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BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

CONTENTS

BOOK 5

"MATHEMATICS, INFORMATICS AND PHYSICS"

VOLUME 10

This is the jubilee 10-th volume of book 5 Mathematics, Informatics and Physics. The beginning was in Spring, 2001, when the colleagues of the former section Mathematics and Physics decided to start publishing our own book of the Proceedings of the Union of Scientists – Ruse. The first volume included 24 papers. Through the years there have been authors not only from the Angel Kanchev University of Ruse but as well as from universities of Gabrovo, Varna, Veliko Tarnovo and abroad – Russia, Greece and USA.

Since the 6-th volume the preparation and publishing of the papers began to be done in English.

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IDENTITIES OF $M_2(E)$ ARE IDENTITIES FOR CLASSES OF SUBALGEBRAS OF $M_n(E)$ AS WELL

Tsetska Rashkova

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Abstract: *Vishne gave in [13] the explicit form of two polynomial identities of degree 8 for the matrix algebra $M_2(E)$, where E is the Grassmann algebra. In the paper we show that some subalgebras of $M_n(E)$ for arbitrary $n>2$ satisfy these two identities as well. By a programme written in the system Mathematica a computer testing the value of the corresponding polynomials is done for one of the considered subalgebras in the case $n=3$ over the finite dimensional Grassmann algebra E_4.*

Keywords: *Grassmann algebra, T-ideal, standard polynomial, multilinear identities, pattern.*

PRELIMINARIES

Let E denote the infinite dimensional Grassmann algebra on a countable dimension vector space over a field K of characteristic zero. A common presentation of E is the following one:

$$E=\langle e_1,e_2,\ldots \mid e_ie_j+e_je_i=0,\ i,j=1,2,\ldots\rangle.$$

We list some well known facts concerning the algebra E:

Proposition 1. [8, Corollary, p. 437] *The T-ideal $T(E)$ is generated by the identity $[x_1,x_2,x_3]=[[x_1,x_2],x_3]=0$, where $[x_1,x_2]=x_1x_2-x_2x_1$.*

Proposition 2. [2, Lemma 6.1] *The algebra E satisfies $S_n(x_1,\ldots,x_n)^k=0$ for all $n,k\geq 2$, where $S_n(x_1,\ldots,x_n)$ is the standard polynomial of degree n.*

The algebra E is in the mainstream of resent research in PI theory. Its importance is connected with the structure theory for the T-ideals of identities of associative algebras developed by Kemer. In [7, Theorem 1.2] he proved that any T-prime T-ideal can be obtained as the T-ideal of identities of one of three algebras, among which is the algebra $M_n(E)$.

Basic results, concerning both the algebras E and $M_n(E)$ could be found in [3,4,6].

In [1, p.356] some open questions in PI theory were stated. We mention two of them:

- Describe the identities of minimal degree of $M_n(E)$.
- Find a set of identities that generate the T-ideal of the identities of $M_n(E)$.

A. Popov and U. Vishne put the beginning of the investigations on the topic. In [10] Popov proved that the algebra $M_n(E)$ has no identities of degree $4n-2$. In [13] Vishne described an efficient way to use the $Sym(n)$-module structure of the ideal of multilinear identities in the computation of polynomial dentities of degree n of a given algebra. The method was used to show that $M_2(E)$ has identities of degree 8, but of no smaller degree. Two explicit identities of degree 8 were shown in [13].

In [9] using the system *Mathematica* a computer proof of the result of Vishne was given by Rashkova and Mihova.

Here we consider classes of subalgebras of $M_n(E)$, introduced in [11, 5]. They all satisfy the identity $X_4[X_1, X_2, X_3] = 0$. It appeared that if the identity $X_4[X_1, X_2, X_3] = 0$ holds in an algebra, the algebra satisfies the identities of degree 8, described by Vishne in [13], too. The proof of this statement is given below.

At the beginning we need some details from [13] for describing the considered multilinear polynomials of degree 8.

A *pattern* is a finite sequence of the letters A, B. If π is a pattern with a appearances of A and b of B, we denote by $\pi(x_1,\ldots,x_a;y_1,\ldots,y_b)$ the product of variables where the $x's$ and $y's$ are combined according to π. For example $ABBA(x_1,x_2;y_1,y_2) = x_1y_1y_2x_2$. A coefficient in front of a pattern π means that the monomial should be multiplied by that coefficient.

Let

$$P_\pi^+ = \sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)\pi(x_{\sigma(1)},\ldots,x_{\sigma(a)};y_{\tau(1)},\ldots,y_{\tau(b)}),$$

$$P_\pi^- = \sum_{\sigma\in Sym(a),\tau\in Sym(b)} sign(\sigma)sign(\tau)\pi(x_{\sigma(1)},\ldots,x_{\sigma(a)};y_{\tau(1)},\ldots,y_{\tau(b)})$$

and

$$\mathrm{P} = \begin{pmatrix} +AAAABAAB, & +AABBAAAA, & -AABAAAAB, \\ -AAAABBAA, & -BAABAAAA, & +BAAAABAA \end{pmatrix}.$$

We construct the polynomial

$$T_1(x_1,\ldots,x_6;y_1,y_2) = \sum_{\pi\in\mathrm{P}} (P_\pi^- + P_\pi^+). \qquad (1)$$

In it only the monomials with y_1 preceeding y_2 appear.

For

$$\mathrm{PP} = \begin{pmatrix} -AAABAABB, & -AABBAABA, & +ABBAABAA, \\ +AAABBAAB, & +AABAABBA, & -ABAABBAA \\ -ABBAAAAB, & +BAABBAAA, & -BAAAABBA \\ +ABAAAABB, & -BBAABAAA, & +BBAAAABA \end{pmatrix}$$

we consruct analogously

$$T_2(x_1,\ldots,x_5;y_1,y_2,y_3) = \sum_{\pi\in\mathrm{PP}} (P_\pi^- + P_\pi^+). \qquad (2)$$

The polynomial T_2 has only the monomials in which the order of y_1, y_2, y_3 is even.

Proposition 3. [13, Corollary 4.2] *T_1 and T_2 are multilinear identities of degree 8 of $M_2(E)$.*

Now we give some examples of subalgebras of $M_n(E)$, considered in [11, 5], satisfying an identity of degree 4.

Proposition 4. [11, Corollary 3] The subalgebra of $M_{2n}(E)$ of $2n\times 2n$ matrices

having n rows with entries all equal to α and n rows with entries all equal to β satisfies the identity $X_4[X_1, X_2, X_3] = 0$.

Proposition 5. [5, Theorems 4, 5, 7] The following algebras

$$A1=\left(\begin{array}{cccccc} x_1 & \ldots & x_1 & kx_1 & \ldots & kx_1 \\ x_2 & \ldots & x_2 & kx_2 & \ldots & kx_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ x_{2n} & \ldots & x_{2n} & kx_{2n} & \ldots & kx_{2n} \end{array} \right), x_i \in E, k \in K),$$

$$A2=\left(\begin{array}{cccc} y_1 & y_2 & \ldots & y_n \\ \alpha_2 y_1 & \alpha_2 y_2 & \ldots & \alpha_2 y_n \\ \ldots & \ldots & \ldots & \ldots \\ \alpha_n y_1 & \alpha_n y_2 & \ldots & \alpha_n y_n \end{array} \right), y_j \in E, \alpha_k \in K) \text{ and}$$

$$A3=\left(\begin{array}{ccccccc} z_1 & 0 & \ldots & \ldots & \ldots & 0 & z_1 \\ 0 & z_2 & 0 & \ldots & 0 & z_2 & 0 \\ \ldots. & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots & 0 & z_{n+1} & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & z_{2n} & 0 & \ldots & 0 & z_{2n} & 0 \\ z_{2n+1} & 0 & \ldots & \ldots & \ldots & 0 & z_{2n+1} \end{array} \right), z_s \in E)$$

satisfy the identity $X_4[X_1, X_2, X_3] = 0$.

THE THEOREM AND ITS PROOF

Here we prove the following

Theorem. Any algebra $\mathbb{A}$, satisfying the identity $x_4[x_1, x_2, x_3] = 0$, satisfies as well the identities $T_1(x_1, \ldots, x_6; y_1, y_2) = 0$ and $T_2(x_1, \ldots, x_5; y_1, y_2, y_3) = 0$ from (1) and (2), respectively.

Proof of the Theorem:

At the beginning we give a way of presenting the identity of degree 4 by means of the pattern notation. Really the identity $0 = y_1[x_1, x_2, y_2]$ could be written as

$$\begin{aligned} 0 &= y_1[x_1, x_2]y_2 - y_1y_2[x_1, x_2] \\ &= \sum_{\sigma \in Sym(2), \pi = BAAB} sign(\sigma)\pi(x_{\sigma(1)}x_{\sigma(2)}; y_1, y_2) \\ &- \sum_{\sigma \in Sym(2), \pi = BBAA} sign(\sigma)\pi(x_{\sigma(1)}x_{\sigma(2)}; y_1, y_2). \end{aligned}$$

For short we use the notation

$$BAAB - BBAA = 0. \quad (3)$$

I. We write P as a 2×3 matrix $P = (a_{ij})$.

Thus we get $a_{11} + a_{21} = AAAA(BAAB - BBAA) = 0$. This really means that the

polynomial, corresponding to the pattern $BAAB - BBAA$, is multiplied on the left by x_3, after two consequitive substituions (123) of the indices of x's we multiply the corresponding sum on the left by x_4, again substitutions of the indices (1234) take place, multiplication on the left by x_5 follows, circular substitution of the indices of x's takes place, multiplication on the left by x_6 follows and a circular substitution of the the indices of x's finishes the operations.

The result is that $\sum_{\pi = AAAABAAB - AAAABBAA} (P_\pi^- + P_\pi^+) = 0$.

The next analogous consequences of the identity $0 = y_1[x_1, x_2, y_2]$ we'll be skeched in the same formal way.

If in (3) we substitute the first B by $AABAA$, we get $AABAAAAB - AABAABAA = AABAAAAB - AABBAAAA$, as (3) holds, i.e. $a_{12} + a_{13} = 0$. For $a_{22} + a_{23} = 0$ we substitute in (3) the first B by BAA and multiply on the right the new identity by AA.

As a final result we get that $T_1(x_1, \ldots, x_6; y_1, y_2) = 0$ is an identity for $\mathbb{A}$.

II. In order to prove the second identity we use again the formal notation $PP = (b_{ij})$ as a 4×3 matrix and the above shorter way of presenting the corresponding consequences of the identity $0 = y_1[x_1, x_2, y_2]$.

Obviously

$$\begin{aligned} b_{12} + b_{22} &= AA(BAAB - BBAA)BA = 0 \\ b_{32} + b_{42} &= (BAAB - BBAA)BAAA = 0 \\ b_{11} + b_{21} &= -AAA(BAAB - BBAA)B = 0 \\ b_{13} + b_{23} &= -A(BAAB - BBAA)BAA = 0 \end{aligned}.$$

In (3) we substitute the first B by BAA and then multiply on the right by BA. At the end we have to change the indices of y's in order to cover the even substitutions of y_1, y_2, y_3. Thus we come to

$b_{33} + b_{43} = (BAAAAB - BAABAA)BA = (BAAAAB - BBAAAA)BA = 0$.

At last we multiply (3) on the right by B, then we substitute the first B by $ABAA$. We get

$b_{31} + b_{41} = (ABAAAAB - ABAABAA)B = (ABAAAAB - ABBAAAA)B = 0$.

The result is that $T_2(x_1, \ldots, x_5; y_1, y_2, y_3) = 0$.

This ends the proof of the Theorem.

COMPUTER TESTING THE VALIDITY OF THE THEOREM

Some results in investigating properties either of the Grassmann algebra E or $M_n(E)$ were suggested or tested using a programme in the system of computer algebra *Mathematica* for manipulating with finite dimensional subalgebras E_k and the matrix algebra $M_2(E_k)$ for small k [9].

