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SIMPLE COMPONENTS OF SEMISIMPLE GROUP ALGEBRAS OF FINITE P-GROUPS WITH MINIMAL COMMUTANTS

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Abstract: *Let G be a finite p -group with a commutant of order p and let K be a field of characteristic different from p. Then the group algebra KG is semisimple and Artinian and, by the classical theorem of Wederburn-Artin, KG will be decomposed in a direct sum of a finite number matrix rings over skew field. In this paper we describe the ideal which is generated from any minimal central idempotent e. This ideal is isomorphic to a matrix ring over a skew field A with identity e. The central idempotents are described in [5].*

The determination of the structure of the mentioned ideal is equivalent to (i) the determination of the dimension of the matrix ring and (ii) the determination of the skew field A. In this paper we solve the mentioned problems (i) and (ii).

Keywords: *p-groups, commutants, matrix rings, ideals*

INTRODUCTION

In 2004 year Ferraz [2] determines the number of the simple components of a group algebra FG over a field F, when the characteristic of F does not divide the order of the group G.

Let K be a real quadratically field and let U be a central division algebra of the quaternions over the field K. In [8] sufficient conditions are given which ensure U to be a subset of a simple component of the group algebra QG of the finite group G over the field Q of the rational numbers.

In [1] the authors consider a group G of order $p_1 p_2$, where p_1 and p_2 are prime and a finite field F_q of q-elements, such that q is relatively prime with $p_1 p_2$. They construct the set of the primitive central idempotents of the semisimple group algebra $F_q[G]$. The authors establish the structure of this algebra and its automorphism group.

Ferraz and Milies [3] find a method for the computation of the number of the simple components of the semisimple finite abelian group algebra and establish all possible cases, when this number is minimal.

Herman, Olteanu and Del Rio [4] consider a finite group G and the finite component of the rational group algebra QG corresponding to a given character. By this component the authors investigate the isomorphism of a cyclic cyclotomic algebra.

In this paper we suppose that G is a finite p-group with a commutant of order p and K is a field of characteristic different from p. For any minimal central idempotent e of KG we establish the construction of the ideal KGe. This ideal is isomorphic to a matrix ring over a skew field A with identity e. We determine the dimension of the matrix ring and the skew field A. For the proof of our two main results we use essentially the concepts of quadratically dependent and quadratically independent field and symplectic pair, the latter is early defined in [6].

PRELIMINARY RESULTS

Definition 2.1. *Let G be a finite p-group with a commutator subgroup G' of order p and c be a generating element of G'. If the equation $x^{p^k} = c$ has a solution in G and the* *equation* $x^{p^{k+1}}=c$ *does not have solution in* G*, then the number* p^k *is called a height of* c *in the group* G *and it is denoted by* $h_G(c)=p^k$*. If this equation is considered in the center* Z *of the group* G*, then this number is called a height of* c *in* Z *and it is denoted by* $h_Z(c)=p^k$*. If* $h_Z(c)=p^k$*, then either* $h_G(c)=p^k$ *or* $h_G(c)=p^{k+1}$*.*

Definition 2.2. *If* $h_Z(c)=h_G(c)$*, then the group* G *is called a group of a central type. If* $ph_Z(c)=h_G(c)$*, then we will call the group* G *a group of a non-central type.*

Definition 2.3. *Let* G *be a finite* p*-group and let* G' *have order* p*. Suppose, that* c *is a generating element of* G'*. Let* $A=\{a_1,a_2,...,a_s,b_1,b_2,...,b_s,c_1,c_2,...,c_r\}$ *be a system of generating elements of* G*, which has the following properties:*

