and on one side of the page. Together with the printed copy the author should submit a disk (or send an e-mail copy to: desi@ami.uni-ruse.bg).

3. Compulsory requirements on formatting:

font - Ariel 12;

paper Size - A4;

page Setup - Top: 20 mm, Bottom: 15 mm, Left: 20 mm, Right: 20mm;

Format/Paragraph/Line spacing - Single;

Format/Paragraph/Special: First Line, By: 1 cm;

Leave a blank line under Header - Font Size 14;

Title should be short, no abbreviations, no formulas or special symbols - Font Size 14, centered, Bold, All Caps;

One blank line - Font Size 14;

Name and surname of author(s) - Font Size: 12, centered, Bold;

One blank line - Font Size 12;

Name of place of work - Font Size: 12, centered;

One blank line;

abstract – no formulas - Font Size 10, Italic, 5-6 lines ;

keywords - Font Size 10, Italic, 1-2 lines;

one blank line;

text - Font Size 12, Justify;

references;

contact address - three names of the author(s) scientific title and degree, place of work, telephone number, Email - in the language of the paper.

4. At the end of the paper the authors should write:

The title of the paper;

Name and surname of the author(s);

abstract; keywords.

Note: If the main text is in Bulgarian or Russian, parts in item 4 should be in English. If the main text is in English, they should be in Bulgarian and have to be formatted as in the beginning of the paper.

5. All mathematical signs and other special symbols should be written clearly and legibly so as to avoid ambiguity when read. All formulas, cited in the text, should be numbered on the right.

6. Figures (black and white), made with some of the widespread software, should be integrated in the text.

7. Tables should have numbers and titles above them, centered right.

8. Reference sources cited in the text should be marked by a number in square brackets.

9. Only titles cited in the text should be included in the references, their numbers put in square brackets. The reference items should be arranged in alphabetical order, using the surname of the first author, and written following the standard. If the main text is in Bulgarian or Russian, the titles in Cyrillic come before those in Latin. If the main text is in English, the titles in Latin come before those in Cyrillic. The paper cited should have: for the first author – surname and first name initial; for the second and other authors – first name initial and surname; title of the paper; name of the publishing source; number of volume (in Arabic figures); year; first and last page number of the paper. For a book cited the following must be marked: author(s) – surname and initials, title, city, publishing house, year of publication.

10. **The author(s) and the reviewer, chosen by the Editorial Board, are responsible for the contents of the materials submitted**.

Important for readers, companies and organizations

1. Authors, who are not members of the Union of Scientists - Ruse, should pay for publishing of materials.

2. Advertising and information materials of group members of the Union of Scientists – Ruse are published free of charge.

3. Advertising and information materials of companies and organizations are charged on negotiable (current) prices.

Editorial Board

ISSN 1314-3077