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Union of Scientists - Ruse
16, Konstantin Irechek Street
7000 Ruse
BULGARIA
Phone: (++359 82) 828 135,
(++359 82) 841 634
E-mail: suruse@uni-ruse.bg
web: suruse.uni-ruse.bg

Contacts with Editor
Phone: (++359 82) 888 738
E-mail: zzdravkova@uni-ruse.bg

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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 12

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RESULTS FROM COMPUTER MATHEMATICS EDUCATION FOR MOTIVATED STUDENTS AT RUSE UNIVERSITY[3]

Stefka Karakoleva, Ivan Georgiev, Slavi Georgiev, Pavel Zlatarov

Angel Kanchev University of Ruse

Abstract: *The article presents the education in Computer Mathematics for motivated students at the University of Ruse and the participation of MATLAB-team from University of Ruse in the Third National Olympiad in Computer Mathematics. The educational program for motivated students is discussed. Some mathematical problems and their solutions in MATLAB and MuPAD are also presented.*
Keywords: *Computer Mathematics, CAS, MATLAB, MuPAD, education, learning*.

INTRODUCTION

The Computer Algebra Systems (CAS), such as MATLAB, Mathematica, Maple, MuPAD, provide computational and visual power to solve many problems faster and more effectively than classical educational paper-and-pencil methods do. A professional support of such software and its ability to solve effectively most of the tasks of mathematics, allows us to teach motivated students in mathematics by using Matlab and MuPAD in Ruse University.

COMPUTER MATHEMATICS FOR MOTIVATED STUDENTS

The goals and requirements of mathematics education for motivated students are based on their knowledge of the foundations of mathematics and computer software systems [2]. The optional course [4] „Academic Research in Computer Mathematics” aims at familiarizing students with the application of a contemporary system for mathematical computations and visualization and developing skills to solve independently various mathematical problems with applications in engineering. Emphasis is placed on the practical use of mathematics in all fields of science, natural law and finance.

The usage of the system for mathematical computations MATLAB [8] and its symbolic package MuPAD [7] provides the speed, visibility and practical orientation of the course.

The course includes study and practical use of commands and functions of the MATLAB system for solving problems in Linear algebra, Geometry, Complex numbers, Functions, Differential and integral calculus, Differential equations, Fourier series and others.

The theoretical part of the material is being studied independently by recommended textbooks [3,7,8]. The knowledge is applied during practical exercises in a computer lab with Internet and installed MATLAB. Each student has an individual assignment for research and working on a non-trivial mathematical problem with practical application, which should be solved by MATLAB. The assignment then is presented on paper and on electronic format and submitted to the colloquium on the subject.

Depending on the completeness, quality and scientific level of the results obtained, the report is presented as a report at a student scientific session or as a scientific publication. Students with excellent results participate in the University of Ruse team in the

[3]This paper contains results of the work on project No 2015-FPHHC-03, financed by “Scientific Research” Fund of Ruse University.

National Student Olympiad in "Computer Mathematics".

After completing the course the students are expected to have competence in the field of applying MATLAB computer system in different branches of mathematics and to have acquired skills for successful applications both in engineering theory and practice.

NATIONAL STUDENT OLYMPIAD OF COMPUTER MATHEMATICS

The National Student Olympiad in Computer Mathematics [1,5,6] (CompMath) is a mathematical contest for university students enrolled on Bachelor or Master degree courses, which is organized once per year. CompMath aims at raising the students' interest in Mathematics and Computer Mathematics systems, as well as creating conditions for sharing experience among students and their tutors. The participation in the Olympiad is individual. The contestants are divided into two groups according to their subject area:

- Group A – Mathematics, Informatics and Computer Science;
- Group B – Engineering and Natural Sciences.

The objective of the contest is to solve 30 mathematical problems with the help of computer mathematical systems. The duration time is four full hours. Ranking is done separately for each group in a descending order of the points obtained.

The National Committee awards golden, silver and bronze medals in an approximate ratio of 1:2:3 to up to 50% of the contestants who have achieved the highest score within their respective group. The National Committee issues certificates of participation in CompMath to all contestants and their team leaders.