1. *For every* $i\in\{1,2,...,s\}$ *it holds:* $a_i^{-1}b_ia_i=b_ic_i$*, and all other elements of* A *commute together.*
2. *If* G *is of a central type, then the cyclic group* $<c_1>$ *contains* G' *and the cyclic subgroup, which are generated from other generating elements, does not contain* G'*. If the group* G *is of a non-central type and either* $p\neq 2$ *or* $h_G(c)\geq 4$*, then the cyclic group* $<b_1>$ *contains* G' *and the other cyclic subgroups, which are generated from* A*, do not contain* G'*. If the group* G *is of a non-central type,* $p=2$ *and* $h_G(c)=2$*, then* $<b_1>$ *contains* G' *and at most one of the subgroups* $<a_i,b_i>$ *is isomorphic to the quaternions group of order eight.*
3. *Any element of the group* G *is represented uniquely as a product* $\prod_{\lambda=1}^{s}\prod_{\mu=1}^{s}\prod_{\nu=1}^{s}a_\lambda^{\alpha_\lambda}b_\mu^{\beta_\mu}c_\nu^{\gamma_\nu}$*, where the exponents* $\alpha_\lambda,\beta_\mu,\gamma_\nu$ *take values from 0 to the order of the corresponding coset in the factor group* G/G'*, with the exception of the element* c_1 *for a group of central type and the element* b_1 *for a group of a non-central type. In this case the exponents take values from 0 to the order of* c_1 *(to the order of* b_1*, respectively). Then we will say, that the elements* $a_1,a_2,...,a_s,b_1,b_2,...,b_sc_1,c_2,...,c_r$ *form a symplectic basis of the group* G*.*

Exactly the third property entitles us to call it a basis, since by these elements we obtain unique representation of the elements of G.

Definition 2.4. [6] *We will say, that the elements* $a,b\in G$ *form a symplectic pair* (a,b)*, if the following conditions hold:*

1. $[b,a]=c$*, where* $G'=<c>$*;*
2. *the element* a *has a minimal order in the coset* $aC_G(b)$*;*
3. *the element* b *has a minimal order in the coset* $bC_G(a)$*.*

Definition 2.5. *If the idempotent* e_0 *corresponds to the identity character of* G' *in* $K(G/G')$*, then the central idempotent of* KG*, which is a continuation of the idempotent* e_0*, is called an idempotent of the first type. If the idempotent* e_0 *corresponds to non-identity character of* KZ*, then the central idempotent of* KG*, which is a continuation of the idempotent* e_0*, is called an idempotent of the second type.*

Definition 2.6. *Let* χ *be a character of an abelian group. Then we denote by* $K(\chi)$ *the extension of the field* K*, which is obtained by the joining of the value of the character* χ*.*

Definition 2.7. *Let* K *be a field of characteristic, different from two. We call the field* K *a quadratically dependent field, if the equation* $x^2+y^2+1=0$ *has a solution in* K*. Otherwise we call it a quadratically independent field.*

Examples:

The fields R and Q are quadratically independent fields, since the indicated equation does not have a solution in these fields.

The field C is obviously a quadratically dependent field.

Any finite field of characteristic, different from two, is a quadratically dependent (we can easily prove that the indicated equation has always a solution).

Definition 2.8. *Let* e *be a minimal central idempotent of* KG *of second type with a corresponding character* χ *and let* (a,b) *be a symplectic pair belonging to a symplectic basis of the group* G*. Then we define an algebra* $A(a,b)$ *over the field* $K(\chi)$*, corresponding to the symplectic pair* (a,b)*, by the following way: a basis of* $A(a,b)$ *will be the various products* $a^i b^j$*, where* $i,j\in\{0,1,2,...,p-1\}$*. We define the multiplication in the algebra* $A(a,b)$ *by:* $\left(a^i b^j\right)\left(a^{i_1}b^{j_1}\right)=a^{i+i_1}b^{j+j_1}c^{ji_1}$ *,* $i_1,j_1\in\{0,1,2,...,p-1\}$*, where* $G'=<c>$ *and taking into consideration that* $a^p,b^p\in K(\chi)$*.*

Theorem 2.9. *Let* e *be a minimal central idempotent of second type of the algebra* KG*, which corresponds to a character* χ *and B is a symplectic basis of the group* G*, such that the non-central elements of B are distributed in the following symplectic pairs:* $(a_1,b_1),...,(a_s,b_s)$*. Then the ideal* KGe*, which is regarded as an algebra with identity* e*, is represented as a tensor product by the following way:*

$$KGe=A(a_1,b_1)e\otimes_{K(\chi)}A(a_2,b_2)e\otimes_{K(\chi)}...\otimes_{K(\chi)}A(a_s,b_s)e. \quad (1)$$