For the purposes of the paper this programme was modified for working in the 3×3 matrix algebra $A3(E_4)$ (the algebra $A3$ is defined in Proposition 5). The entries of the

matrices from $A3(E_4)$ are written as vectors, each with 16-th coordinates. The notation $(\alpha_1,\alpha_2,...,\alpha_{16})$ stands for the Grassmann element

$$\alpha_1+\alpha_2e_1+\alpha_3e_2+\alpha_4e_1e_2+\alpha_5e_3+\alpha_6e_1e_3+\alpha_7e_2e_3+\alpha_8e_1e_2e_3+\alpha_9e_4+\alpha_{10}e_1e_4$$
$$+\alpha_{11}e_2e_4+\alpha_{12}e_1e_2e_4+\alpha_{13}e_3e_4+\alpha_{14}e_1e_3e_4+\alpha_{15}e_2e_3e_4+\alpha_{16}e_1e_2e_3e_4.$$

The coefficients α_i for the nonzero entries are random integers of a preliminary given interval.

The programme calculates the value of the polynomial B_{12} (corresponding to the pattern $BAAB-BBAA$) and those of T_1 и T_2 from (1) and (2), respectively. The computer realization of the last two polynomials is done recurrently using 35 operators for T_1 and 60 for T_2. The details are given in [12]. Here we give the presentation of a matrix in $A3(E_4)$ and some final values only:

```
x1={{Array[a1,16],{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
Array[a1,16]},{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},Array[b1,16],
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{Array[c1,16],
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},Array[c1,16]}}
{{{20,-6,-12,17,-6,-4,4,11,-9,5,-14,-11,18,-18,-19,12},
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
{20,-6,-12,17,-6,-4,4,11,-9,5,-14,-11,18,-18,-19,12}},
{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},
{1,-19,-14,-18,-2,15,12,19,7,0,19,4,-8,9,18,-6},
{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{4,-17,-10,13,19,0,20,4,-1,-
19,-17,15,7,4,3,-1},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{4,-17,-
10,13,19,0,20,4,-1,-19,-17,15,7,4,3,-1}}}

B12[x1_,x2_,y1_,y2_]:=y2⊗(S2[x1,x2]⊗y1)-y2⊗(y1⊗S2[x1,x2])
B12[x1,x2,y1,y2]
{{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0}},{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0
,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}}}

T1[x1_,x2_,x3_,x4_,x5_,x6_,y1_,y2_]:=A[x1,x2,x3,x4,x5,x6,y1,y2]+
B[x1,x2,x3,x4,x5,x6,y1,y2]-CH[x1,x2,x3,x4,x5,x6,y1,y2]-
DH[x1,x2,x3,x4,x5,x6,y1,y2]-
EH[x1,x2,x3,x4,x5,x6,y1,y2]+FH[x1,x2,x3,x4,x5,x6,y1,y2]
```

The parts of the polynomial $T_1(x_1,\ldots,x_6;y_1,y_2)$ are defined in [12]. The correspondence with the steps in the proof of the theorem is the following: The polynomial $a_{11}+a_{21}$ is realized in *Mathematica* by the polynomial $A(x_1,\ldots,x_6;y_1,y_2)-DH(x_1,\ldots,x_6;y_1,y_2)$, the expression $a_{12}+a_{13}$ leads to $B(x_1,\ldots,x_6;y_1,y_2)-CH(x_1,\ldots,x_6;y_1,y_2)$ and the computer construction of $a_{22}+a_{23}$ is the polynomial $FH(x_1,\ldots,x_6;y_1,y_2)-EH(x_1,\ldots,x_6;y_1,y_2)$.

Here we give the result for $a_{12} + a_{13}$ only:

```
B[x1,x2,x3,x4,x5,x6,y1,y2]
{{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-
2438631710136000},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,-
2438631710136000}},{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,2438631710136000},{0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,2438631710136000}}}

CH[x1,x2,x3,x4,x5,x6,y1,y2]
{{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-
2438631710136000},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,-
2438631710136000}},{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0
,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0
,0,0,0,0,0,0,0,0,0,0,0,0,2438631710136000},{0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,2438631710136000}}}
```

Thus we get the final result for the first identity:

```
T1[x1,x2,x3,x4,x5,x6,y1,y2]
{{{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0
,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}},{{0,0,0,0,0,0,0,0,0,0,0,0,
0,0,0,0},{0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0}}}
```

The second identity is tested analogously. The polynomials considered in the proof of the theorem correspond to respective parts of the polynomial $T_2(x_1,\ldots,x_5;y_1,y_2,y_3)$.

The programme ran on a computer AMD Athlon(tm) II X4 630 with 3,25 GB of RAM.

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ТЪЖДЕСТВА ЗА $M_2(E)$ СА ТЪЖДЕСТВА ЗА ПОДАЛГЕБРИ НА $M_n(E)$

Цецка Рашкова

Русенски университет "Ангел Кънчев"

Резюме: *В [13] Вишне дава явната форма на две полиномни тъждества от степен 8 за матричната алгебра $M_2(E)$, където E е Грасмановата алгебра. В статията се доказва, че те са тъждества и за някои подалгебри на $M_n(E)$ при произволно $n>2$. Чрез програма на системата Mathematica е направено компютърно пресмятане стойността на съответните полиноми за една от разглежданите матрични алгебри при $n=3$ над крайномерната Грасманова алгебра E_4.*

Ключови думи*: Грасманова алгебра, T-идеал, стандартен полином, полилинейни тъждества, мотив.*

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POLYNOMIAL IDENTITIES OF THE 3x3 MATRICES OVER THE FINITE DIMENSIONAL GRASSMANN ALGEBRA

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***Abstract**: In the paper we consider some polynomial identities in the 3 by 3 matrix algebra over the finite dimensional Grassmann algebra, related to the standard polynomial.*

***Keywords**: Matrix algebras, Polynomial identities, Grassmann algebra.*

INTRODUCTION

Let K be a field of characteristic 0 and V be a vector space with ordered basis $\{e_1, e_2, ...\}$.

The associative algebra $G(V)$ with defining relations $e_i e_j + e_j e_i = 0$, for all $i, j = 1,2,...$is called Grassmann algebra(or Exterior algebra). The basic elements $e_i \in V$, $i = 1,2,...$ are generators of $G(V)$. The elements of the algebra $G(V)$ will be called Grassmann elements. The defining relations allow to rearrange the products of the generators, and $e_{\sigma(i_1)} \dots e_{\sigma(i_n)} = (sign\,\sigma) e_{i_1} \dots e_{i_n}$ for any permutation σ of $i_1, \dots, i_n$.

The basis of $G(V)$ is $B = \{1\} \cup \{e_{i_1} e_{i_2} \dots e_{i_m} \mid 1 \le i_1 < i_2 < \dots < i_m, m = 1,2,...\}$. The number m defines the length of the basic element $a = e_{i_1} e_{i_2} \dots e_{i_m}$. If a Grassmann element is a linear combination of monomials $e_{i_1} e_{i_2} \dots e_{i_m}$, when m is even, it will be called even Grassmann element. Similarly, when m is odd, it will be called odd Grassmann element.

If dim $V = n$ the corresponding finite dimensional algebra $G(V_n)$ with basis $B_n = \{1, e_1, e_2, e_1 e_2, e_3, e_1 e_3, e_2 e_3, e_1 e_2 e_3, ..., e_1 e_2 ... e_n\}$ will be denoted by G_n.

POLYNOMIAL IDENTITIES

In the paper we consider some polynomial identities in the matrix algebra over the finite dimensional Grassmann algebra.

The polynomial $s_n(x_1, x_2, ..., x_n) = \sum_{\sigma \in S_n} sign(\sigma)\, x_{\sigma(1)} x_{\sigma(2)} ... x_{\sigma(n)}$, where S_n is the symmetric group of degree n is called a standard polynomial. The standard polynomial is multilinear and alternating, namely $s_n(\dots, x, \dots, y, \dots) = -s_n(\dots, y, \dots, x, \dots)$.

In 1950, S. A. Amitsur and J. Levitzki proved [1]: Over any field K, the $n \times n$ matrix algebra $M_n(K)$ satisfies the standard identity of degree $2n$. It does not satisfy polynomial identities of lower degree and, up to a multiplicative constant, $s_{2n} = 0$ is the only mutilinear polynomial identity of degree $2n$ for $M_n(K)$.

The expression $[x_1, x_2] = x_1 x_2 - x_2 x_1$ is called commutator of x_1 and x_2. Inductively a longer commutator is defined by $[x_1, \dots, x_{n-1}, x_n] = [[x_1, \dots, x_{n-1}], x_n], n = 3,4, \dots$.

Proposition 1 [6, Corollary, p.437]. *The T-ideal $T(G)$ is generated by the identity*

$[[x_1, x_2], x_3] = [x_1, x_2, x_3] = 0$.

Proposition 2 [3, Lemma 6.1]. *The algebra* $G(V)$ *satisfies the identity* $s_n^k(x_1, x_2, ..., x_n) = 0$ *for all* n, $k > 1$.

Proposition 3 [5, Exercise 5.3]. *For* $G_n = G(V_n)$ *over* n*-dimensional vector space* V_n, $n > 1$, *all identities follow from the identity* $[x_1, x_2, x_3] = 0$ *and the standard identity* $s_{2p}(x_1, x_2, ..., x_{2p}) = 0$, *where* p *is the minimal integer with* $2p > n$.

PI-degree of an algebra is called the smallest degree of the multilinear identities, which the algebra satisfies.

Proposition 4 [3, Lemma 3.2]. *Let* R *be an algebra with* $PI\deg(R) = r$, *then* $PI\deg(M_n(R)) \geq nr$. *In particular,* $PI\deg(M_n(G)) \geq 3n$.

Proposition 5 [8, Theorem]. *Let* $M_n(G)$ *be the matrix algebra of order* n *over the (infinite dimensional) Grassmann algebra. Then* $M_n(G)$ *has no identities of degree* $4n - 2$.

Proposition 6 [2, Lemma, p.1509]. *The algebra* $M_n(G)$ *satisfies the identity* s_{2n}^k *for some* $k > 1$ *but satisfies neither* s_{2n} *nor identities of the form* s_m^k *for any* k *when* $m < 2n$.

Proposition 7 [4, Proposition 2.1]. *Let* $f_1, \ldots, f_d \in K\langle x_1, \ldots x_m \rangle$ *be elements of the T-ideal of identities of* M_n. *If* $d > \frac{1}{2}n^2 m$, *then* $f_1 \ldots f_d = 0$ *is an identity on* $M_n(G)$.

Some identities to 2 by 2 matrix algebra over the finite dimensional Grassmann algebra, related to the standard polynomial, are investigated in [7].

Proposition 8 [7, Theorem 1]. The algebra $M_2(G_n)$ satisfies the identity $s_4^p(x_1, x_2, x_3, x_4) = 0$, where p is the minimal integer with $2p > n$.

Proposition 9 [5, Exercise 2.8]. *The* $n \times n$ *matrix algebra* $M_n(K)$ *satisfies the identity of algebraicity*

$$d_{n+1}(1, x, x^2, \ldots, x^n, 1, y_1, ..., y_n, 1) = \sum_{\sigma \in Sym\{0,1,\ldots,n\}} sign(\sigma)\, x^{\sigma(0)} y_1 x^{\sigma(1)} y_2 \ldots x^{\sigma(n-1)} y_n x^{\sigma(n)} = 0,$$

and the identity $s_n([x, y], [x^2, y], \ldots, [x^n, y]) = 0$.

In [9] some identities in $M_3(G)$ are explored.

Proposition 10 [9, Proposition 12]. *The commutator of two symmetric matrices* A_1 *and* A_2 *of type* $\begin{pmatrix} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{pmatrix}$ *in* $M_3(G)$ *is nilpotent with index of nilpotency* ≤ 3.

The commutator of any two matrices of type $\begin{pmatrix} \alpha & \beta & \beta \\ -\beta & \alpha & \beta \\ -\beta & -\beta & \alpha \end{pmatrix}$ *in* $M_3(G)$ *is nilpotent with index of nilpotency* ≤ 3.

More facts, concerning the index of nilpotency of commutators of length 2 for some types upper triangular matrices over the Grassmann algebra can be seen in [9].

POLYNOMIAL IDENTITIES, RELATED TO THE STANDARD POLYNOMIAL IN $M_3(G_n)$ AND $M_k(G_n)$

Proposition 8 has an analogue in the algebra $M_3(G_n)$.

Theorem 1. The polynomial $s_6^p(x_1, x_2, x_3, x_4, x_5, x_6)$, where p is the minimal integer with $2p > n$, is an identity of $M_3(G_n)$.