INVOLVEMENT OF MOTIVATED STUDENTS FROM UNIVERSITY OF RUSE IN THE THIRD NATIONAL STUDENT OLYMPIAD OF COMPUTER MATHEMATICS

Nine students from the University of Ruse - two in Group A and seven in Group B, participated in the Third National Olympiad in Computer Mathematics at Hisar, Bulgaria. Seventy nine students from 9 Bulgarian Universities participated in the Olympiad. The students from the University of Ruse made worthy performances, demonstrated creative thinking and showed new ideas. The second year student in Financial Mathematics Slavi Georgiev received a Golden medal in Group A and the third year student in Computer Systems and Technology Pavel Zlatarov won a Bronze medal in Group B.

The preparation of all students from the University of Ruse was serious and thorough, and was held by the team leaders Stefka Karakoleva and Ivan Georgiev from the Department of "Applied Mathematics and Statistics". The team received moral and financial support from the Rector and Vice Rectors of the University of Ruse and partial financial assistance from the Union of Bulgarian Mathematicians – branch Ruse.

SELECTED PROBLEMS AND THEIR SOLUTIONS

The following examples are given to illustrate the advantages in applying Computer Algebra Systems (CAS) for solving mathematical problems. These problems are part of the problems solved by Slavi Georgiev (Problems 1-6) and Pavel Zlatarov (Problems 4,7 and 8) during the Third National Olympiad in Hisar, Bulgaria.

Problem 1. (CompMath14/A-5) [6] Simplify the expression

$$2(a+b)^{-1}(ab)^{\frac{1}{2}}\left(1+\frac{1}{4}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^{2}\right)^{\frac{1}{2}}$$

if a and b are real numbers.

Solution. The ordinary simplification does not give a result. Let's examine the domain of the expression. The parameters a and b are non-zero real numbers and they have the same sign. If we use the `IgnoreAnalyticConstraints` option, we notice that in deradicalization the case with negative sign is omitted. So let us get the partially simplified expression before that step. For simplicity now we will simplify the subexpression

$$2\left(1+\frac{1}{4}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2\right)^{\frac{1}{2}}$$

in MuPAD notebook (Fig.1) using command

```
simplify(2*(1+((sqrt(a/b)-sqrt(b/a))^2) /4)^(1/2))
```

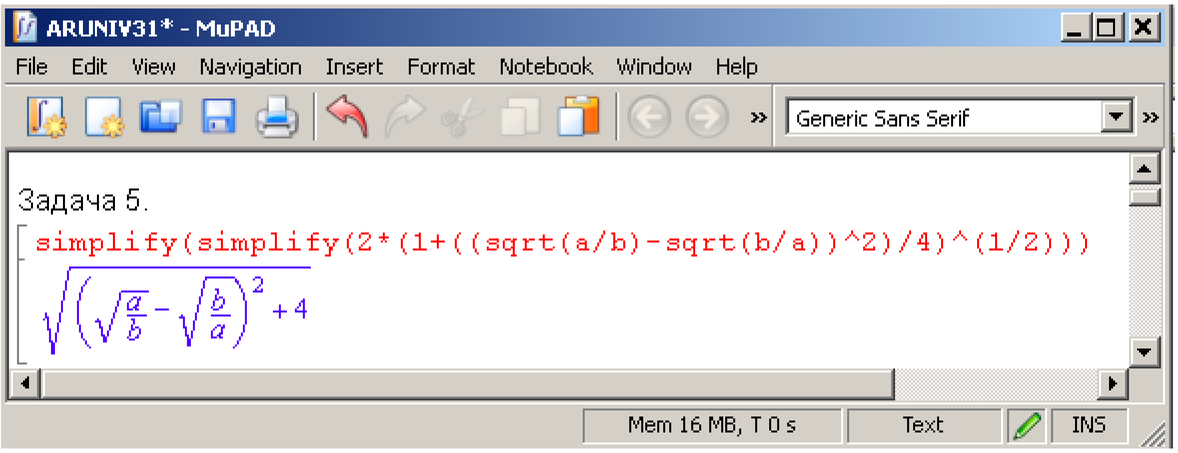

Fig.1. Solution of the Problem 1, CompMath14, Group A

It is obvious that the result equals $\sqrt{\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)^2}=\pm\left(\sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}}\right)$. When we multiply it with the omitted subexpression $(a+b)^{-1}(ab)^{\frac{1}{2}}$, we get ± 1 as a result.