Proof. For any $i,j\in\{1,2,...,s\}$ the dimension of the algebra $A(a_i,b_i)e$ over $K(\chi)$ is p^2. Then the dimension of the tensor product will be p^{2s}. The elements $a_1^{\alpha_1}b_1^{\beta_1}a_2^{\alpha_2}b_2^{\beta_2}...a_s^{\alpha_s}b_s^{\beta_s}$, where $\alpha_1,\beta_1,...,\alpha_s,\beta_s\in\{0,1,...,p-1\}$, are a basis of KGe and the number of these basis elements is also p^{2s}. Therefore, the tensor product and the ideal KGe have the same dimension and the multiplication of the basis elements is accomplished by an identical way. Hence (1) holds.

It remains to clarify the structure of any algebra, which is included in (1).

Lemma 2.10. *Let* K *be a field of characteristic, different from two and let* A *be the quaternions algebra over* K *with a basis* $1,a,b,ab$ *with relations:* $a^2=-1,b^2=-1,ba=-ab$*. Then the algebra* A *is isomorphic to* $M(2,K)$*, if and only if, the field* K *is a quadratically dependent field. The quaternions algebra is a skew field with a dimension 4, if and only if,* K *is a quadratically independent field.*

Proof. Let K be a quadratically dependent field. Consequently, the equation $x^2+y^2+1=0$ has a solution in K. Consider the correspondences:

$a\rightarrow\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, $b\rightarrow\begin{pmatrix}x & y\\ y & -x\end{pmatrix}$, $1\rightarrow\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. Then $ab\rightarrow\begin{pmatrix}y & -x\\ -x & -y\end{pmatrix}$.

These matrices fulfill the relations, which satisfy the elements a and b. Hence, this correspondences can be continue by a linearly to a homomorphism of A in $M(2,K)$. It remains to prove, that this homomorphism is an isomorphism. Since $1, a, b$ and ab form a basis of A, then the images of these elements will form a basis of the algebra $M(2,K)$, if we prove, that these images are linear independent. Suppose, that the indicated images are linear dependent, i.e. there exists a linear combination of above four matrixes with coefficients $\lambda_1, \lambda_2, \lambda_3$ and λ_4, such that this combination is equal to the zero matrix. Then we obtain the following system:

$$\begin{vmatrix} \lambda_1 + \lambda_3 x + \lambda_4 y = 0 \\ \lambda_2 + \lambda_3 y - \lambda_4 x = 0 \\ -\lambda_2 + \lambda_3 y - \lambda_4 x = 0 \\ \lambda_1 - \lambda_3 x - \lambda_4 y = 0 \end{vmatrix}.$$

The coefficients determinant before the unknowns in this system is equal to -4, i.e. it is different from zero. Hence, the system has only a zero solution. Consequently, the given matrixes are linear dependent. If K is a quadratically dependent field, then $A \cong M(2,K)$.

Conversely, if $A \cong M(2,K)$, then we will prove, that the field K is a quadratically dependent field. The isomorphism $A \cong M(2,K)$ implies that the images of the basis elements of A will be some matrices, which satisfy the same relations. Let

$$a \to \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_3 & -\alpha_1 \end{pmatrix},\ b \to \begin{pmatrix} \beta_1 & \beta_2 \\ \beta_3 & -\beta_1 \end{pmatrix},\ ab \to \begin{pmatrix} \alpha_1\beta_1 + \alpha_2\beta_3 & \alpha_1\beta_2 - \alpha_2\beta_1 \\ \alpha_3\beta_1 - \alpha_1\beta_3 & \alpha_3\beta_2 + \alpha_1\beta_1 \end{pmatrix}.$$

Then we obtain the system:

$$\begin{vmatrix} \alpha_1^2 + \alpha_2\alpha_3 + 1 = 0 \\ \beta_1^2 + \beta_2\beta_3 + 1 = 0 \\ 2\alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 = 0 \end{vmatrix},$$

since $a^2 + 1 = 0,\ b^2 + 1 = 0,\ (ab)^2 + 1 = 0.$

If at least one of the elements $\alpha_2, \alpha_3, \beta_2$ and β_3 is equal to zero, then the equation $x^2 + y^2 + 1 = 0$ has a solution. Therefore, the field K is a quadratically dependent field. Further we will suppose, that these elements are different from zero. By a simple calculations we obtain, that K is a quadratically dependent field.