Proof. Let $X = \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_5 & \alpha_6 \\ \alpha_7 & \alpha_8 & \alpha_9 \end{pmatrix}$ be a matrix with entries Grassmann elements from $G_n = G(V_n)$ and

$$\alpha_i = \alpha_{i_0} + \alpha_{i_1}e_1 + \alpha_{i_2}e_2 + \alpha_{i_3}e_3 + \cdots + \alpha_{i_n}e_n + \alpha_{i_{n+1}}e_1e_2 + \cdots + \alpha_{i_{2^n-1}}e_1e_2\ldots e_n,$$

$i = 1,\ldots,9,\ a_{i_s} \in K,\ s = 0,1,\ldots,2^n - 1.$

We can express X as follows

$$X = \begin{pmatrix} \alpha_{1_0} & \alpha_{2_0} & \alpha_{3_0} \\ \alpha_{4_0} & \alpha_{5_0} & \alpha_{6_0} \\ \alpha_{7_0} & \alpha_{8_0} & \alpha_{9_0} \end{pmatrix} + \sum_{j=1}^{n} \begin{pmatrix} \alpha_{1_j} & \alpha_{2_j} & \alpha_{3_j} \\ \alpha_{4_j} & \alpha_{5_j} & \alpha_{6_j} \\ \alpha_{7_j} & \alpha_{8_j} & \alpha_{9_j} \end{pmatrix} e_j + \cdots +$$

$$+ \begin{pmatrix} \alpha_{1_{n+1}} & \alpha_{2_{n+1}} & \alpha_{3_{n+1}} \\ \alpha_{4_{n+1}} & \alpha_{5_{n+1}} & \alpha_{6_{n+1}} \\ \alpha_{7_{n+1}} & \alpha_{8_{n+1}} & \alpha_{9_{n+1}} \end{pmatrix} e_1e_2 + \cdots + \begin{pmatrix} \alpha_{1_{2^n-1}} & \alpha_{2_{2^n-1}} & \alpha_{3_{2^n-1}} \\ \alpha_{4_{2^n-1}} & \alpha_{5_{2^n-1}} & \alpha_{6_{2^n-1}} \\ \alpha_{7_{2^n-1}} & \alpha_{8_{2^n-1}} & \alpha_{9_{2^n-1}} \end{pmatrix} e_1e_2\ldots e_n.$$

Let $X_i = \begin{pmatrix} \alpha_{1_i} & \alpha_{2_i} & \alpha_{3_i} \\ \alpha_{4_i} & \alpha_{5_i} & \alpha_{6_i} \\ \alpha_{7_i} & \alpha_{8_i} & \alpha_{9_i} \end{pmatrix}$, $i = 0,1,\ldots,2^n - 1.$ Then

$$X = X_0 + X_1e_1 + X_2e_2 + X_3e_3 + \cdots + X_ne_n + X_{n+1}e_1e_2 + \cdots + X_{2^n-1}e_1e_2\ldots e_n. \quad (1)$$

We consider six matrices $X^{(k)}$, $k = 1,\ldots,6$ of type (1).

$X^{(k)} = X_0^{(k)} + X_1^{(k)}e_1 + X_2^{(k)}e_2 + X_3^{(k)}e_3 + \cdots + X_n^{(k)}e_n + X_{n+1}^{(k)}e_1e_2 + \cdots + X_{2^n-1}^{(k)}e_1e_2\ldots e_n,$ $k = 1,\ldots,6$, and transform the standard polynomial, using its multilinearity, as follows:

$$s_6\left(X^{(1)},\ldots,X^{(6)}\right) =$$

$$s_6(X_0^{(1)} + X_1^{(1)}e_1 + \cdots + X_n^{(1)}e_n + X_{n+1}^{(1)}e_1e_2 + \cdots + X_{2^n-1}^{(1)}e_1e_2\ldots e_n, \ldots,$$

$$X_0^{(6)} + X_1^{(6)}e_1 + \cdots + X_n^{(6)}e_n + X_{n+1}^{(6)}e_1e_2 + \cdots + X_{2^n-1}^{(6)}e_1e_2\ldots e_n) =$$

$$= s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) +$$
$$+ \sum_{j=1}^{n}\Big[s_6\left(X_j^{(1)} e_j, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) + s_6\left(X_0^{(1)}, X_j^{(2)} e_j, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) +$$
$$+ s_6\left(X_0^{(1)}, X_0^{(2)}, X_j^{(3)} e_j, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) + s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_j^{(4)} e_j, X_0^{(5)}, X_0^{(6)}\right) +$$
$$+ s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_j^{(5)} e_j, X_0^{(6)}\right) + s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}\ X_0^{(4)}, X_0^{(5)}, X_j^{(6)} e_j\right)\Big] + S =$$

$$= s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) +$$
$$\sum_{j=1}^{n}\Big[s_6\left(X_j^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) e_j + s_6\left(X_0^{(1)}, X_j^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) e_j +$$
$$+ s_6\left(X_0^{(1)}, X_0^{(2)}, X_j^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) e_j + s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_j^{(4)}, X_0^{(5)}, X_0^{(6)}\right) e_j +$$
$$+ s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_j^{(5)}, X_0^{(6)}\right) e_j + s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_j^{(6)}\right) e_j\Big] + S.$$

Here S is the sum of products of matrices with entries from K and the basic elements $e_1 e_2 ... e_m,\ m \geq 2.$

Each of the matrices $X_j^{(k)}$ for $k = 1, \ldots, 6$ and $j = 0,1, \ldots, n$ is a matrix with entries from K. Then due to Amitsur-Levitzki theorem it follows

$$s_6\left(X_0^{(1)}, X_0^{(2)}, X_0^{(3)}, X_0^{(4)}, X_0^{(5)}, X_0^{(6)}\right) = 0;$$
$$s_6\left(X_0^{(1)}, \ldots, X_j^{(k)}, \ldots, X_0^{(6)}\right) = 0,\ \forall k = 1, \ldots, 6;\ j = 0,1, \ldots, n.$$

Then $s_6\left(X^{(1)}, \ldots, X^{(6)}\right) = S$ and $s_6^p = S^p$. Since S is a sum of matrices multiplied by the basic elements $e_1 e_2 ... e_m,\ m \geq 2$, then S^p will be a sum of matrices multiplied by the elements $e_1 e_2 ... e_q,\ q \geq 2p.$ If $2p > n$ then in the element $e_1 e_2 ... e_q,\ q \geq 2p > n$ there is at least one repeated generator and $e_1 e_2 ... e_q = 0$. Hence $s_6^p = 0.$ This completes the proof.

We can generalize the above theorem for the matrix algebra $M_k(G_n)$. The additional difficulties are only of technical nature.

Theorem 2. The polynomial $s_{2k}^p\left(x_1, x_2, x_3, x_4, x_5, x_6\right)$, where p is the minimal integer with $2p > n$, is an identity of $M_k(G_n)$.

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ПОЛИНОМНИ ТЪЖДЕСТВА В МАТРИЧНАТА АЛГЕБРА ОТ ТРЕТИ РЕД НАД КРАЙНОМЕРНА ГРАСМАНОВА АЛГЕБРА

Антоанета Михова

Русенски университет „Ангел Кънчев”

***Резюме**: В статията са разгледани тъждества, свързани със стандартния полином, в матричната алгебра от трети ред над крайномерна Грасманова алгебра.*

***Ключови думи**: Матрична алгебра, Полиномни тъждества, Грасманова алгебра.*

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ON THE EXISTENCE OF MULTIPLE PERIODIC SOLUTIONS OF FOURTH-ORDER SEMILINEAR DIFFERENTIAL EQUATIONS

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Abstract: *This paper is focused on periodic solutions of fourth-order nonautonomous semi-linear parabolic equations arising in the dynamics of populations. These equations play also an important role in modeling bi-stable systems, related to studying spatial patterns. Analytical results on the existence of multiple solutions have been presented, using the theorem for minimization and Clark's theorem.*

Keywords: Fourth-order ODE, periodic solutions, minimization theorem, Clark's theorem.

INTRODUCTION

Mathematical models are used to study a number of phenomena, observed in complex natural systems. The so called model equations play a key role in those models. Historically, in 1936-1939 Kolmogorov-Petrovski-Piskunov's equations [8] appear for studying the evolution of species, and Brugers' equations for studying the processes of distribution of different types of flat waves.

Later in 1977-1987, a fourth order model equation was introduced, now known as the extended Fisher - Kolmogorov (EFK) equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial^4 u}{\partial x^4} + f(u), \quad f(u) = u - u^3 \, , \gamma > 0 \, ,$$

proposed by Coullet, Elphick & Repaux [3] and Dee & Saarloos [4]. The EFK equation appeared when different systems were described such as phase transition model in two-dimensional system close to the point of Lifshitz [7], [17]. Stationary solutions of the autonomous equation have been extensively studied in many works of Peletier and his collaborators [10,11,12,13,14]. Existence of stationary periodic solutions and homoclinics of the non-autonomous equation has been obtained by J. Chaparova, L. Sanchez, S. Tersian, T.Gyulov [5,6,16].

This paper focuses on the non-autonomous semi-linear ODE

$$u^{iv} - mu'' + a(x)u|u|^{p-2} - b(x)u|u|^{q-2} = 0 \, , \tag{1}$$

subject to boundary conditions

$$u(0) = u''(0) = u(L) = u''(L) = 0 \, . \tag{2}$$

In our previous works [1, 2] the problem (1)-(2) is examined in case $1 < p < q \, , q > 2$.

We suppose $a(x)$ and $b(x)$ are positive, even, continuous $2L$-periodic functions,

$m > 0$ and $1 < q < p$.

The boundary value problem (1)-(2) has a variational structure and its solutions could be obtained as critical points of the energy functional

$$J(u) = \int_0^L \left[\frac{1}{2}u''^2 + \frac{m}{2}u'^2 + \frac{1}{p}a(x)|u|^p - \frac{1}{q}b(x)|u|^q \right] dx, \tag{3}$$

in the Sobolev space

$$X = H^2(0,L) \cap H_0^1(0,L)$$
$$= \left\{ u \in L^2(0,L) : u' \in L^2(0,L), u'' \in L^2(O,L), u(0) = u(L) = 0 \right\}.$$

The space X is Hilbert space with the scalar product

$$(u,v) = \int_0^L (u''v'' + u'v' + uv)dx, \quad u, v \in X,$$

and norm $\|u\| = \sqrt{(u,u)}$. By the Poincaré inequality

$$\int_0^L u^2 dx \le \frac{L^2}{\pi^2} \int_0^L (u')^2 dx, \quad u \in X,$$

$$\|u\|_1 = \left(\int_0^L \left[(u'')^2 + m(u')^2 \right] dx \right)^{1/2}$$

is an equivalent norm in X. For simplicity we will denote $\|.\|_1$ by $\|.\|$.

EXISTENCE RESULTS

We find the critical points of the functional using the minimization theorem:

Theorem (Minimization theorem [9]). Let J be a functional bounded from below in the reflexive Banach space X. Also let J be weakly lower semi-continuous X and possess a bounded minimizing sequence. Then there is an element $u \in X$, such that $J(u) = \inf_X J$.

Proposition 1. Let $1 < q < p$. Then the functional $J(u)$ is bounded from below on X and there exists a minimizer $u \in X$ of the functional J, i. e. $J(u) = \inf_X J$.

Proof. First we show that the functional is bounded from below. Let $a(x) \geq a_1 > 0$, $b(x) \leq b_2$. Then for every $u \in X$ we have

$$J(u) \geq \frac{1}{2}\|u\|^2 + \frac{a_1}{p}\int_0^L |u|^p \, dx - \frac{b_2}{q}\int_0^L |u|^q \, dx \quad .$$

Denote by $f(u)$ the function $f(u) = \frac{a_1}{p}|u|^p - \frac{b_2}{q}|u|^q$. Then

$$f'(u) = a_1 u|u|^{p-2} - b_2 u|u|^{q-2}$$
$$= u|u|^{q-2}\left(a_1|u|^{p-q} - b_2\right).$$

The minimum of the function $f(u)$ is obtained for $|u_1| = \left(\frac{b_2}{a_1}\right)^{1/(p-q)}$ and it is

$$f_{\min}(u_1) = \left(\frac{b_2^p}{a_1^q}\right)^{1/(p-q)} \cdot \frac{(q-p)}{pq} = k.$$

Taking into account the fact that $1 < q < p$, it follows that $k < 0$.
Thus for any $u \in X$ we obtain

$$J(u) \geq \frac{1}{2}\|u\|^2 + kL \geq kL, \tag{4}$$

hence $J(u)$ is bounded from below.

