# PROCEEDINGS

## of the Union of Scientists - Ruse

Book 5

## Mathematics, Informatics and Physics

Volume 13, 2016

RUSE

# PROCEEDINGS
## OF THE UNION OF SCIENTISTS - RUSE



**Union of Scientists - Ruse**
16, Konstantin Irechek Street
7000 Ruse
BULGARIA
Phone: (++359 82) 828 135,
(++359 82) 841 634
E-mail: suruse@uni-ruse.bg
web: suruse.uni-ruse.bg

**Contacts with Editor**
Phone: (++359 82) 888 738
E-mail: zzdravkova@uni-ruse.bg

# PROCEEDINGS
**of the Union of Scientists – Ruse**

**ISSN 1314-3077**

**The Ruse Branch of the Union of Scientists in Bulgaria** was founded in 1956. Its first Chairman was Prof. Stoyan Petrov. He was followed by Prof. Trifon Georgiev, Prof. Kolyo Vasilev, Prof. Georgi Popov, Prof. Mityo Kanev, Assoc. Prof. Boris Borisov, Prof. Emil Marinov, Prof. Hristo Beloev. The individual members number nearly 300 recognized scientists from Ruse, organized in 13 scientific sections. There are several collective members too – organizations and companies from Ruse, known for their success in the field of science and higher education, or their applied research activities. The activities of the Union of Scientists – Ruse are numerous: scientific, educational and other humanitarian events directly related to hot issues in the development of Ruse region, including its infrastructure, environment, history and future development; commitment to the development of the scientific organizations in Ruse, the professional development and growth of the scientists and the protection of their individual rights.

The Union of Scientists – Ruse (US – Ruse) organizes publishing of scientific and popular informative literature, and since 1998 – the "Proceedings of the Union of Scientists- Ruse".

**BOOK 5**

**"MATHEMATICS, INFORMATICS AND PHYSICS"**

**VOLUME 13**

CONTENTS

**Mathematics**



**Informatics**

**BOOK 5**

**"MATHEMATICS, INFORMATICS AND PHYSICS"**

**VOLUME 13**

web: suruse.uni-ruse.bg

# EFFICIENCY OF ALLOCATION TABLE METHOD FOR SOLVING TRANSPORTATION MAXIMIZATION PROBLEM

**Md Sharif Uddin[1,2], M. Nazrul Islam[1], Iliyana Raeva[2] , Aminur Rahman Khan[1]**

*[1]Jahangirnagar University, [2]Angel Kanchev University of Ruse*

***Abstract:*** *A maximization transportation problem can be solved by the traditional transportation algorithms. In this paper, solution procedure for solving transportation maximization problems using the newly introduced Allocation Table Method has been illustrated. This study carried out to justify the efficiency of Allocation Table Method for solving maximization transportation problems. During this process, it is observed that the Allocation Table Method (ATM) is an efficient procedure for solving transportation maximization problems.*

***Key words:*** *Allocation Cell Value, Allocation Table Method, Maximization Problem, Optimum Solution, Transportation problems.*

## INTRODUCTION

In general, Transportation Problem (TP) is known as a minimization problem as its objective is to schedule shipments of a single commodity from a number of sources to a number of destinations with minimal transportation cost. Transportation model is famous in Operations Research for its vast application in the various fields of real life. It can be formulated as a Linear Programming Problem, because of its special structure. The TP is originally introduced by Hitchcock [1] in 1941. Efficient methods of solution derived from the simplex algorithm were flourished, primarily by Dantzig [2] in1951 and then by Charnes, Cooper and Henderson [3] in 1953.

To describe the transportation problem, following notations are to be used:

$m$ Total number of sources/origins
$n$ Total number of destinations
$S_i$ Amount of supply at source $i$
$d_j$ Amount of demand at destination $j$
$c_{ij}$ Unit transportation cost from source $i$ to destination $j$
$x_{ij}$ Amount to be shipped from source $i$ to destination $j$

Using the above notations network representation of the transportation problems is shown in Fig. 1.