Let us investigate the last identity. Due to the fact that the radical is always positive, we use the negative sign when a and b are negative, and respectively – we get the positive sign (or no sign) when a and b are positive. So the final answer of the task is:

$$\begin{cases} 1, & if\ a>0\ and\ b>0, \\ -1, & if\ a<0\ and\ b<0. \end{cases}$$

Problem 2. (CompMath14/A-18) [6] Calculate

$$\lim_{n\to\infty}\left(\sqrt[n+1]{(n+1)!}-\sqrt[n]{n}\right)$$

Solution. We can just solve the limit in MuPAD (Fig.2):

```
limit(((n + 1)!)^(1 / (n + 1)) - (n!)^(1 / n), n = infinity)
```

When we simplify the result, we obtain: e^{-1}

The solution is very similar, when we use MATLAB functionality. We just declare n as symbolic variable and write in the MuPAD Notebook:

```
>> simplify(limit(factorial(n + 1)^(1/(n + 1))-factorial(n)^(1/n),n,inf))
ans = exp(-1)
```

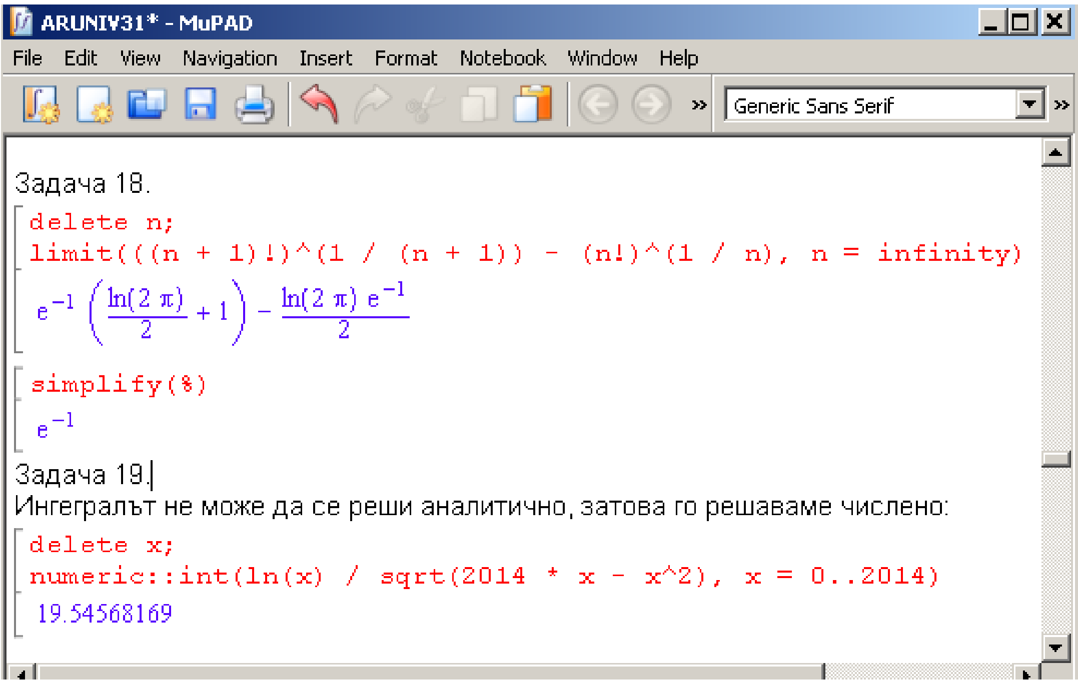

Fig.2. Solutions of the Problems 2 and 3, CompMath14, Group A

Problem 3. (CompMath14/A-19) [6] Calculate the integral

$$\int_0^{2014} \frac{\ln x \, dx}{\sqrt{2014x - x^2}}$$

Solution. As we could see, the given integral is unsolvable in quadratures. So we will find its approximate value numerically:

```
>> quad(@(x) log(x) ./ sqrt(2014 .* x - x.^2), 0, 2014)
ans = 19.5456837503445
```

NB. In the future versions of MATLAB, the function «*quad*» and its family will be replaced by «*integral*» function.

If we use MuPAD, the solution (Fig. 2) would look like:

```
numeric::int(ln(x) / sqrt(2014 * x - x^2), x = 0..2014)
```

19.54568169

Problem 4. (CompMath14/A-28,B-26) [6] Find the 2014th digit of the number 2014^{2014}.

Solution 1 in MATLAB (S.Georgiev)

The easiest way (in MATLAB) to solve the problem is to stringify the number, because it is very convenient to check the digit at the given position:

```
>> symNumber = sym('2014^2014');
>> strNumber = char(symNumber);
>> strNumber(2014)
ans = 2
```

Solution 2 in MuPAD (P.Zlatarov)

An easy way of solving this problem would be converting the number to a string and simply getting the character at the 2015-th position.