From the last inequality we obtain $J(u) \to \infty$ as $\|u\| \to \infty$ which means that the functional is coercive, i.e. it possesses a bounded minimizing sequence (u_n).

Next we show that the functional is weakly lower semi-continuous. We present the functional as a sum of two functionals,

$$J(u) = \frac{1}{2}\|u\|^2 + \int_0^L \left[\frac{a(x)}{p}|u|^p - \frac{b(x)}{q}|u|^q\right]dx = \Phi_1(u) + \Phi_2(u).$$

$\Phi_1(u)$ is a convex continuous functional. Therefore, it is weakly lower semi-continuous.

Since (u_n) is bounded we assume that $u_n \rightharpoonup u$ weakly. By Sobolev embedding theorem it follows that $u_n \to u$ in $C^1[0,L]$. Then $\Phi_2(u_n) \to \Phi_2(u)$, i.e. $\Phi_2(u)$ is weakly continuous.

Thus the functional $J(u)$ is weakly lower semi-continuous in X. The existence of the minimizer $u \in X$ such that $J(u) = \inf_X J$ follows by the general minimization theorem.

Proposition 2. Let $1 < q < p$, $q < 2$. Then the minimizer of the functional J is the non-zero function, which is a non-zero solution of problem (1)-(2).

Proof. We use the test function $u_0(x) = t.\sin\frac{\pi x}{L}$ which depends on a real parameter $t > 0$. We have

$$J(u_{0(x)}) = \frac{t^2\pi^4}{2L^4}\int_0^L \frac{1-\cos\frac{2\pi x}{L}}{2}dx + \frac{mt^2\pi^2}{2L^2}\int_0^L \frac{1+\cos\frac{2\pi x}{L}}{2}dx$$

$$+\frac{t^p}{p}\int_0^L a(x)\left(\sin\frac{\pi x}{L}\right)^p dx \_ \frac{t^q}{q}\int_0^L b(x)\left(\sin\frac{\pi x}{L}\right)^q dx.$$

Denote

$$c_1 = \frac{\pi^2\left(\pi^2 + 2mL^2\right)}{4L^3} > 0,\ c_2 = \frac{1}{p}\int_0^L a(x)\left(\sin\frac{\pi x}{L}\right)^p dx > 0 \text{ and}$$

$$c_3 = \frac{1}{q}\int_0^L b(x)\left(\sin\frac{\pi x}{L}\right)^q dx > 0.$$

Thus for a sufficiently small $t > 0$ we have

$$J(u_0) = t^q\left(c_1 t^{2-q} + c_2 t^{p-q} - c_3\right) < 0,$$

since $q < 2$ and $1 < q < p$.

As $\inf_X J < J(u_0) < 0 = J(0)$ it follows that the minimizer of J is a nontrivial function, which ends the proof.

We use the following theorem to prove the main result in this paper:

Theorem(Clark [15]). Let X be a real Banach space with a functional $J \in C^1(X, R)$ bounded from below, even and satisfying the Palais-Smale condition. Suppose $J(0) = 0$. Let there be a set $K \subset X$ such that K is homeomorphic to the sphere S^{n-1} in R^n, $n \in N$, by an odd map, and $\sup_K J < 0$. Then the functional J possesses at least n distinct pairs of critical points.

Theorem 1. Let $1 < q < p$ and $q < 2$. Then problem (1)-(2) has infinitely many pairs of solutions.

Proof. For the functional J we know that $J \in C^1(X, R)$, J is even, $J(0) = 0$ and it is bounded from below on X. To satisfy the hypotheses of Clark's theorem, it remains the Palais-Smale condition (PS) and the geometric condition to be proved.

Let (u_n) be a (PS) – sequence of J, i. e.

$(J(u_n))$ be bounded and $J'(u_n) \to 0$ as $n \to \infty$.

We show that the sequence (u_n) is bounded. Indeed from (4), for $M = -kL > 0$, we have

$$J(u) \geq \frac{1}{2}\|u\|^2 - M \Rightarrow J(u_n) + M \geq \frac{1}{2}\|u_n\|^2$$

Since $(J(u_n))$ is bounded there is $C > 0$ such that $|J(u_n)| \leq C$. Then

$$C + M \geq J(u_n) + M \geq \frac{1}{2}\|u_n\|^2.$$

Thus the sequence (u_n) is bounded, and we may assume (going to a subsequence if necessary) that $u_n \rightharpoonup u$ weakly in X.

By Sobolev embedding theorem $u_n \to u$ in $C^1[0, L]$ and $\|u_n\| \to \|u\|$ (see [2]).

Hence $\|u_n\| \to \|u\|$ and $u_n \rightharpoonup u$ in X implies $u_n \to u$ in X.

We show that $\sup_K J(u) < 0$ for $\forall \eta \in N$, where

$$K = \left\{ \lambda_1 \sin\frac{\pi x}{L} + \lambda_2 \sin\frac{2\pi x}{L} + ... + \lambda_\eta \sin\frac{\eta \pi x}{L} : \lambda_1^2 + \lambda_2^2 + ... + \lambda_\eta^2 = \rho^2 \right\}$$

It is clear that the odd mapping $H : K \to S^{\eta-1}$, defined by

$$H\left(\lambda_1 \sin\frac{\pi x}{L}+...+\lambda_\eta \sin\frac{\eta\pi x}{L}\right)=\left(-\frac{\lambda_1}{\rho},...,-\frac{\lambda_\eta}{\rho}\right),$$

is a homeomorphism between K and $S^{\eta-1}$, [2].

The set K is a subset of the finite-dimensional space

$$X_\eta = Span\left\{\sin\frac{\pi x}{L}, \sin\frac{2\pi x}{L}, ..., \sin\frac{\eta\pi x}{L}\right\},$$

equipped with the norm

$$\left\|\lambda_1 \sin\frac{\pi x}{L}+\lambda_2 \sin\frac{2\pi x}{L}+...+\lambda_n \sin\frac{\eta\pi x}{L}\right\|^2_{X_\eta} = \lambda_1^2+\lambda_2^2+...+\lambda_\eta^2.$$

For each $w\in K$ we have

$$\frac{1}{2}\int_0^L (w'')^2\,dx = \frac{L}{4}\left(\frac{\pi}{L}\right)^4 \sum_{k=1}^{\eta}\lambda_k^2 k^4$$

$$\le \frac{L}{4}\left(\frac{\eta\pi}{L}\right)^4 \sum_{k=1}^{\eta}\lambda_k^2 = \frac{L}{4}\left(\frac{\eta\pi}{L}\right)^4 \|w\|^2_{X_\eta},$$

$$\frac{m}{2}\int_0^L (w')^2\,dx = \frac{mL}{4}\frac{\pi^2}{L^2}\sum_{k=1}^{\eta}\lambda_k^2 k^2$$

$$\le \frac{mL}{4}\left(\frac{\eta\pi}{L}\right)^2 \sum_{k=1}^{\eta}\lambda_k^2 = \frac{mL}{4}\left(\frac{\eta\pi}{L}\right)^2 \|w\|^2_{X_\eta},$$

$$\frac{1}{p}\int_0^L a(x)|w|^p\,dx \le \frac{a_2}{p}\|w\|^p_{L^p}, \text{ and } \frac{1}{q}\int_0^L b(x)|w|^q\,dx \ge \frac{b_1}{q}\|w\|^q_{L^q}.$$

Thus

$$J(w)\le \|w\|^q_{X_\eta}\left(k_1\|w\|^{2-q}_{X_\eta}+k_2\|w\|^{p-q}_{X_\eta}-k_3\right),$$

since from the equivalency of the norms in finite-dimensional spaces for each $r>1$ there

are $k_2(\eta) > 0$, $k_3(\eta) > 0$, such that

$$k_3(\eta)\|w\|_{X_\eta} \le \|w\|_{L^r} \le k_2(\eta)\|w\|_{X_\eta}, \forall w \in X_\eta .$$

We choose $\|w\|_{X_\eta}$ sufficiently small and we obtain for $1 < q < 2$ и $q < p$

$$J(w) < 0 .$$

The functional J satisfies all hypotheses of Clark's theorem. Hence J has at least η distinct pairs of critical points. Since η is arbitrary, J has infinitely many pairs of critical points which are solutions of (1)-(2).

CONCLUSION

In this work we establish existence of infinitely many solutions of the boundary value problem (1)-(2). Using appropriate extension we obtain infinitely many $2L-$periodic solutions of the equation (1) which are antisymmetric with respect to $x = 0$ and $x = L$, namely taking the $2L-$periodic extension of the odd extension

$$\bar{u}(x) = \begin{cases} u(x), & x \in [0, L], \\ -u(-x), & x \in [-L, 0), \end{cases}$$

of the solution $u(x)$ for the problem (1)-(2).

The results obtained extend earlier existence results for fourth-order semi-linear ODEs.

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ВЪРХУ СЪЩЕСТВУВАНЕ НА БЕЗБРОЙ МНОГО ПЕРИОДИЧНИ РЕШЕНИЯ НА ПОЛУЛИНЕЙНИ ДИФЕРЕНЦИАЛНИ УРАВНЕНИЯ ОТ ЧЕТВЪРТИ РЕД

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Резюме: *В тази статия разглеждаме периодични решения на полулинейни параболични уравнения от четвърти ред. Това са уравнения, описващи динамика на популациите и играещи важна роля при моделирането на биустойчиви системи, свързани с изучаването на пространствени форми. Представени са аналитични резултати за съществуване на безброй много решения, използвайки теоремата за минимизация и теоремата на Кларк.*

Ключови думи: *ОДУ от четвърти ред, периодични решения, теорема за минимизация, теорема на Кларк.*

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SOME STUDIES ON THE POSSIBILITIES TO PROVIDE EMERGENCY MEDICAL AID CENTRES WITH NEW TRANSPORT VEHICLES

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***Abstract**: The present paper studies the process of buying and decommissioning new transport vehicles in the emergency medical aid centres in the Republic of Bulgaria. The process has been modelled by using an appropriate mathematical apparatus. The characteristics of the random process have been found. The actual number of ambulances in operation has been estimated. The probability has been determined for the number of vehicles to be no less than 1600 for the whole period of supplying ambulances to emergency medical centres, assuming that new cars are not taken out of operation. Conclusions are drawn, respectively.*

***Keywords:** Mathematical modelling, Random processes, Markov's processes, Distribution laws, Emergency medical aid.*

INTRODUCTION

The purchase and utilization of transport vehicles in the emergency medical aid centres (EMA) in Bulgaria is an important part of the overall patient care. The formation of the teams includes a medical practitioner (doctor) and a vehicle (ambulance) by means of which the team moves to the patients in need of emergency care. The transport service accounts for about 30% of the budget of these centres. Ambulances are expensive vehicles and superfluous ones put an extra burden on the budget [3].

In the present study we set ourselves the task to examine the process of transport vehicles' entry into operation and withdrawal from service in the EMA centers in our country through modeling this process with the appropriate mathematical tools.

METHODOLOGY

TASK ONE

Let us suppose that the purchase of new vehicles is done with intensity $\lambda(t) = at$. We shall assume that the intensity of withdrawal of vehicles from the service is $\mu = const$. We shall be looking for the characteristics of the random process $X(t)$ - the number of ambulances in operation at the point of time *t,* if $X(0) = 0$.

It is shown in the reference literature [6] that with vehicle purchasing intensity of $\lambda(t)$ and intensity of withdrawal from service $\mu(t)$, the mathematical expectation of the random process $X(t)$ will be calculated using formula (1):

$$m_x(t) = e^{-\int_0^t \mu(\tau)d\tau}\left(\int_0^t \lambda(x).e^{\int_0^x \mu(\tau)d\tau} dx + m_x(0)\right) \quad (1)$$

For the case under investigation ($X(0) = 0$ and $m_x(0) = 0$), with the given intensity of ambulances being put into and withdrawn from service, equation (1) will acquire the form (2):

$$m_x(t) = e^{-\mu t}\int_0^t ax.e^{\mu x} dx = \frac{a}{\mu^2}\left(\mu t - 1 + e^{-\mu t}\right). \quad (2)$$

The graph of the relationship (2) is shown in Fig. 1 where $a = 40$ and $\mu = 0.3$. The asymptote to the curve is drawn on the same figure, too.

$$Y = kt + n = \frac{a}{\mu}t - \frac{a}{\mu^2} = \frac{a}{\mu}\left(t - \frac{1}{\mu}\right) \tag{3}$$

It is evident from Fig. 1 ($m_x(t)$ denoted as *mx*) that after a period of time $\tau > \frac{3}{\mu}$ has passed since the beginning of the process under study, the addend, containing $e^{-\mu t}$ will tend to zero and then the graph of the dependence $m_x(t)$ will become linear.