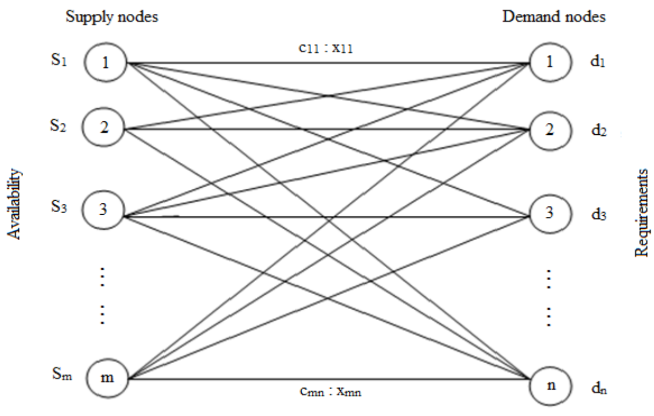


**Fig. 1. Network Diagram for Transportation Problem**

The general and accepted form of the transportation problem is presented by the following scheme:

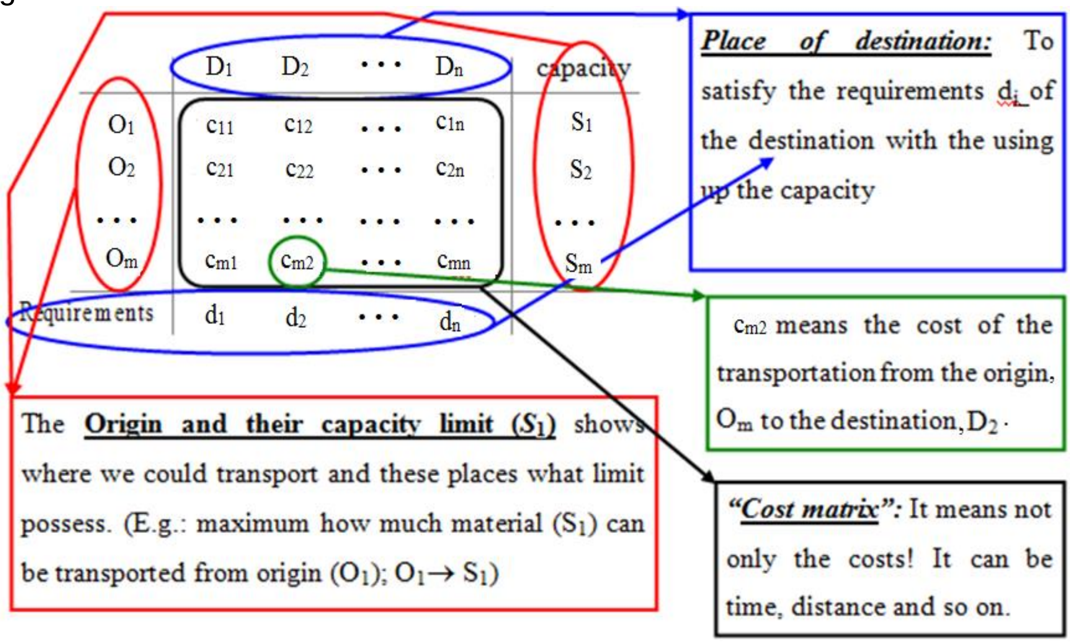


**Fig. 2. Transportation Problem Scheme**

The objective of the model is to determine the unknowns' $x_{ij}$ that will minimize the total transportation cost while satisfying the supply and demand restrictions. Basing on this objective transportation can be formulated as:

Minimize: $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to : $$\sum_{j=1}^{n} x_{ij} \le S_i \; ; \; i = 1, 2, \dots\dots, m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \; ; \; j = 1, 2, \dots\dots, n$$

$x_{ij} \ge 0$, for all $i$ and $j$.

Till now, several researchers studied extensively to solve cost minimizing transportation problems in various ways. The well reputed transportation algorithms like North West Corner Method (NWCM) [4], Least Cost Method (LCM) [4], Vogel's Approximation Method (VAM) [4] and Extremum Difference Method (EDM) [5] have been basically introduced in order to solve transportation problems.