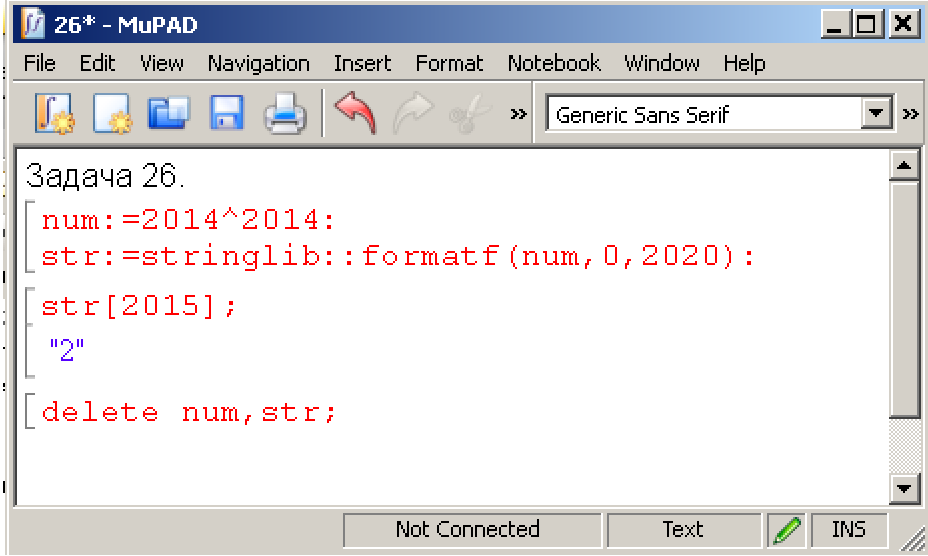

Fig.3. Solution of the Problem B-26, Group B in MuPAD

First, the value of 2014^{2014} must be calculated:

```
num:=2014^2014:
```

We don't need to see the output of this calculation.

Next, the result is converted to a string using the formatf function from MuPAD's stringlib library:

```
str:=stringlib::formatf(num,0,2020):
```

This output does not need to be seen either.

The number is converted to a 2020-character string because we need the digit at the 2015-th position (the first character is usually a space).

Next, the 2015-th character is displayed:

```
str[2015];
```

Thus we get the solution of the problem (Fig.3).

All that's left is to free up the memory that our number and string are taking up:

```
delete num,str;
```

Problem 5. (CompMath14/A-21) [6] Solve the equation

$$\operatorname{arctg} x - \frac{1}{2}\operatorname{arctg}\frac{2x}{1-x^2} = \frac{\pi}{2}$$