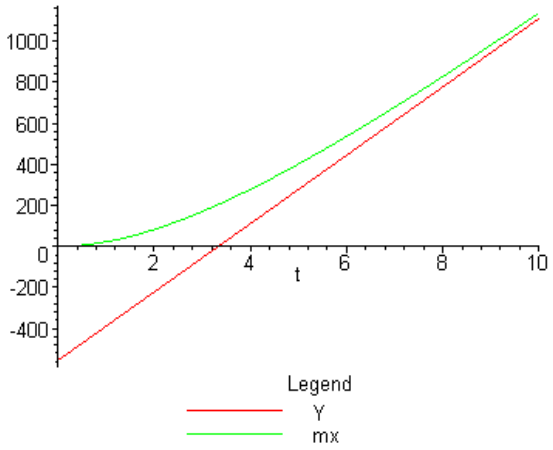

Fig. 1. Dependence of the mathematical expectation on time

As the considered random process is of Poisson type [6], it follows that $m_x(t) = D_x(t)$ and

$$m_x(t) = D_x(t) \approx \frac{a}{\mu}\left(t - \frac{1}{\mu}\right), \quad \left(t > \frac{3}{\mu}\right) \tag{4}$$

The one-dimensional law of distribution of the random process $X(t)$ [1] will be the Poisson law:

$$P\big(X(t) = i\big) = p_i(t) = \frac{[m_x(t)]^i}{i!}e^{-m_x(t)}. \tag{5}$$

We will examine the process of providing the emergency medical aid centres with the necessary means of transport. We will presume that the process of introduction of new vehicles is of Poisson's type with intensity $\lambda(t)$ and it has the form:

$$\lambda(t) = \begin{cases} at & за\ 0 \le t \le t_1 \\ at_1 & за\ t_1 < t \le t_2 \\ 0 & за\ t > t_2 \end{cases} \tag{6}$$

Figure 2 shows the dependence of the intensity of introducing new ambulances on time. During the time interval $(0, t_1)$, the initial provision of the centres with new transport vehicles takes place; in the interval (t_1, t_2) the entry of vehicles is with a constant intensity at_1, and at point t_2 the supply of new ambulances is stopped for some reason (either their

number is sufficient already, or the economic environment deteriorates and there are no funds for new ambulances). Each ambulance is used for a random period of time T (no longer than 15 years), distributed according to an exponential law with a parameter μ.

The problem we set to ourselves to solve is determining the characteristics of the random process $X(t)$ (the number of ambulances in service) – the mathematical expectation $m_x(t)$ and the variance $D_x(t)$ respectively. We will assume that at the initial moment of the examined process, the old ambulances are worn out and are permanently taken out of operation due to serious damage. Then, the following initial conditions will be fulfilled for the new vehicles: $m_x(0) = 0,\ D_x(0) = 0.$

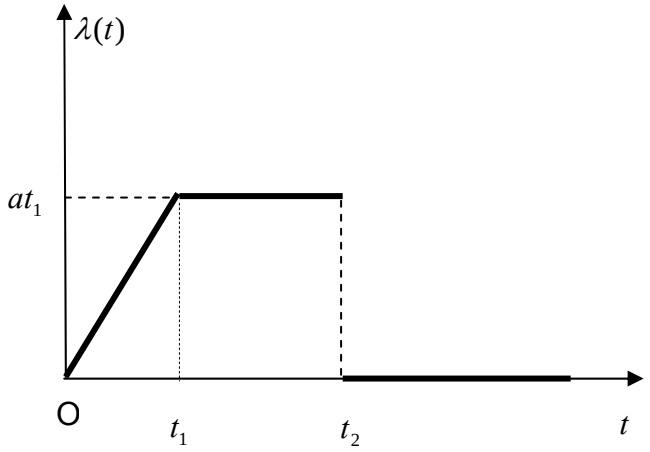

Fig. 2. Dependence of the intensity of introducing new ambulances on time

Taking into account dependence (6), formula (2) will be valid for the mathematical expectation and the variance of the random process $X(t)$ within the $(0,t_1)$ interval, i.e.:

$$m_x(t) = D_x(t) = \frac{a}{\mu^2}\left(\mu t - 1 + e^{-\mu t}\right). \tag{7}$$

To find the same characteristics in the (t_1,t_2) interval, it is necessary to introduce the variable $\tau = t - t_1$. For the initial condition at point t_1, from equation (2) we obtain

$$m_{x|\tau=0} = D_{x|\tau=0} = \frac{a}{\mu^2}\left(\mu t_1 - 1 + e^{-\mu t_1}\right). \tag{8}$$

Since the intensities λ and μ are constant when $\tau > 0$ (for the interval (t_1,t_2)), evidently equation (9) is derived from equation (1).

$$m_x(t) = \frac{\lambda}{\mu}\left(1 - e^{-\mu t}\right) + m_x(0)e^{-\mu t}. \tag{9}$$

After substituting $\lambda(t)$ from (6) in (9), the following dependence is in effect:

$$m_x(\tau) = D_x(\tau) = \frac{at_1}{\mu}(1 - e^{-\mu t}) + m_{x|\tau=0}.e^{-\mu\tau}. \tag{10}$$

Therefore:

$$m_x(t) = D_x(t) = \frac{at_1}{\mu}\left(1 - e^{-\mu(t-t_1)}\right) + \frac{a}{\mu^2}\left(\mu t_1 - 1 + e^{-\mu t_1}\right)e^{-\mu(t-t_1)} =$$

$$= \frac{at_1}{\mu} - \frac{a}{\mu^2}\left(1 - e^{-\mu t_1}\right)e^{-\mu(t-t_1)}, \qquad \left(t_1 < t < t_2\right). \tag{11}$$

For the part of the graph $t > t_2$ (when new ambulances are not provided and $\lambda = 0$), from (9) we obtain

$$m_x(t) = D_x(t) = m_x(t_2).e^{-\mu(t-t_2)}, \quad \left(t > t_2\right), \tag{12}$$

where $m_x(t_2) = \frac{at_1}{\mu} - \frac{a}{\mu^2}\left(1 - e^{-\mu t_1}\right).e^{-\mu(t_2-t_1)}$. (13)

The graph of the dependence $m_x(t) = D_x(t)$ where $t_1 = 5\ years$, $t_2 = 9\ years$ $a = 50\,(1/\ year^2)$, $\mu = 0.3\,amb./\ year$ is shown in Fig. 3.

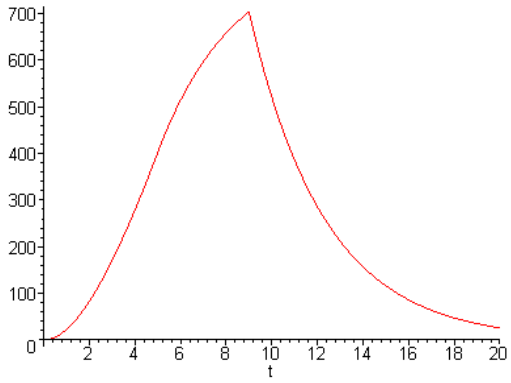

Fig. 3. Dependence of the mathematical expectation on time

The software product MAPLE [5] has been used to draw the graphs in Fig. 1. and Fig. 3.

When $m_x(t) > 20$, one can assert with accuracy sufficient for practical purposes [2] that the one-dimensional process $X(t)$ is normal with the found characteristics. If the average number of ambulances that serve patients in need is $m_x(t)$ and "the three-sigma rule" is applied [4], one can establish that the actual number of cars that are in service in the fleet of the emergency medical aid centres is within the limits

$$m_x(t) \pm 3\sigma(t) = m_x(t) \pm 3\sqrt{D_x(t)} = m_x(t) \pm 3\sqrt{m_x(t)} = m_x(t).\left(1 \pm \frac{3}{\sqrt{m_x(t)}}\right). \tag{14}$$

According to the Ministry of Health of the Republic of Bulgaria, the number of emergency medical aid centers is 186 with a total of 1600 ambulances in them. Then, if $m_x(t) = 1300$ ambulances on average are in service, the following limits will be obtained for the number of ambulances actually used:

$$1300.\left(1 \pm \frac{3}{\sqrt{1300}}\right) \approx 1300 \pm 108. \tag{15}$$

CONCLUSION

From the above results it can be concluded that the actual number of ambulances in operation fluctuates by 8% around their mean value.

TASK TWO

The second problem that the present study sets about to solve is finding the probability for the number of vehicles to be no less than x=1600 for the whole period of supplying ambulances to EMA centres, assuming that there are no withdrawals (taking out of operation) of newly bought ambulances, i.e. when $\mu_i(t) = 0, (i = 1,2,....)$.

In this case, a Markov's process of "pure" multiplication runs in the system and its characteristics are determined by the differential equations [6]

$$\frac{dm_x(t)}{dt} = \sum_{i=0}^{\infty} \left(\lambda_i(t) - \mu_i(t)\right).p_i(t) \tag{16}$$

and $\frac{dD[X(t)]}{dt} = \frac{dD_x(t)}{dt} = \sum_{i=0}^{\infty} \left[\lambda_i(t) + \mu_i(t) + 2\left(i - m_x(t)\right).\left(\lambda_i(t) - \mu_i(t)\right)\right].p_i(t)\,.$ (17)

From (16) and (17) for $\mu_i(t) = 0, (i = 1,2,....)$ the following equations are obtained respectively

$$\frac{dm_x(t)}{dt} = \sum_{i=0}^{\infty} \lambda_i(t).p_i(t) \tag{18}$$

and $\frac{dD_x(t)}{dt} = \sum_{i=0}^{\infty} \lambda_i(t).\left[1 + 2.\left(i - m_x(t)\right)\right].p_i(t).$ (19)

If $\lambda_i(t) = \lambda(t)$ and at the initial point of time $t = 0$ the distribution of probabilities $p_i(0), (i = 0,1,2,3,.....)$ represents Poisson distribution with a parameter $m_x(0)$ and the equations below are fulfilled

$$p_i(0) = \frac{[m_x(0)]^i}{i!}.e^{-m_x(0)},\ i = 0,1,2.....\,, \tag{20}$$

then the one-dimensional law for distribution of the random process $X(t)$ represents Poisson law with a parameter $m_x(t)$, which can be defined by solving the equation

$$\frac{dm_x(t)}{dt} = \sum_{t=0}^{\infty} \lambda(t).p_i(t) = \lambda(t)\,. \tag{21}$$

Therefore $m_x(t) = \int_0^t \lambda(t)dt + m_x(0)\,.$ (22)

By taking into account the way of setting of the intensity of the Poisson flow of purchasing new transport vehicles for the EMA centres (6), then from the dependence (22) one can calculate the mathematical expectation of the total number of supplied cars on the condition that $m_x(0) = 0,$ i.e. at the initial point of the period for supplying the EMA centres, there isn't a single new ambulance there. Then:

$$m_x(\infty) = \int_0^{\infty} \lambda(t)dt = \int_0^{t_2} \lambda(t)dt = \int_0^{t_1} atdt + \int_{t_1}^{t_2} at_1 dt = \frac{at_1^2}{2} + at_1\left(t_2 - t_1\right). \tag{23}$$

By substituting the values of the quantities t_1, t_2, a in (23), i.e. $t_1 = 5 years,\ t_2 = 9\ years,\ a = 50\ (1/year^2)$ with the ones above, we get:

$$m_x(\infty) = m_x(t_2) = 50.\frac{5^2}{2} + 50.5.(9-5) = 1625 \text{ ambulances.} \quad (24)$$

Because in our case the conditions of the Poisson distribution are fulfilled for the random process $X(t)$, then $m_x(0) = D_x(0) = 0,\ p_0(0) = 1$ and $m_x(t_2) = D_x(t_2) = 1625$, from which the result is obtained, namely

$$\sigma_x(t_2) = \sqrt{D_x(t_2)} = \sqrt{1625} = 40{,}31 \text{ ambulances.} \quad (25)$$

CONCLUSIONS

It is known from the literature [2] that when $m_x(t_2) > 20$, one can assert with accuracy sufficient for practical purposes, that the random quantity $X(t_2)$ distributed according to Poisson law, is if fact distributed according to a normal law with a parameter $m_x(t_2) = 1625$ ambulances. Then $\sigma_x(t_2) = 40{,}31$ ambulances. Therefore, the probability we are looking for is

$$P\{X(t_2) > 1600\} = 0{,}5 - \Phi\left\{\frac{x - m_x(t_2)}{\sigma_x(t_2)}\right\} = 0{,}5 - \Phi\left\{\frac{1600 - 1625}{40{,}31}\right\} =$$
$$= 0{,}5 - \Phi\{-0{,}61\} = 0{,}5 + 0{,}2291 = 0{,}7291 \quad (26)$$

By applying "the three-sigma rule", the average number of ambulances supplied for the duration of the period under investigation is in practice defined by the condition

$$m_x(t_2) \pm 3\sigma_x(t_2) = 1625 \pm 3.\sqrt{1625} = 1625 \pm 3.40{,}31 \approx 1625 \pm 120, \quad (27)$$

i.e. it varies between the limits $1505 < X(t_2) < 1745$.