Let K is a quadratically independent field. We will prove that A is isomorphic to a skew field of the quaternions. Let $x = \lambda a + \mu b + \nu ab$ is an element from A (since x must to be a non-zero element, then at least one λ, μ and ν is different from zero). Then $x^2 = -\lambda^2 - \mu^2 - \nu^2$ and $x^2 \neq 0$, because K is a quadratically independent field. The element x is invertible, since $x^2 \neq 0$ and by a division of the last equality with $-\lambda^2 - \mu^2 - \nu^2$, we obtain $x \dfrac{x}{-\lambda^2 - \mu^2 - \nu^2} = 1$. Now we take an arbitrary element $x = \alpha_0 + \alpha_1 a + \alpha_2 b + \alpha_3 ab$, such that at least one $\alpha_0, \alpha_1, \alpha_2$ and α_3 is different from zero. We have prove, that x is invertible. We take an another element $y = \alpha_1 + \alpha_0 a$ and we calculate that $xy=(\alpha_0^2+\alpha_1^2)a+\alpha_1\alpha_2 b+(-\alpha_0\alpha_2+\alpha_1\alpha_3-\alpha_0\alpha_3)ab$. In this expression at least one of the coefficients before a,b and ab must be different from zero. The coefficient before a is $\alpha_0^2+\alpha_1^2$. Suppose $\alpha_0^2+\alpha_1^2=0$. However $\alpha_0\neq 0$. Therefore, we can divide the last equality with α_0^2. We obtain $\left(\frac{\alpha_1}{\alpha_0}\right)^2+1=0$, which is a contradiction, since K is a quadratically independent field. Consequently, xy is an invertible element. Hence, x is an invertible element. Therefore, A is a skew field.

Conversely, let A is isomorphic to the skew field D of the quaternions. Suppose, that K is a quadratically dependent field. We obtain, from above proved, that $A\cong M(2,K)$. However, in the other hand, $A\cong D$, which is a contradiction, since in the skew field of the quaternions there are no zero divisors while in $M(2,K)$ there are zero divisors. Hence, our assumption is not true. Therefore, K is a quadratically independent field. The lemma is proved.

The skew field from Lemma 2.10 is called a skew field of the quaternions over K.

Lemma 2.11. *Let (a,b) be a symplectic pair of the basis of the group G and let e be a minimal central idempotent of KG of the second type, which corresponds to the character χ. Let at least one of the following conditions be fulfilled:*

1. $p\neq 2$;

2. $K(\chi)\neq K$;

3. K is a quadratically dependent field;

4. the key subgroup of $G/Ker\chi$ is non-isomorphic to Q_8.

Then the algebra $A(a,b)e$ is isomorphic to $M(p,K(\chi))$.

Proof. The pair (a,b) is a symplectic pair. Therefore, $a^{p^n}=1, b^{p^m}=1, a^{-1}ba=bc, ac=ca, bc=cb$, where c is a generating element of the commutant G' of the group G. Applying the character χ, we obtain that $K(\chi)$ will be imbedded in $A(a,b)$ such that its image be generated from $a^p e, b^p e$ and ce. These elements generate the field $K(\chi)$, i.e. there exists an imbedding $\varphi: K(\chi)\to A(a,b)$, such that $\chi(a^p)=\varepsilon_k$, where ε_k is a root of the identity and $k\leq n-1$. We have $(a^p e)^{p^{n-1}}=a^{p^n}e=e,\ \chi(b^p)=\varepsilon_t,\ t\leq m-1$. Let, for example, $k\leq t$. Then $\varepsilon_k=\varepsilon_t^{p^{t-k}}$. Hence $a^p e=\varepsilon_t^{p^{t-k}}$. We take p^{t-k}-th degree of the last equality and obtain $\varphi(b^{p^{t-k+1}}e)=\varepsilon_t^{p^{t-k}}$. Then $a^p e b^{p^{t-k+1}}e=1$ and $\varphi((ab^{-p^{t-k}})^p e)=1$. Consequently, the symplectic pair $(ab^{-p^{t-k}},b)$ will generate a new symplectic pair and we can suppose, that $\varphi(a^p e)=1, \varphi(b^p e)=\varepsilon_t, t\leq m-1$. We consider now the following idempotents:

$$e_{ii}=\frac{1}{p}\left(1+b^{-ip^{t-1}}a+b^{-2ip^{t-1}}a^2+...+b^{-(p-1)ip^{t-1}}a^{p-1}\right), i\in\{0,1,2,...,p-1\}.$$

We introduce the following notation: $e_{ij}=e_{ii}b^{p^{t-1}(i-j)}$ for $a^p e b^{p^{t-k+1}}e=1$. One can prove, that these elements have the property: $e_{ij}e_{lk}=\begin{cases} e_{ik}, j=l \\ 0, j\neq l\end{cases}$.

It remains to prove, that e_{ij} are linear independent. We consider $\sum_{i=0}^{p-1}\sum_{j=0}^{p-1}\alpha_{ij}e_{ij}=0.$

We multiply this equality on the left with and on the right with and we obtain:
$\sum_{i=0}^{p-1}\sum_{j=0}^{p-1}\alpha_{ij}e_{li_1}e_{ij}e_{j_1k}=0.$

By the definition of $e_{ij}e_{lk}$, we have summands, which are distinct from zero only if $i_1=i$ and $j_1=j$. Therefore $\alpha_{ij}e_{ij}=0$. However $e_{ij}\neq 0$, since $e_{ii}\neq 0$. Therefore $\alpha_{ij}=0$ for every $a^p eb^{p^{t-k+1}}e=1$. Consequently, we have a linear independent. Hence, we obtain $A(a,b)e\cong M(p,K(\chi))$, since we have p-idempotents e_{ii} over $K(\chi)$.

Lemma 2.12. *Let (a,b) be a symplectic pair, which satisfies the conditions:*

1. $p=2$;

2. $K(\chi)=K$;

3. K is a quadratically independent field;

4. the key subgroup of $G/Ker\chi$ is isomorphic to Q_8.

Then $A(a,b)e\cong D$, where D is a skew field of the quaternions.

Proof. The conditions of the lemma imply that $K(\chi)$ is a quadratically independent field and $a^2\in Z$. Then $a^2\in K(\chi)$. But $K(\chi)=K$ and a^2 will be a root of the identity. The element a^2 can not be the fourth and a heigher root of the identity, since K is a quadratically independent field and, therefore, in K there is not such a root. The element a^2 coincides with G' and since the idempotent e is of the second type, then $\chi(c)=-1$ and $a^2e=ce=-e$. Consequently $a^2e=-e$. Analogously, we obtain $b^2e=-e$. Besides we have $a^{-1}bae=bce=-be,\ bae=-abe,\ (be)(ae)=-(ae)(be),\ (ae)^2=-e,\ (be)^2=-e$.

Therefore, the algebra $A(a,b)e$ is isomorphic to the quaternions algebra. Since K is a quadratically independent field, then by Lemma 2.11, we obtain $A(a,b)e\cong D$.

MAIN RESULTS

Theorem 3.1. *Let e be a minimal central idempotent of KG of the first type and let χ be its corresponding character, which continues the identity character of KG'. Then the ideal KGe is isomorphic to the field $K(\chi)$.*

Proof. We have $KG'e_0\cong K$. Then $KGe\cong K(G/G')$. Since $K(G/G')$ is an abelian group, then $K(G/G')e\cong K(\chi)$ (it follows from the theory of the commutative group algebra).