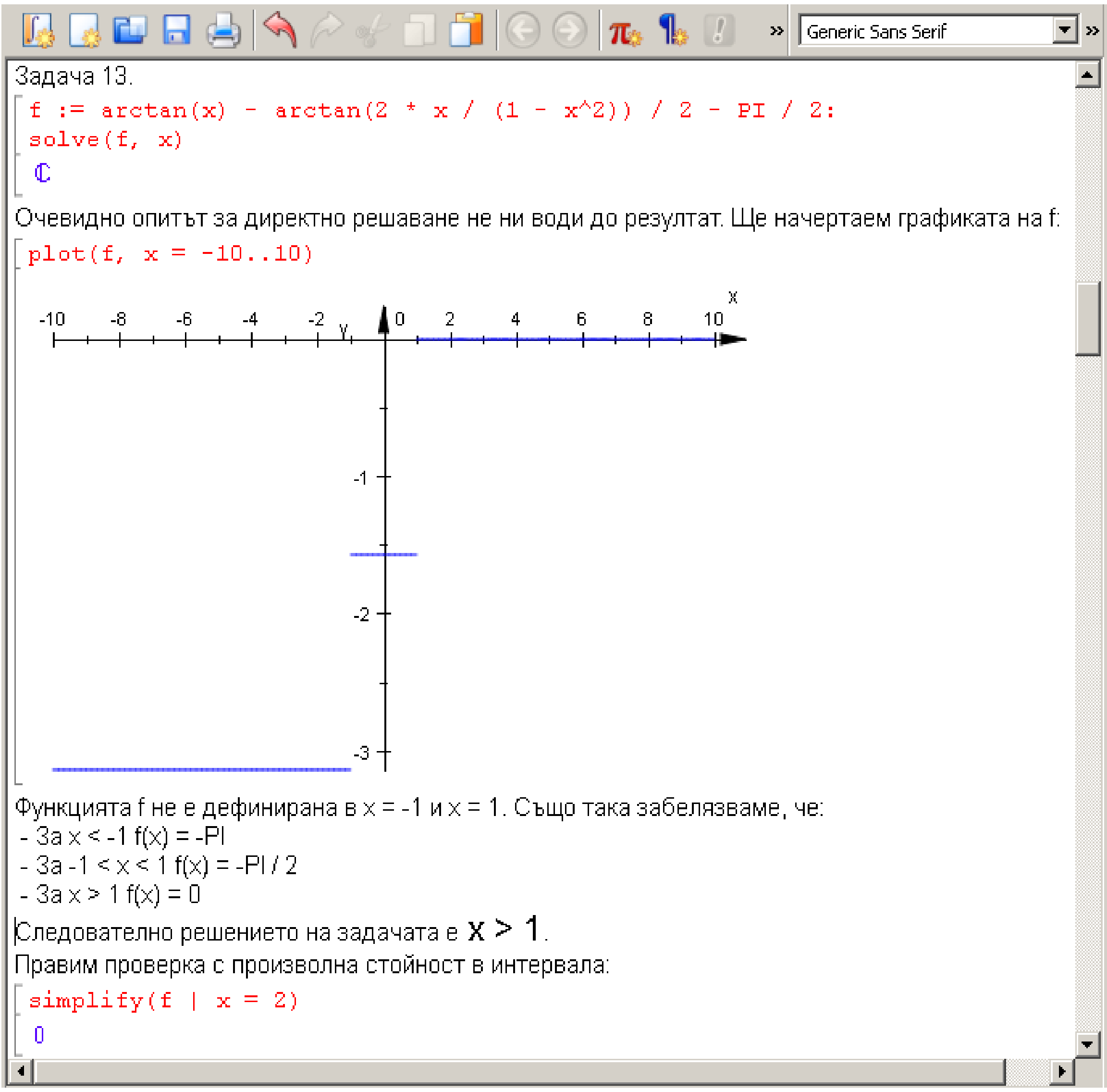

Fig.4. Solution of the Problem 5, CompMath14, Group A

Solution. We see that the ordinary solutions cannot figure out the problem (Fig. 4). So let us investigate the expression. We can tangent the both sides of the equation. With some equivalent transformations the expression is simplified, using the identities about $\mathrm{tg}(\alpha - \beta)$ and $\mathrm{tg}\, 2\alpha$. Next, the tg and $arctg$ functions annihilate each other with caution to the domain. Due to the fact that an identity for all x in the domain is obtained in the end, the whole domain $(x > 1)$ is the solution of the task.

Problem 6. (CompMath14/A-21) [6] Let the triangle ABC be a right isosceles triangle with hypotenuse AB equal to 4. The point D lies on a circle with centre C and radius 1. Find the smallest possible perimeter of the triangle ABD.

Solution. ABC is an isosceles triangle so its sides *AB* and *BC* are equal to $2\sqrt{2}$. If the height from C to AB is CH, so its length equals 2. The condition for smallest perimeter of ABD is satisfied only when D lies where the circle and CH cross. So DH is equal to 1. AD and BD are considered as hypotenuses of AHD and BHD respectively, and they equal $\sqrt{5}$. Finally, the perimeter is P $ABD = AB + AD + BD = 2\sqrt{5} + 4$, (Fig. 5).