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НЯКОИ ИЗСЛЕДВАНИЯ ВЪРХУ ВЪЗМОЖНОСТИТЕ ЗА ОБОРУДВАНЕ НА ЦЕНТРОВЕТЕ ЗА СПЕШНА МЕДИЦИНСКА ПОМОЩ С НОВИ ТРАНСПОРТНИ СРЕДСТВА

Веселина Евтимова

Русенски университет „Ангел Кънчев”

***Резюме**: В настоящата работа е изследван процесът на закупуване и снемане от експлоатация на нови транспортни средства в центровете за спешна медицинска помощ в Република България. Процесът е моделиран като е използван подходящ математически апарат. Намерени са характеристиките на случайния процес. Определен е реалният брой на линейките в експлоатация. Намерена е вероятността за целия период на доставка на нови автомобили в центровете за спешна медицинска помощ броят на линейките да бъде не по-малко от 1600 като се счита, че няма откази на закупените нови автомобили. Направени са съответните изводи.*

***Ключови думи:** Математическо моделиране, Случайни процеси, Марковски процеси, Закони на разпределение, Спешна медицинска помощ*

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SYSTEM FOR MODELING OF AMBIGUOUS SEMANTICS

Iliyana Raeva

Angel Kanchev University of Ruse

Abstract: *The paper presents a system for linguistics analysis of a natural language text. Based upon a correct syntax analysis, the system generates the semantics of each word, phrase or sentence and solves ambiguities by the selection of appropriately chosen meanings of the ambiguous words selecting the words from semantic nests. The program creates an attributive value matrix for each processed sentence and the relative logical form. The system is to be applied for processing and correcting documents and it can be used as an educational system.*

Keywords*: semantics, logical form, ambiguities, attributive value matrix*

1. INTRODUCTION

The Word Sense Disambiguation (WSD) is a long standing process in computational linguistics. A system solving the problem should be possible to receive as an input undefined text and to select of each word the most appropriate meaning being of acceptable accuracy and efficiency. The most common approach to eliminate ambiguity of the word is the use of the context of the word according to the entered text. For this purpose we use the information for any of the word's significance [4].

2. CONTEMPORARY ALGORITHMS FOR WSD

The algorithms for WSD can be categorized depending on the method, used for the acquisition of knowledge. Here's a sample categorization [2]:

- **Base of knowledge**: WSD using information from a comprehensive dictionary/lexicon or a knowledge base. The dictionary could be computer readable dictionary or encyclopedia (such as LDOCE or WordNet), or manually written. An example is the well known system of McRay;
- **Based on corpora:** WSD using information obtained from the study of several corpuses. This approach could be divided as:

  =tagged on corpora: Information is collected from the corpus the semantic ambiguity of which has been already removed. An example of this is the bilingual corpus of Brown [1] for training statistical WSD algorithms;

  =untagged corpora: Information is collected from the "raw" corpus, from which the semantic ambiguity was not removed.

The modern methods, applying knowledge of the world as selectors (selection preferences) to solve this problem, do not use effectively yet the available knowledge bases. Moreover, their effectiveness decreases when enriching knowledge by increasing the conceptual connections. The effective removement of the ambiguity of the words' meaning requires taking into consideration the dynamic context in processing the sentence in order to find the right set of selectors. In this sense, the system proposed by Kavi Mahesh, Sergej Nirenberg, Stephen Beale (Computing Research Laboratory, New Mexico State University, USA) is such an introductory operator (inference operator), which is the most accurate in the context of WSD Mikrokosmos semantic analyzer.

The method used in the system retains its efficiency even in a high broad knowledge base with a high degree of coupling between the concepts.

3. DESCRIPTION OF THE MODEL – LOGICAL STRUCTURE

The syntax of the language, processing the system, is described by the following contacts free grammar:

$$L=<T, H, A, R> ,$$

where

T = {Noun, Name. Pronominal, Adjective, Adverb, Preposition, Verb, Number, Article, λ};
H = {S. VP, PP, NP, NP2, NP3, Art1, Num1, Adj1, Adv1. λ};
A= {S};
R= {

S-> NP, VP
NP -> Art1, Num1, Adj1, Noun
NP-> Name
NP-> Pronominal
VP-> Verb,, Adv1, NP2
NP2->NP3, PP
NP2-> NP3
NP3->NP
NP3->PP
PP-> Preposition NP
Art1-> Art
Art1-> λ
Num1-> Number
Num1-> λ
Adj1->Adjective
Adj1-> λ
Adv1->Adverb
Adv1-> λ }

The check of the syntax of this context free grammar is implemented by the following preceding matrix:

	Noun	Name	Pron.	Adj.	Prep.	Verb	Adv.	Numb.	Dumb (space)	Art
	0	0	0	0	1	1	0	0	1	0
	0	0	0	0	1	1	0	0	1	0
	0	0	0	0	1	1	0	0	1	0
	1	0	0	0	0	0	0	0	0	0
	1	1	1	1	0	0	0	1	0	1
	1	1	1	1	1	0	1	1	0	1
	1	1	1	1	1	0	0	1	0	1
	1	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0
	1	0	0	1	0	0	0	1	0	0

Although the system processes sentences in which the ambiguous words are nouns, verbs or adjectives, it could be considered as a suitable model for examining the efficiency of the algorithm, since the counted parts of the speech are the majority of the ones in natural language.

The set value function belonging to a semantic nest is defined on the key activities "ambiguous word + test" by a factor of compatibility. This determines the percentage of statistical probability, which is the ratio of the number of word's uses with a specific meaning to the total number of cases in which the words are found with any of their meaning.

As a heuristic criterion, when an **adjective** is considered, the given order follows: first is the noun to be explained, then the verb in the sentence comes and finally, if no longer used as a criterion, the subject of the sentence follows.

For the "criterion" of the **verb** of the sentence, the noun from the following the verb noun phrases is used.

If the ambiguous word is a noun, it is determined by comparing consequentively:

- with its modificator (only with the adjective), if any;
- then with the verb, if it is "to be" this comparison becomes pointless;
- if you have not defined the exact meaning yet and the question concerns the noun from the object noun phrases, the word is juxtaposed with the subject of the sentence.

If after the application of the rules set out above, the ambiguity is not yet removed,

the verb and the subject of the previous sentence are used. This is not implemented in the model system by purely technical reasons.

If the result of the comparison is a coefficient 0, it terminates and such an option is considered impossible (meaningless). As a criterion in the demonstration model pronouns are not used, even if they fall within one of the above-described key positions because the anaphoric reference is not processed and therefore undefined data must not be kept.

For example the combination "be" and "blue" is unclear as "he" is undefined in the context phrase.

It is enough to use only the previous sentence of the processed text to remove the remaining uncertainties in the meanings of the words as it in turn uses the previous sentence as well and thus indirectly the whole preceding text is processed to remove the ambiguity. To understand the meaning of a word a larger context of a sentence or two is not necessary. Therefore rarely we have to wait for the next sentence in order fully to understand the meaning of the foregoing one.

Therefore, using words from the following sentences as a criterion to remove ambiguity is unjustified and it unnecessary delays and complicates the algorithm.

The system is self-learning in enriching the vocabulary and the relationships between words, so it is not dependent significantly on the type of the processed text.

The method uses a set of 10 dictionaries in which information for the type of the respective word and for some of its fundamental characteristic parameters is stored (not all of them are used in the system).

They are the following:

1. Adjectives [Category (A 15), Head (A 20)];
2. Adverb [Category (A 15), Head (A 20)];
3. Names [Category (A 15), Head (A 20), Spirit (A 1), Number (A 1)];
4. Nouns [Category (A 15), Head (A 20), Spirit (A 1), Number (A 1), Count(A 1)];
5. Numbers [Category (A 15), Head (A 20), Type (A 1)];
6. Prepositions [Category (A 15), Head (A 20)];
7. Pronominal [Category(A 15), Head(A 20), Type (A 1), Person (N), Number(A 1)];
8. Verbs [Category(A 15), Head(A 20), Tense (A 25), Person (N), Number(A1), Infinitive(A 20)];
9. Meanings [Word (A 25), Meaning(A 50), Category (A 15)];
10. Fuzzy [Word1 (A 30), Meaning1 (A 50), Word 2 (A30), Meaning 2 (A 50), Coefficient (S)],

where:
A is a text type and the number after it indicates how many bytes are allocated to it;
N is an integer;
S is a type (short); t and f (true, false); s and p are numbers (singular, plural); type o and c (ordinary, cardinal); Person (person) 1, 2 and 3;
“Coefficient” shows the percentage of statistical probability.

The total coefficient of variants of the sentence from a given text is calculated as the product of the coefficients of compatibility of the couples word - criterion and an ambiguous word. The percentage of probability of a given option compared with the other versions of the same sentence in the same text is calculated as follows:

in 2 options	in 3 options	in 4 options
x + y = 100	$\begin{cases} x+y+z=100 \\ \frac{x}{y}=\frac{a}{b} \\ \frac{y}{z}=\frac{b}{c} \end{cases}$ =>	$\begin{cases} x+y+z+w=100 \\ \frac{x}{y}=\frac{a}{b} \\ \frac{y}{z}=\frac{b}{c} \\ \frac{z}{w}=\frac{c}{d} \end{cases}$ =>
$x=\frac{100a}{a+b}$	$\begin{cases} x=\frac{100a}{a+b+c} \\ y=\frac{100b}{a+b+c} \\ z=\frac{100c}{a+b+c} \end{cases}$	$\begin{cases} x=\frac{100a}{a+b+c+d} \\ y=\frac{100b}{a+b+c+d} \\ z=\frac{100c}{a+b+c+d} \\ w=\frac{100d}{a+b+c+d} \end{cases}$
$y=\frac{100b}{a+b}$		

Here x, y, z and w are the values of the probabilities of the respective options in percentages, while a, b, c and d are the common coefficients of the respective options.

4. PROGRAM RESTRICTIONS

Restrictions that are imposed on the type of sentences, processed by the system, are as follows:

* The statements must be in English, be simple, non-negative and grammatically correct.

* There can be only one adjective and / or number, which explains (modifies) the noun and they are before the noun in the stated order.

* There cannot be separate adjectives or numbers.

* A noun cannot be used as a clarification (modifier) of a noun, if an adjective is used it has to be after the noun.

* After the verb maximum two noun phrases could be present.

* Facilitator names and pronouns have no modifiers.

* There cannot be definite articles of own names.

* The verb can be only one word in present or past tense.

The text, processing the system, has the following limitations:

* Each sentence can have maximum two ambiguous words that can be nouns or verbs.

* The system processes no more than five sentences.

* Each ambiguous word can have maximum two meanings.

Input data

The system accepts input text that can be entered manually at startup or by using a standard text file, introduced previously. The program allows the entered text to be saved as a text file with extension * txt. In further processing if unknown words or combinations of words are available, the system requires the introduction of additional data for the corresponding word.