Now-a-days, many researchers are developing new methods for solving cost minimization transportation problems [6-8]. Again these methods may be used to solve maximization transportation problems [9] and also time minimization transportation Problem [10]. The maximization problem can be converted into an equivalent minimization problem by multiplying the given profit matrix by -1. The converted problem can then be solved by any usual method. Finally, obtain the maximum profit by the relation Max z = - {Min (-z)}.

In this paper the newly introduced allocation table method (ATM) [8] has been studied to describe the procedure of this method in solving maximization transportation problems. The procedure has been elaborated and also been justified by solving a good number of numerical problems. During this progression it is observed that ATM is an efficient procedure for solving transportation problems.

**ALLOCATION TABLE METHOD (ATM) FOR SOLVING THE TRANSPORTATION MAXIMIZATION PROBLEM**

Recently developed ATM for solving cost minimizing transportation problems is illustrated below how it is to be used in case of solving transportation maximization problems.

- Step-1: Construct a Transportation Table (TT) from the given transportation problem.
- Step-2: Ensure whether the TP is balanced or not, if not, make it balanced.
- Step-3: Select Minimum Odd Cost (MOC) from all the cost cells of TT. If there is no odd cost in the cost cells of the TT, keep on dividing all the cost cells by 2 (two) till obtaining at least an odd cost in the cost cells.
- Step-4: Form a new table which is to be known as allocation table (AT) by keeping the MOC in the respective cost cell/cells as it was/were, and subtract selected MOC only from each of the odd cost valued cells of the TT. Now all the cell values are to be called as Allocation Cell Value (ACV) in AT.
- Step-5: Now identify the maximum ACV and allocate minimum of supply/demand at the place of selected ACV in the AT. In case of same ACVs, select the ACV where maximum allocation can be made. Again in case of same allocation in the ACVs, choose the maximum cost cell which is corresponding to the cost cells of TT formed in Step-1 (i.e. this maximum cost cell is to be found out from the TT which is constructed in Step 1). Again if the cost cells and the allocations are equal, in such case choose the nearer cell to the minimum of demand/supply which is to be allocated. Now if demand is satisfied delete the column and if it is supply delete the row.
- Step-6: Repeat Step 5 until the demand and supply are exhausted.
- Step-7: Now transfer this allocation to the original TT.
- Step-8: Finally calculate the total profit of the TT. This calculation is the sum of the product of cost and corresponding allocated value of the TT.

**NUMERICAL ILLUSTRATION**

**Example 1.**

Consider the following profit maximization transportation problem (Table 1).

| Machines | **Products** | | | **Capacity** |
|---|---|---|---|---|
| | **$P_1$** | **$P_2$** | **$P_3$** | |
| **$M_1$** | **10** | **15** | **12** | **50** |
| **$M_2$** | **6** | **9** | **20** | **30** |
| **$M_3$** | **21** | **13** | **7** | **20** |
| **$M_4$** | **23** | **2** | **25** | **60** |
| **Demand** | **80** | **70** | **10** | |

**Table 1. Data of Example 1.**

**Solution of Example 1.**

Formation of allocation table and allocation in the various cells for Example 1 is shown in Table 2 given below.

| Machines | Products | | | Capacity |
|---|---|---|---|---|
| | **$P_1$** | **$P_2$** | **$P_3$** | |
| **$M_1$** | 10 | 50<br>8 | 12 | **50** |
| **$M_2$** | 6 | 20<br>2 | 10<br>20 | **30** |
| **$M_3$** | 20<br>14 | 6 | 7 | **20** |
| **$M_4$** | 60<br>16 | 2 | 18 | **60** |
| **Demand** | **80** | **70** | **10** | |

**Table 2. Allocation table and allocation in the various cells**

Final allocation to obtain the maximum profit is shown in Table 3 after shifting the allocation to the original problem.

| Machines | Products | | | Capacity |
|---|---|---|---|---|
| | **$P_1$** | **$P_2$** | **$P_3$** | |
| **$M_1$** | 10 | 50<br>15 | 12 | **50** |
| **$M_2$** | 6 | 20<br>9 | 10<br>20 | **30** |
| **$M_3$** | 20<br>21 | 13 | 7 | **20** |
| **$M_4$** | 60<br>23 | 2 | 25 | **60** |
| **Demand** | **80** | **70** | **10** | |