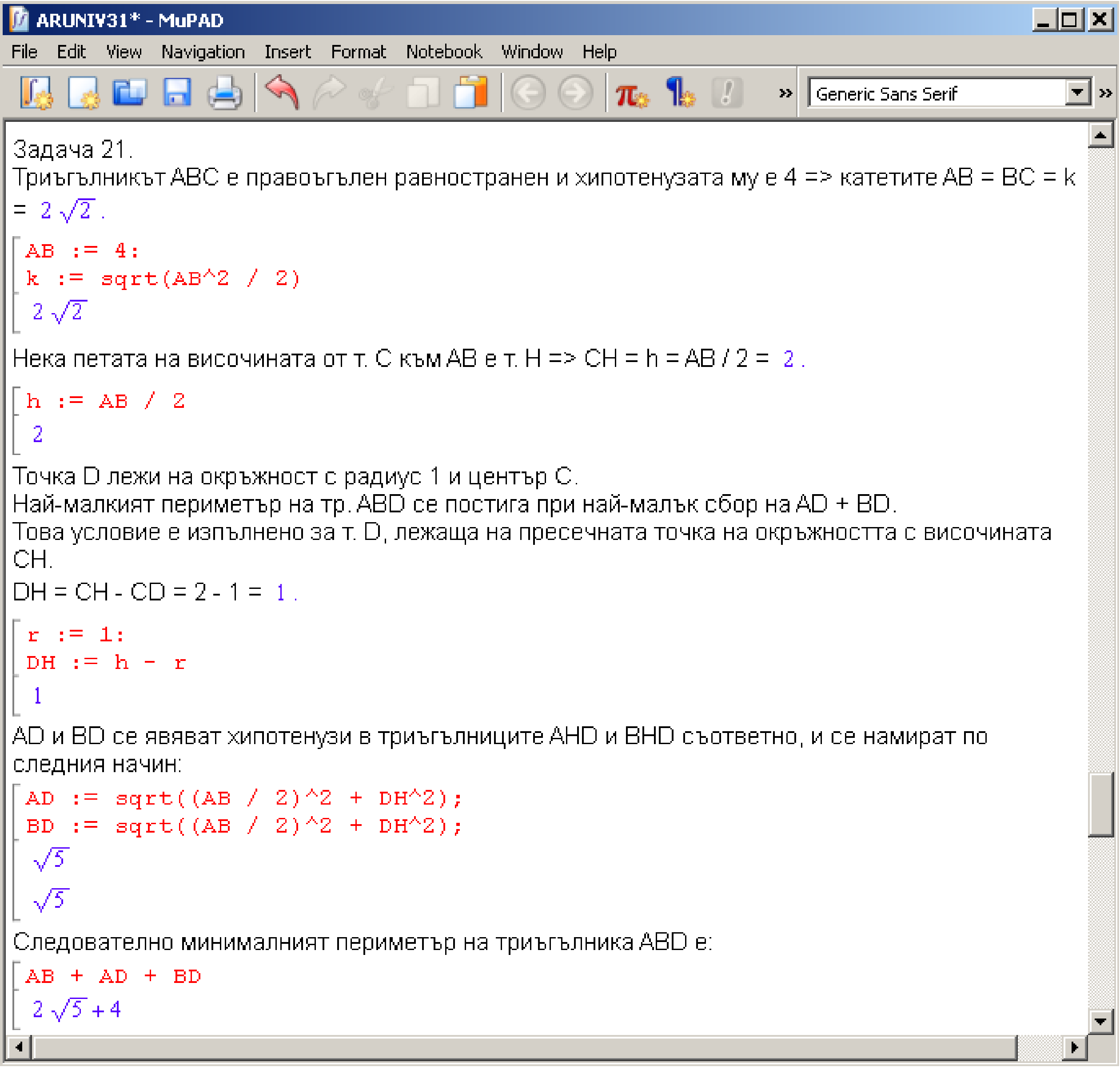

Fig.5. Solution of the Problem 6, CompMath14, Group A

Problem 7. (CompMath14/B-21) [6] Find all four-digit numbers, that are equal to the sum of the forth degrees of their digits.

Solution. One of the easiest ways to find the solution is to write a MATLAB function.

The code, followed by an explanation, is listed on Figure 6.

Four-digit numbers are in the 1000-9999 range, so this exact range will be processed using a for-cycle with a step equal to 1. The first digit is separated, using a modulo division by 10:

```
d1 = mod(i,10); %4
```

The second digit is separated in a similar way:

```
d2 = mod((i-d1)/10,10); %3
```

Here, the number currently processed is divided by 10, and `d1` is subtracted from it, as the first digit has already been removed.