Output data

Output program displays:

• Atributive Value Matrix (AVM) as a tree for each variant of sentence;

• logical form of each variant ;

• the coefficients of consistency of words-criteria's and ambiguous words;

• the common probability of sentence variants providing a possibility for determining the minimum level of probability of the sentence under which the sentence is rejected. If the coefficient of consistency is 1 it is possible to be determined whether in the logical form it will appear as a multiplier and whether the couple of words with this coefficient to be shown.

5. CONCLUSION

The applications could be implemented in systems for correcting and processing of documents (contracts), training systems, expert systems, etc.

To every processing sentence and to every noun phrase a logical form is mapped. Thus, based on the syntax, a certain type of semantic ambiguities are solved, arising from the words in the sentences.

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СИСТЕМА ЗА МОДЕЛИРАНЕ НА НЕЕДНОЗНАЧНИ СЕМАНТИКИ

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Резюме: *В статията е представена система за лингвистичен анализ на естестено-езиков текст. На основата на коректен синтактичен анализ, системата задава семантика на отделни думи, фрази или изречения, като решава неедноznачности чрез избор на подходящо избрани значения на неедноznачните думи чрез избор на думите от семантични гнезда.*

Програмата задава атрибутивно-стойностна матрица на всяко обработвано изречение, както и съответната логична форма. Системата е предназначена за обработка и корекция на документи и може да служи като система за обучение

Ключови думи: *семантика, логична форма, неедноznачност, атрибутивно-стойностна матрица*

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USE OF COMPUTER GAMES AS AN EDUCATIONAL TOOL

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***Abstract**: This paper is related to a research developed in collaboration by three higher education institutions – University of Ruse, New Bulgarian University and South-West University "Neofit Rilski". The research is supported by National fund: Bulgarian science fund, project ДФНИ-И01/10. The major objective of the project is to research innovative methods for assessing competence in e-learning environments. The specific article presents an overview of using computer games as an educational tool for providing the development of cognitive skills and making possible the socio cultural interactions. It defines games as an instructional medium for experiential learning, team building and easier understanding of abstract concepts.*

***Keywords**: e-learning, computer games, teaching, learning.*

INTRODUCTION

Constantly emerging new technologies challenge the field of education. Proper use of contemporary educational technologies can enhance learning and teaching. However, to be helpful, this technologies need to be incorporated in advanced pedagogical methods, which on the other hand stimulate collaboration, communication and mobility.

Dramatic changes in information and communication technologies (ICT) provide a powerful strength for the growth of e-learning. E-learning has become the certain trend for education with the educational technologies like:

- Smartphones and mobile computers.
- Networking software (Facebook, Twitter, Skype).
- Learning applications and open educational resources.
- Tools for personal learning environments - collaborative tools (blogs, wikis, authoring software).
- Virtual reality and immersive environments (virtual worlds).
- Media production and distribution tools.

According to Moore & Anderson using computers and accessing the Web in universities, using quality e-learning resources, broadband connections and Web 2.0 networking applications, provide conditions for increasing student autonomy and learning effectiveness [8].

In an e-learning environment the role of the teacher has shifted from content provider to facilitator of students' learning. The teacher plays a key role in e-learning environment. Their mission is to track learner's progress during the course and provide him with support. They should help and encourage the learners, track their progress, guide them, answer their questions, and so on. Ertmer [2] argued that teacher's pedagogical beliefs about the value and role of technology will determine students' attitudes towards using technology [2].

1. COMPUTER GAME - DEFINITION

Computer games are a new challenge to the teaching community, providing the development of cognitive skills and making possible the socio cultural interactions [6]. It is an instructional medium offering strong degrees of cognitive efficiencies for experiential learning, team building and easier understanding of abstract concepts. The following definitions of games are provided by different researchers:

- Gredler defined educational games as unique opportunities for students to experience activities within a cognitive domain in which new knowledge can be introduced [5].
- Computer game is a game to be played using technological devices throughout human - computer interaction [9, 4].
- According to Zimmerman the game is a fictional interactive activity without obligations, with rules, a defined time and space and quantifiable outcomes. The game play is the free space of movement within a rigid structure [10].
- Another term used by researchers is Serious Games. A serious game is a game designed for a primary purpose other than pure entertainment. The "serious" adjective is generally pretended to refer to products used by industries like defense, education, scientific exploration, health care, emergency management, city planning, engineering, religion, and politics [11].

2. USING COMPUTER GAMES AS AN EDUCATIONAL TOOL – STRENGTHS AND WEAKNESSES

The interest in using computer games for educational purposes has also increased over the last decade, with researchers identifying key pedagogical features that make good computer games inherently strong learning tools. A well designed game can teach higher order thinking skills such as strategic thinking, interpretative analysis, problem solving, plan formulation and execution and adaptation to rapid change [12]. These are the skills employers increasingly seek in workers and new workforce entrants. Games can also serve as powerful tools for teaching practical and technical skills. In addition, today's students who have grown up with the new ICT are especially ready to take advantage of the educational games.

The following *benefits* of gaming have been recognized by the Federation of American Scientists, the Entertainment Software Association and the National Science Foundation convened a National Summit on Educational Games [3]:

Higher Order Skills:

- Think strategically about their positioning, analyze opponent strengths and weaknesses, plan how to achieve game goals and execute those plans.
- Master resource management - managing people, money, food and natural resources - and learn to acquire and apply force multipliers such as knowledge and technology.
- Interact with systems and understand the interaction of variables.
- Multi-task, manage complexity, respond to rapidly changing scenarios and make decisions.
- Learn compromise and trade-off in satisfying the needs of diverse constituencies.
- Manage complex relationships.
- Exercise leadership, team building, negotiation and collaboration.

Experiential Learning

The Experiential Learning promotes active learning, including planning, reflection and acquire theoretical knowledge in the following manner:

- Learning takes place as part of a sequence of steps where a learner starts by actively taking part in an educational process that provides a concrete experience.
- Personal reflection on the experience.
- This reflection is then followed by the application of known theories to the experience or the derivation of rules from it.

- Learning is used to inform, modify and plan the next learning activity.

Practical Skills Training:

Through games and simulations learners can exercise practical skills. This allows learners to train and practice without risking life or breaking up expensive equipment in the classroom. The learners can try again and again to acquire the required knowledge and skills.

High Performance Situations:

Games and simulations show promise in training individuals for high-performance situations that require complex and multicomponent decision-making. Characteristics of such situations include: rapidly evolving; ambiguous scenarios; time and performance pressures; the need for judgment and high consequence for errors.

Rarely Used Skills:

Simulations are particularly important for reinforcing skills that are seldom used. For example: to practice students' responses to natural disaster situations.

Developing Expertise:

Games offer a way to learn how experts cope with the problems, creating mental models or templates that they apply to different situations in their work. Games and simulations provide an opportunity to develop mental models more quickly to augment real-world experiences.

Collaborative Learning and Team Building:

Games offer opportunities to information sharing, goal-directed cooperation and the spontaneous formation of relations - all critical skills demanded from the today's employers.

Educational computer games:

- Develop skills for effective work team, especially in decision-making, exercising judgment and solving problems under pressure.
- Offer opportunities for teams to develop shared knowledge and shared mental models that support implicit communications.
- Help teams improve communications among members by allowing them to develop information flows and practice task sequencing.
- Provide feedback - one of the benefits of computer games is the ability to provide the interaction and feedback that is crucial to the experiential learning cycle and to the whole learning process.

Problem-Based Learning

Problem-based learning involves small groups of students working together on real-life and cross-disciplinary problems. The teacher is facilitator rather than subject expert. Resources are provided to the students but information on how to cope with the problem is not provided. This encourages students taking more responsibility for their own learning and learning in a real-world context [1].

Disadvantages of using computer games in education

Despite all the benefits that learning with computer games provides, there are some disadvantages. For instance [7]:

- High development costs.

- Slow change in educational institutions for adopting new innovations and new learning technologies.
- Some teachers have very negative attitudes about giving up textbooks in order to use educational gaming products.
- The specific educational values that are related to educational standards have not been proven.
- Games are especially good at teaching skills, which are not typically evaluated in traditional assessments.
- Lack of access to required technologies.

CONCLUSION

The aim of this article was to present a short overview about the educational value of computer games in learning context. It states that using computer games as an education tool contributes to development of the human society and facilitating ways of social interaction. It is one of the most common forms of entertainment. The value of computer games lies in the opportunity to combine the play with pedagogical advantages and to promote changes in cognitive, behavioral and psychomotor skills.

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ИЗПОЛЗВАНЕ НА КОМПЮТЪРНИ ИГРИ КАТО СРЕДСТВО ЗА ОБУЧЕНИЕ

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Русенски университет "Ангел Кънчев"

***Абстракт**: Тази статия е свързана с изследване, проведено в сътрудничество с три висши учебни заведения – Русенски университет Ангел Кънчев, Нов Български Университет и Югозападен университет Неофит Рилски. Изследването е подпомогнато от Национален фонд Научни изследвания, проект ДФНИ-И01/10. Основната цел на проекта е изследване на иновационни методи за оценка на компетенции в среди за електронно обучение. Конкретната статия представя въведение в използването на компютърните игри като средство за обучение.*

***Ключови думи**: електронно обучение, компютърни игри, преподаване, учене*

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EDUCATIONAL COMPUTER GAMES FOR DIFFERENT TYPES OF LEARNING

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***Abstract**: This paper is related to a research developed in collaboration with three higher education institutions – University of Ruse, New Bulgarian University and South-West University "Neofit Rilski". The research is supported by National Fund: Bulgarian Science Fund, project ДФНИ-И01/10. The major objective of the project is to investigate innovative methods for assessing competence in e-learning environments. The article identifies the different types of computer games available and the kind of learning that can be supported with computer games*

***Keywords**: E-learning, computer games, common game types, teaching, learning*

INTRODUCTION

The use of information technologies and multimedia in education has significantly changed peoples' learning processes. Results from a number of research studies indicate that appropriately designed multimedia instruction enhances students' learning performance in different subjects [4].

Computer games demonstrate effective pedagogical techniques that can be used in learning. Game environment enables new forms of knowledge interaction previously unavailable within the traditional curricula. It is powerful instructional tool - from illustrating principles in physics to practicing surgery on a virtual patient. The importance of providing such learning environments to science and engineering students is obvious.

The claims about using computer games in education are confirmed by the fact that this use :

- Is based on strict learning theories and principles.
- Provides engagement for the learner.
- Provides personalized learning environments.
- Supports 21st century skills.
- Provides an environment for authentic and relevant assessment.

1. GAME PLATFORMS

There are distinctions among different electronic hardware. Digital games can be console-based or computer-based. The latest generation of consoles includes Microsoft's Xbox 360, Sony's Playstation 3 and Nintendo's Wii along with handheld devices such as the Nintendo DS and Sony PSP. While some games are console specific, many games can be played across multiple platforms [2].

This paper describes digital gaming, regardless of whether it is on a handheld, a console or a PC.

2. MODES OF PLAY

In addition games can be characterized by the modes of play - some are single-player and some are multiplayer; some are handheld and some are online.

- **Single-Player mode -** single-player mode allows one player to play on one platform.
- **Multiplayer mode - a**lthough PCs and handheld games allow only a single player's experience, console games support multiplayer modality. All game consoles

are manufactured with multiple controller ports and thus more than one player can play at a time.

- **Network mode –** network mode allows multiple players to play a single game simultaneously. The computers must be networked together, but the game may run either on a server on the network or through peer-to-peer connection.
- **Online mode –** online games are a subset of networked games with sufficient distinct characteristics. This subset is treated as a separate type of games according to these characteristics.

3. GAME GENRES

Games come from many different genres, including first-person shooters, role-playing, action, adventure, card, puzzle and sports. Some are single-player and some are multiplayer; some are handheld and some are online.

Computer games, like books and movies, come in a number of different genres. The literature offers the following categorization:

Adventure

Adventure-style games are typically story-driven and have one or more central characters. These games are perceived mostly like movies and can rely heavily on dialogue, exploration and logical problem solving to move the player through the narrative [5].

A classic example in this genre is the interactive fiction game *Myst*. The tasks in the game may be relevant to the curriculum and the learning process, often in terms of motivation, as in the case of *Civilization*, a widely popular and researched game that involves geography, history and politics [1].

Shooting /Action

This category includes a different gameplay perspectives and subgenres. These games can include First Person Shooters (FPS) or other fighting games. Usually action games consist of tests of players' dexterity, reaction time and quick-wittedness under pressure.