**Table 3.** Final allocation after shifting the allocation to original

- Finally, maximum profit according to allocation table method is,

(50x15+20x9+10x20+20x21+60x23) = 2930

**Example 2.**

Four products are produced by three machines and their profit margins are given in the following Table 4. Find a suitable plan of production in machines so that the capacities and requirements are satisfied and the profit is maximized.

| Machines | Products | | | | Capacity |
|---|---|---|---|---|---|
| | **$P_1$** | **$P_2$** | **$P_3$** | **$P_4$** | |
| **$M_1$** | 16 | 14 | 11 | 25 | **140** |
| **$M_2$** | 18 | 29 | 12 | 27 | **180** |
| **$M_3$** | 14 | 23 | 16 | 12 | **70** |
| **Demand** | **60** | **100** | **150** | **80** | |

**Table 4. Data of Example 2.**

**Solution of Example 2.**

Formation of allocation table and allocation in the various cells for Example 2 is shown in Table 5 given below.

| Machines | Products | | | | Capacity |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | |
| $M_1$ | 16 | 14 | 80<br>11 | 60<br>14 | 140 |
| $M_2$ | 60<br>18 | 100<br>18 | 12 | 20<br>16 | 180 |
| $M_3$ | 14 | 12 | 70<br>16 | 12 | 70 |
| Demand | 60 | 100 | 150 | 80 | |

**Table 5. Allocation table and allocation in the various cells.**

Final allocation to obtain the maximum profit is shown in the Table 6 after shifting the allocation to the original problem.

| Machines | Products | | | | Capacity |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | |
| $M_1$ | 16 | 14 | 80<br>11 | 60<br>25 | 140 |
| $M_2$ | 60<br>18 | 100<br>29 | 12 | 20<br>27 | 180 |
| $M_3$ | 14 | 23 | 70<br>16 | 12 | 70 |
| Demand | 60 | 100 | 150 | 80 | |

**Table 6**. Final allocation after shifting the allocation to the given problem.

- Finally, maximum profit according to allocation table method is,

(80x11+60x25+60x18+100x29+20x27+70x16) = 8020

**Example 3.**

Consider the following profit maximization transportation problem (Table 7).

| Machi-nes | Products | | | | Capaci-ty |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | |
| $M_1$ | 6 | 4 | 1 | 5 | 14 |
| $M_2$ | 8 | 9 | 2 | 7 | 18 |
| $M_3$ | 4 | 3 | 6 | 2 | 7 |
| Demand | 6 | 10 | 15 | 8 | |

**Table 7. Data of the Example 3.**

**Solution of Example 3.**

Formation of allocation table and allocation in the various cells for Example 3 is shown in Table 8 given below.

| Machines | Products | | | | Capacity |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | |
| $M_1$ | 6 | 4 | 8<br>1 | 6<br>4 | **14** |
| $M_2$ | 6<br>8 | 10<br>8 | 2 | 2<br>6 | **18** |
| $M_3$ | 4 | 2 | 7<br>6 | 2 | **7** |
| **Demand** | **6** | **10** | **15** | **8** | |

**Table 8. Allocation table and allocation in the various cells.**

Final allocation to obtain the maximum profit is shown in the Table 9 after shifting the allocation to the original given problem.

| Machines | Products | | | | Capacity |
|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | |
| $M_1$ | 6 | 4 | 8<br>1 | 6<br>5 | **14** |
| $M_2$ | 6<br>8 | 10<br>9 | 2 | 2<br>7 | **18** |
| $M_3$ | 4 | 3 | 7<br>6 | 2 | **7** |
| **Demand** | **6** | **10** | **15** | **8** | |

**Table 9.** Final allocation after shifting the allocation to the given problem.

- Finally, maximum profit according to allocation table method is,

(8x1+6x5+6x8+10x9+2x7+7x6) =232

**Example 4.**

Consider the following profit maximization transportation problem (Table 10).