Theorem 3.2. *Let K be a field of characteristic different from p and let G be a p-group with a commutant of order p. Suppose, that e is a minimal central idempotent of KG of the second type and χ is the corresponding character of e. Then for the ideal KGe of KG the following cases hold:*

A) If at least one of the following conditions is fulfilled:

1) $p\neq 2$;

2) $K(\chi)\neq K$;

3) K is a quadratically independent field;

4) the key subgroup of $G/Ker\chi$ is isomorphic to Q_8,

then KGe is isomorphic to the matrix ring $M(n,K(\chi))$, where $K(\chi)$ is the field of the character χ, and $n=\sqrt{\frac{|G|}{|Z|}}$.

B) If the following conditions are fulfilled:

1') $p=2$;

2') $K(\chi)=K$;

3') K is a quadratically independent field;

4') the key subgroup of $G/Ker\chi$ is isomorphic to Q_8,

then $KGe\cong M(n,D)$, where D is the skew field of the quaternions over K and $n=\frac{1}{2}\sqrt{\frac{|G|}{|Z|}}$.

Proof.

Case A. We have $KGe\cong A(a_1,b_1)e\otimes_{K(\chi)}A(a_2,b_2)e\otimes_{K(\chi)}...\otimes_{K(\chi)}A(a_s,b_s)e.$

On the other hand we use lemma 2.12 for all factors. We obtain $A(a_i,b_i)e\cong M(p,K(\chi))$. We know from [7, Consequence b, p.211] that $M(n,L)\otimes_L M(m,L)\cong M(nm,L)$. Therefore,

$$KGe\cong A(a_1,b_1)e\otimes_{K(\chi)}A(a_2,b_2)e\otimes_{K(\chi)}...\otimes_{K(\chi)}A(a_s,b_s)e\cong M(p^s,K(\chi))=M(n,K(\chi)),$$

since $p^s=\sqrt{\frac{|G|}{|Z|}}=n.$

Case B. We have $KGe\cong A(a_1,b_1)e\otimes_{K(\chi)}A(a_2,b_2)e\otimes_{K(\chi)}...\otimes_{K(\chi)}A(a_s,b_s)e.$

Lemma 2.12 implies, that we have $A(a_i,b_i)e\cong D$ exactly for one factor and all other factors are isomorphic to $M(2,K)$, which are $s-1$. We have

$$KGe\cong D\otimes_K M(2,K)\otimes_K ...\otimes_K M(2,K).$$

Again, from [7, Consequence b, p.211], we obtain

$$KGe\cong A(a_1,b_1)e\otimes_{K(\chi)}A(a_2,b_2)e\otimes_{K(\chi)}...\otimes_{K(\chi)}A(a_s,b_s)e\cong D\otimes_K M(2^{s-1},K).$$

We have $D\otimes_K M(2^{s-1},K)\cong M(2^{s-1},D)$.

Since $2^s=\sqrt{\frac{|G|}{|Z|}}$, then $2^{s-1}=\frac{1}{2}\sqrt{\frac{|G|}{|Z|}}$. The proof is completed.

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ПРОСТИ КОМПОНЕНТИ НА ПОЛУПРОСТИ ГРУПОВИ АЛГЕБРИ НА КРАЙНИ Р-ГРУПИ С МИНИМАЛНИ КОМУТАНТИ

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Резюме: *Нека G е крайна p-група с комутант от ред p и нека K е поле с характеристика, различна от p. Тогава груповата алгебра KG е полупроста и Артинова и съгласно класическата теорема на Ведербърн-Артин, тя се разлага в директна сума на краен брой матрични пръстени над тяло. В тази работа определяме идеал, породен от централен идемпотент e. Този идеал е изоморфен на матричен пръстен над тяло A с единица e. Централните идемпотенти са определени в [5]. Определянето на строежа на споменатия идеал е еквивалентно на (i)определяне на размерността на матричния пръстен и (ii)определяне на тялото A. В тази работа даваме решение на проблемите (i) и (ii).*

Ключови думи: *p-групи, комутанти, матрични пръстени, идеали*

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