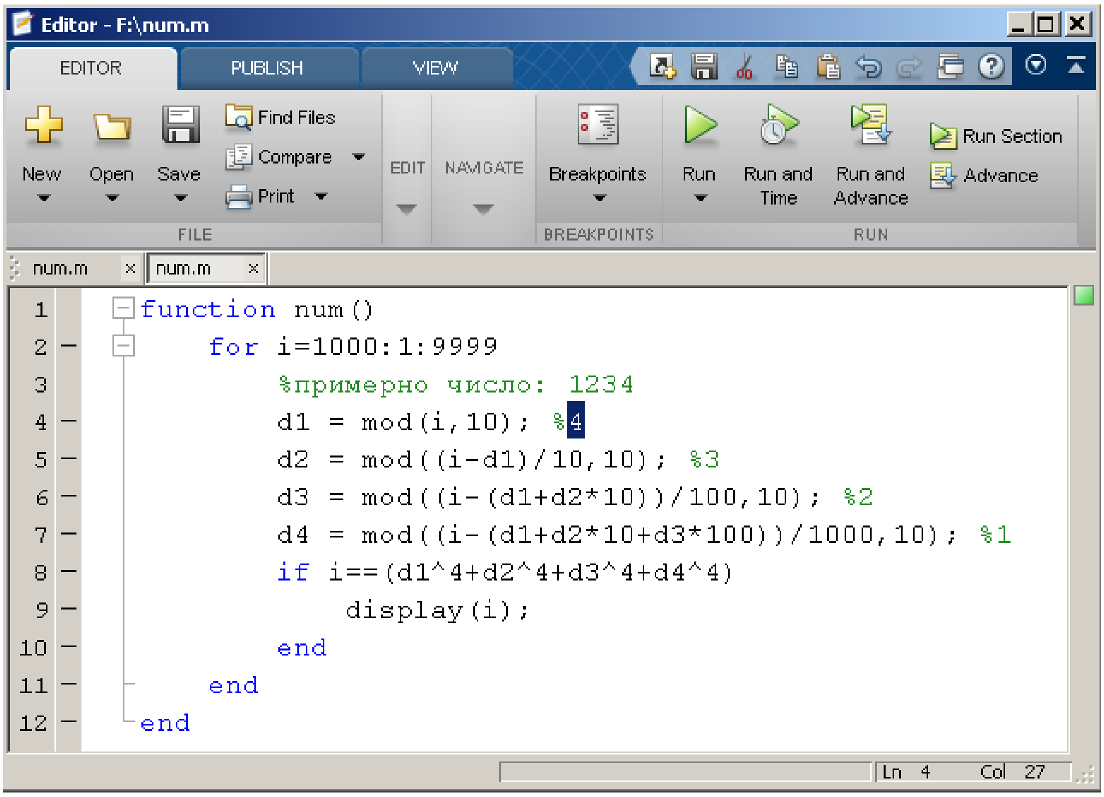

Fig.6. The code of the function for the Problem 6, CompMath14, Group B

Next is the third digit:

```
d3 = mod((i-(d1+d2*10))/100,10); %2
```

The number is divided by 100, and `d2*10+d1` are subtracted, since the first and second digit have been removed previously.

Finally, the fourth digit:

```
d4 = mod((i-(d1+d2*10+d3*100))/1000,10); %1
```

The expression follows the same logic as the previous ones.

Now that all four digits of the currently processed number are separated, all that's left is to compare it to the sum of its digits' fourth power and display it if so:

```
if i==(d1^4+d2^4+d3^4+d4^4)
    display(i);
```

Running the function in MATLAB with

```
>> num()
```

we see that 1634, 8208 and 9474 form the solution of this problem.

Problem 8. (CompMath14/B-12) [6] Compare the value of the expression

$$arctg\,\frac{1}{2} + arctg\,\frac{1}{5} + arctg\,\frac{1}{8}$$

with the number $\pi/4$.

Solution. To do so, a new variable should be allocated in a MuPAD Notebook first. Deleting its contents is optional, but recommended, to ensure no other values interfere with the calculation of this problem.

```
delete f;
```

Then, the value of the given expression is calculated, using MuPAD's built-in trigonometric functions:

```
f:=arctan(1/2)+arctan(1/5)+arctan(1/8);
```

As the result is not particularly clear in this format, it's a good idea to convert it to a floating-point number:

`float(f)`

It's also a good idea to convert $\pi/4$ to a floating-point number:

float(PI/4)

At first glance, we see that the numbers are equal. To prove this, we can use MuPAD's `testeq` function:

`testeq(f,PI/4)`

Hence, the expression is equal to $\pi/4$.