| Machines | Products | | | | | | Capacity |
|---|---|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | |
| $M_1$ | 35 | 41 | 28 | 16 | 20 | 12 | **320** |
| $M_2$ | 14 | 21 | 28 | 30 | 15 | 24 | **180** |
| $M_3$ | 45 | 18 | 17 | 29 | 26 | 19 | **200** |
| $M_4$ | 21 | 23 | 16 | 11 | 22 | 20 | **300** |
| $M_5$ | 41 | 16 | 15 | 17 | 21 | 28 | **300** |
| **Demand** | **225** | **225** | **200** | **200** | **275** | **175** | |

**Table 10. Data of Example 4.**

**Solution of Example 4.**

Formation of allocation table and allocation in the various cells for Example-4 is shown in Table 11 given below.

| Machines | Products | | | | | | Capacity |
|---|---|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | |
| $M_1$ | 24 | 225 / 30 | 95 / 28 | 16 | 20 | 12 | **320** |
| $M_2$ | 14 | 10 | 28 | 180 / 30 | 4 | 24 | **180** |
| $M_3$ | 200 / 34 | 18 | 6 | 18 | 26 | 8 | **200** |
| $M_4$ | 10 | 12 | 25 / 16 | 11 | 275 / 22 | 20 | **300** |
| $M_5$ | 25 / 30 | 16 | 80 / 4 | 20 / 6 | 10 | 175 / 28 | **300** |
| **Demand** | **225** | **225** | **200** | **200** | **275** | **175** | |

**Table 11. Allocation table and allocation in the various cells.**

The final allocation to obtain the maximum profit is shown in Table 12 after shifting the allocation to the original problem.

| Machines | Products | | | | | | Capacity |
|---|---|---|---|---|---|---|---|
| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | |
| $M_1$ | 35 | 225 / 41 | 95 / 28 | 16 | 20 | 12 | **320** |
| $M_2$ | 14 | 21 | 28 | 180 / 30 | 15 | 24 | **180** |
| $M_3$ | 200 / 45 | 18 | 17 | 29 | 26 | 19 | **200** |
| $M_4$ | 21 | 23 | 25 / 16 | 11 | 275 / 22 | 20 | **300** |
| $M_5$ | 25 / 41 | 16 | 80 / 15 | 20 / 17 | 21 | 175 / 28 | **300** |
| **Demand** | **225** | **225** | **200** | **200** | **275** | **175** | |

**Table 12. Allocation table and allocation in the various cells.**

- Finally, maximum profit according to allocation table method is,

(225x41+95x28+180x30+200x45+25x16+275x22+25x41+80x15+20x17+175x28) =40200

**Results and Discussion**

To analyze the performance of allocation table method, various problems have been solved and a comparative study also was carried out among the results obtained by various methods including the ATM, which is shown in Table 13 given below.

| Method | Total Profit | | | |
|---|---|---|---|---|
| | Ex. 1 | E | Ex. 3 | Ex. 4 |
| North West Corner Method | 1290 | 5 | 137 | 3237 |
| Least Cost Method | 2810 | 8 | 232 | 4020 |
| Vogel's Approximation | 2930 | 8 | 232 | 4020 |
| Extremum Difference Method | 2930 | 8 | 232 | 4020 |
| Proposed Approach (ATM) | 2930 | 8 | 232 | 4020 |
| Optimum Solution | 2930 | 8 | 232 | 4036 |

**Table 13. Results obtained by various methods.**

From the above Table 13 it is observed that the allocation table method yields results which are also effective to solve transportation problem.

## CONCLUSION

Obtaining an initial feasible solution is the prime condition to find the optimal solution for a transportation problem. There is no unique method which can be claimed as the best solution procedure to obtain optimal solution for transportation problems. But the efficiency and effectiveness of the initial feasible solution finding procedure depends on few factors, like, the procedure is complicated or not, initial feasible solution is nearer to optimal or not, is it time consuming or not etc. Considering all these factors, the allocation method can be used to find the initial basic feasible solution of the maximization transportation problems as like as the other traditional methods.

## REFERENCES

[1] F.L. Hitchcock, "The distribution of a Product from Several Sources to Numerous Localities", *Journal of Mathematics and Physics*, vol. 20, pp. 224-230, 1941.