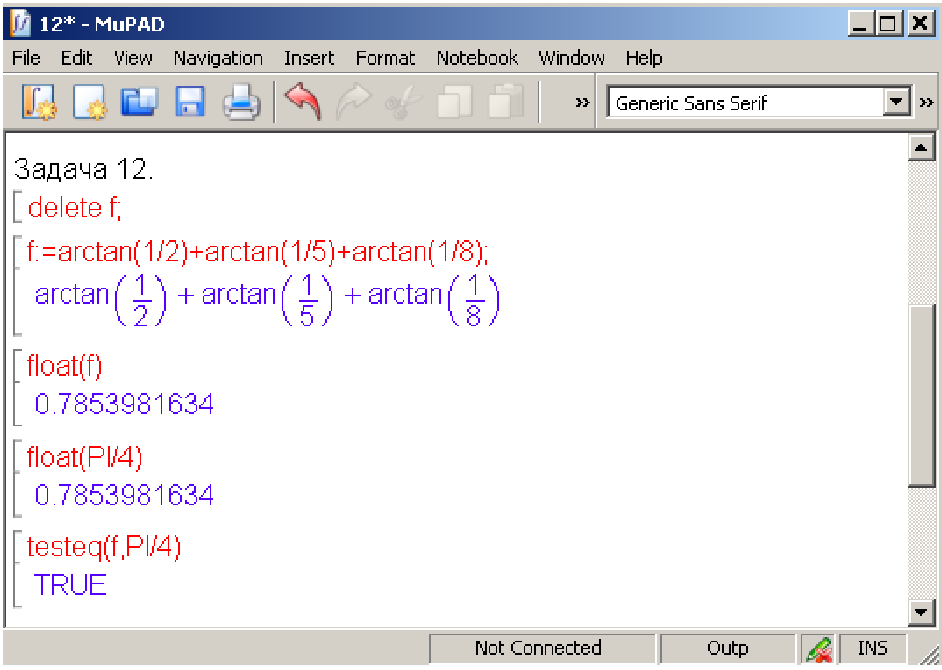

Fig.7. Solution of the Problem 7, CompMath14, Group B

CONCLUSION

The practical experience in training motivated students in Computer Mathematics with MATLAB and MUPAD and the results achieved in the Third National Computer Olympiad in Mathematics are a clear indication that the use of computer algebra systems in education is necessary as the only way out of the crisis in education and presents a real alternative to the traditional system of education.

The approach proposed by the authors provokes creative thinking and experimental spirit in the process of solving mathematical problems and supports the development of algorithmic and analytical way of thinking by the students that is needed for modeling and solving of practical problems.

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CONTACT ADDRESSES

Assistant Prof. Stefka Karakoleva
Department of Applied Mathematics and Statistics
Angel Kanchev University of Ruse
8, Studentska Str.
7017 Ruse, Bulgaria
Phone: (+359 82) 888 60
E-mail: skarakoleva@uni-ruse.bg
E-mail: skarakoleva@gmail.com

Asistant Prof. Ivan R. Georgiev
Department of Applied Mathematics and Statistics
Angel Kanchev University of Ruse
8, Studentska Str.
7017 Ruse, Bulgaria
Phone: (+359 82) 888 424
E-mail: irgeorgiev@uni-ruse.bg

Slavi G. Georgiev
Fac. of Natural Sciences and Education Electronics
Student in Financial Mathematics

Angel Kanchev University of Ruse
8, Studentska Str.
7017 Ruse, Bulgaria
Phone: +359 88 90 21 262
E-mail: georgiev.slavi.94@gmail.com

Pavel St. Zlatarov
Fac. of Electrical Engineering, and Automation
Student in Computer Systems and Technology
Angel Kanchev University of Ruse
8, Studentska Str.
7017 Ruse, Bulgaria
Phone: +359 88 53 08 826
E-mail: pzlatarov@gmail.com

РЕЗУЛТАТИ ОТ ОБУЧЕНИЕТО ПО КОМПЮТЪРНА МАТЕМАТИКА ЗА МОТИВИРАНИ СТУДЕНТИ В РУСЕНСКИЯ УНИВЕРСИТЕТ

Стефка Караколева, Иван Георгиев, Слави Георгиев, Павел Златаров

Русенски университет "Ангел Кънчев"

Резюме: *Статията представя обучението по компютърна математика за мотивирани студенти в Русенския университет и участието на отбора по MATLAB от Русенския университет в Третата национална студентска олимпиада по компютърна математика. Разгледана е учебната програма за мотивирани студенти. Представени са някои математически задачи и техните решения с MATLAB и MuPAD.*

Ключови думи: *Компютърна математика, СКА, MATLAB, MuPAD, преподаване, обучение.*

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