[2] G.B. Dantzig, *Application of the Simplex Method to a Transportation Problem, Activity Analysis of Production and Allocation,* (T.C. Koopmans ed), New York: John Wiley and Sons, pp. 359-373, 1951.

[3] A. Charnes, W.W. Cooper and A. Henderson, *An Introduction to Linear Programming*, John Wiley & Sons, New York, 1953.

[4] Hamdy, A. Taha, *Operations Research: An Introduction*, 8th Edition, Pearson Prentice Hall, Upper Saddle River, New Jersey 07458, pp. 193-221, 2007.

[5] H.S. Kasana and K.D. Kumar, *Introductory Operations Research: Theory and Applications*, Springer International Edition, New Delhi, pp. 221-243, 2005.

[6] Aminur Rahman Khan, Adrian Vilcu, Nahid Sultana and Syed Sabbir Ahmed, "Determination of Initial Basic Feasible Solution of a Transportation Problem: A TOCM-SUM Approach", *Buletinul Institutului Politehnic Din Iasi, Romania, Secţia Automatica si Calculatoare*, Tomul LXI (LXV), Fasc. 1, pp. 39-49, 2015.

[7] Aminur Rahman Khan, Adrian Vilcu, Md. Sharif Uddin and Florina Ungureanu, "A Competent Algorithm to find the initial Basic Feasible Solution of Cost Minimization Transportation Problem", *Buletinul Institutului Politehnic Din Iasi, Romania, Secţia Automatica si Calculatoare*, Tomul LXI (LXV), Fasc. 2, pp. 71-83, 2015.

[8] Mollah Mesbahuddin Ahmed, Aminur Rahman Khan, Md. Sharif Uddin and Faruque Ahmed, "A New Approach to Solve Transportation Problems", *Open Journal of Optimization*, vol. 5, no. 1, pp. 22-30, 2016 .

[9] Md. Amirul Islam, M. Sharif Uddin, S. M. Mahmudul Hasan, Farhana Rashid andM. Abdul Malek, "Profit Maximization of a Manufacturing Company: An Algorithmic

Approach", *Jahangirnagar Journal of Mathematics & Mathematical Sciences*, vol. 28, pp. 29-37, 2013.

[10] M.M. Ahmed, Islam, Md.A., Katun, M., Yesmin, S. and Uddin, Md.S., "New Procedure of Finding an Initial Basic Feasible Solution of the Time Minimizing Transportation Problems", *Open Journal of Applied Sciences*, vol. 5, pp. 634-640, 2015.

**CONTACT ADDRESSES**
*Dr. Md. Sharif Uddin, M. Nazrul Islam, Aminur Rahman Khan*
Department of Mathematics
Jahangirnagar University
Savar, Dhaka-1342, Bangladesh
Phone: +8801726452777
E-mail: msharifju@yahoo.com; www.omrg.education

Assistant prof. PhD Iliyana Raeva
Department of Applied Mathematics an  and Statistics
Angel Kanchev University of Ruse
8, Studentska Str.
7017 Ruse,BULGARIA
Phone: (+359 82) 888 606
E-mail: iraeva@uni-ruse.bg

## ЕФЕКТИВНОСТ НА ТАБЛИЧНИЯ МЕТОД НА РАЗПРЕДЕЛЕНИЕ ЗА РЕШАВАНЕ НА МАКСИМИЗАЦИОННАТА ТРАНСПОРТНА ЗАДАЧА

**Md Sharif Uddin[1,2], M. Nazrul Islam[1], Илияна Раева[2] , Aminur Rahman Khan[1]**

*Jahangirnagar University, Русенски университет "Ангел Кънчев"*

***Резюме:*** *Проблемът за максимизиране на транспортните разходи може да бъде решен чрез традиционните алгоритми. В настоящия доклад е използван нов табличен метод за разпределение (Allocation Table Method – ATM) при решаване на транспортната максимизационна задача. Изследвана е и е доказана ефективността на ATM за решаване на максимизационната транспортна задача.*

***Ключови думи:*** *Разпределяне на стойност на клетка, Табличен метод на разпределение, Максимизационна задача, Оптимално решение, Транспортна задача.*

ISSN 1314-3077
9 771314 307000