In shooter games, players typically aim and fire at moving objects to destroy them. This involves the development of fast hand to eye coordination and may be important in training areas associated with the police or military. In most cases the player operates virtual mechanical devices and has to accomplish some objective (e.g. drive a vehicle, fire a weapon or use a tool) [1].

Puzzle

Puzzle games primarily involve problem-solving, including words, logic and mathematics. These types of games are based on traditional puzzles. Games that involve logic, problem solving, pattern matching or all of the above fall into this game type. For example: Tetris, Bejeweled, Sudoku etc.

Strategy

Strategy games involve the player making strategic decisions within a scenario in order to meet the goal of the game, which is usually completing a level or solving a particular problem [7].

There are many good examples of this type of games, in the areas of history, economy, management, ecology, society, etc. Very popular and successful examples are: Civilization and Age of Empires.

Simulation

In simulation games, the player operates a model or simulation that behaves according to a programmed set of rules. Many simulation games focus on some elements

of realism, thus forcing players to understand and remember complex principles and relations and progress by trial-and-error [1].

These games are able to teach flying a plane up in the sky, steering a submarine deep in the ocean, etc. Good examples are *Flight Simulator*, *Train Simulator*, *SimCity*, etc.

Social simulation games are also a large component in the simulation genre, for example The Sims. Another free simulation game for Business Project Management training is INNOV8, developed by IBM a few years ago and now reaching version 2 [8].

Role-Playing

A Role-Playing Game is a game in which the participants take the roles of fictional characters. The player can perform different activities - solving quests, fighting, treasure hunting, and interacting with other characters.

In the context of learning, role-playing games are useful for providing a context for building collaborative skills, social interaction and negotiation, management of complex systems (e.g. character statistics), strategy and working through scenarios.

Sports

Sports games allow the player to simulate taking part in a sporting event or tournament. Sporting games can be used to practice the actual skills of a sport, tactics, rules and the ability to think and make decisions quickly.

Virtual Worlds

3D virtual worlds can provide opportunities for high sensory immersive experiences, with authentic contexts and activities for experiential learning, simulation and role-play, including the creation of complex environments and scenarios.

A well-known example is Second Life which enables users to interact with each other through avatars. Players (residents) can explore the world, meet other residents, socialize, participate in individual and group activities and create and trade virtual property and services with one another.

Traditional and Casual Games

Examples of such games are Chess, solitaire and card games, online 2D or 3D spaces with obstacles to overcome. Casual games are divided into many small levels that follow the same pattern. They can be played for a few minutes at a time and easily stopped and restarted.

British Education Communications and Technology Agency (BECTA) presents examples of different genres of games along with brief description (Table 1):

Table1. Examples of different genres of games along with brief description [6]

Genre	Examples	Description
Action adventure	Tomb Raider, Soul Reaver	Combines elements of combat, platform games, problem solving and exploration.
Fighting games	Tekken 3, WWF	Most popular on consoles, game play is based on two or more opponents attempting to knock the other out.
Management games	Championship Manager 2001-2002, City Trader, Zoo Tycoon	Usually based on economic management in a simulated environment. The player must raise funds to pay for maintenance, wages, research, a new striker, etc.
Platform games	Rayman, Lego Alpha Team, Abe's Odyssey	The player must complete levels by avoiding various obstacles, jumping onto platforms or using objects with special properties.
Racing games	Grand Turismo 3, Wip3out, Grand Prix 3	The realism can vary from approximate simulations of rallies using real map data, to arcade-style races, where realism is sacrificed to provide a greater sense

		of speed and present feats of driving impossible in reality.

Real time strategy	Command and Conquer, Sudden Strike, Stronghold	The player will normally command groups of units and gather resources to fund further expansion. Units move in 'real time' synchronous with the opposition's units.
Role playing games	Fallout, Baldur's Gate	The player controls a single character or group of characters. Game play is usually based around exploration and completion of quests.
Simulation games	IL2 Sturmovik, Train Simulator, Flight Unlimited	Simulation games can provide very accurate reconstructions of modern or historical vehicles.
World-building games	*SimCity*3000, Civilisation 3, Black and White, The Sims	This category covers a wide range of game styles. Essentially, the player must manipulate either a character or an environment to encourage development and progress.

4. TYPES OF LEARNING THAT CAN BE SUPPORTED WITH COMPUTER GAMES

As already have been stated computer games offer effective pedagogical techniques that can be used in learning. Gagne describes five categories of learning with a gamut of elements that could be developed with computer games (Table 2).

Table2. Categories of Learning [3]

Category	**Description**
Intellectual skill	Concepts, rules and relationships and making discriminations (e.g. using algebra to solve a mathematical puzzle).
Cognitive strategy	Personal techniques for thought and action (e.g. developing a mental model of a problem).
Verbal information	Relating facts (e.g. recalling the names of the bones in the hand).
Motor skill	Actions that use the muscles (e.g. dancing).
Attitude	Beliefs and feelings (e.g. choosing to read detective fiction).

CONCLUSION

We live in a world of constantly emerging new technologies that challenge the field of education while at the same time present exciting opportunities. There is no doubt about the benefits of using computer games to support teaching and learning. This article first presented an overview of game platforms, player modes and genres (adventure, platform, puzzle, role play, shooter, sports, strategy, etc) and highlighted types of learning that can be supported with computer games.

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ОБРАЗОВАТЕЛНИ КОМПЮТЪРНИ ИГРИ ЗА РАЗЛИЧНИ ТИПОВЕ УЧЕНЕ

Валентина Войноховска, Светлозар Цанков, Румен Русев

Русенски университет "Ангел Кънчев"

***Абстракт**: Тази статия е свързана с изследване, проведено в сътрудничество с три висши учебни заведения – Русенски университет Ангел Кънчев, Нов Български Университет и Югозападен университет Неофит Рилски. Основната цел на проекта е изследване на иновационни методи за оценка на компетенции в среди за електронно обучение. Статията представя описание на различни типове компютърни игри и разглежда видовете учене, които могат да бъдат подпомагани от тях.*

***Ключови думи**: Електронно обучение, компютърни игри, типове компютърни игри, преподаване, учене*

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CREATING AN E-TEXTBOOK FOR THE COURSE WORKSHOP ON COMPUTER NETWORKS AND COMMUNICATION

Victoria Rashkova, Metodi Dimitrov

Angel Kanchev University of Ruse

***Abstract**: The IT development gives further opportunities to the distance learning through e-textbooks. E-textbooks are undoubtedly of great convenience as learners could control studying in accordance with their speed and aptitude for learning. E-textbooks assist the process of comprehension and training: learners could choose relevant fields of interests; they could zoom out images and test the knowledge gained at any step of learning. Video tutorials, dictionaries, abbreviations list are also available, as well as a good number of useful links for those who are eager to learn.*

***Key words**: distance learning, e-textbook, graphical user interface, visual programming environment, user form.*

I. INTRODUCTION

E-textbooks become more popular with the development of the information technologies. The replacement of paper textbooks with electronic ones is owed to the following advantages:

- e-textbooks could be continually updated;
- they are cheaper having no issuance expenditures;
- they are accessible and interactive;
- the learner could determine the speed of studying, depending on his aptitude and knowledge gained to the moment;
- e-textbooks offer various tools for visualization and easier perception of the material taught including tasks, tests, images, video tutorials, as well as additional useful links for the advance learners;
- e-textbooks provide mobility, as they could be transferred (copied) from PC to PC; they could be used on tablets, e-readers and other digital devices which support relevant technology and have adequate displays.

It is confirmed that the more senses in the process of learning are involved, the better and long-lasting is the knowledge gained. In general, an individual memorizes:

- 10% of what is read;
- 20% of what is heard;
- 30% of what is seen;
- 50% of what is heard and seen.

Since the e-textbooks combine different visualization tools it is evident from the above stated statistics that they are much more convenient and helpful in the comprehension of the teaching material. Nowadays learners could find plenty of e-textbooks on the web.

An e-textbook is featured with the following items:

- Easy and straightforward design.
- Extensive workspace for content visualization.
- Various buttons with links to different forms.
- Tests for self-study and knowledge self assessment.
- An option for video clip addition.

- Alphabetical list of abbreviations and terminological dictionary.
- Buttons with links to additional information.

II. SYSTEM DESIGN

The application is designed with Windows Forms which is a standard library of .NET Framework for constructing windows based graphical user interface (GUI) for desktop applications, including SharePoint and Cloud. The Windows Forms Framework provides a straightforward object-oriented group of classes that enables the development of mighty applications for Windows [2, 3].

The form is part of the screen, usually in a rectangular shape, intended for visualization of user information. The forms could be standard windows, dialogue fields or areas for displaying diagrams/charts. Each form includes appearance determination controls, behavior determination methods and user interaction events.

Presentation of all forms of software application for creating an E-textbook and their inheritance is shown in Fig.1.

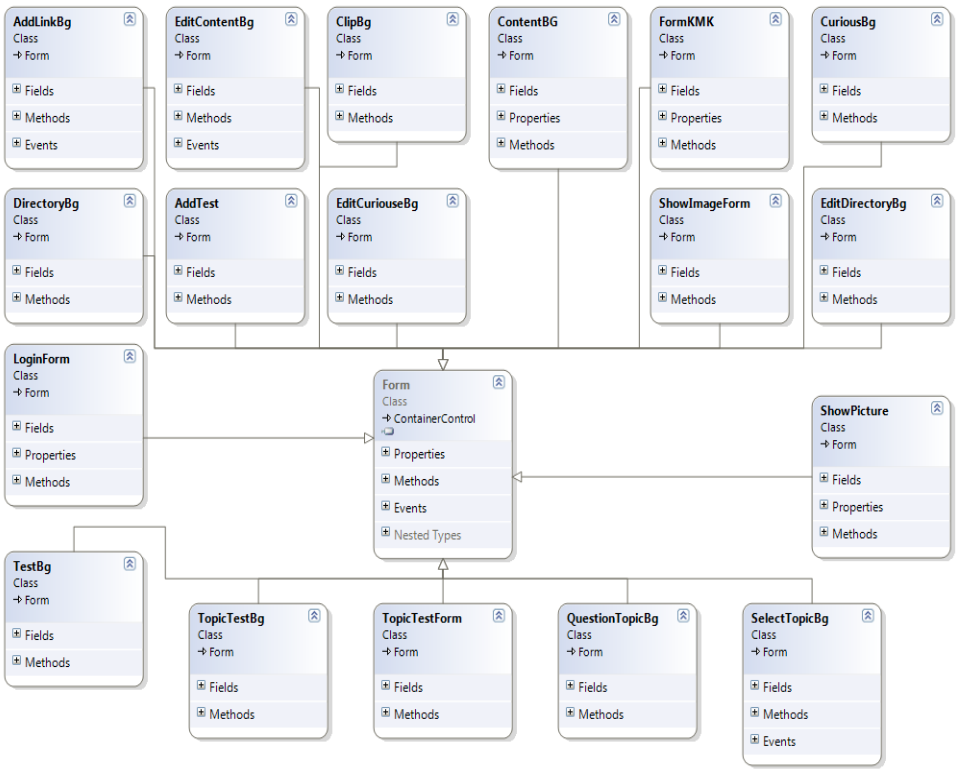

Fig.1. Inheritance used for E-Textbook forms

The e-textbook, described herein, has been created with the help of Microsoft Visual Studio 10. The application is implemented in C#, being one of the leading modern programming languages. As a component of the Microsoft .NET technology, it is an object-oriented programming language, developed by Microsoft as a part of the .NET software platform. The C# programming language elaboration was focused on the development of a modern object-oriented programming language in general use. C# stepped on its predecessors: C++, Java, as well as Delphi, VB .NET and C to some extent. It is as mighty as C++ having the capacity for a fast continual development like Visual Basic and Java. C# involves a set of classes' definitions which contain methods and the methods themselves include the programming logics, i.e. the instructions which a computer executes [1].

III. E-TEXTBOOK OPTIONS

The system provides the following options:

- Choosing a language: the system is designed to be run in two languages – Bulgarian and English. The language could be chosen with the start of the application (Fig.2).

Fig.2. Choosing a language

- Choosing the logon rights: immediately after the language choice, the user selects the option to access the system. Two options are provided for running the system – by user's (student's) or administrator's account (Fig.3).

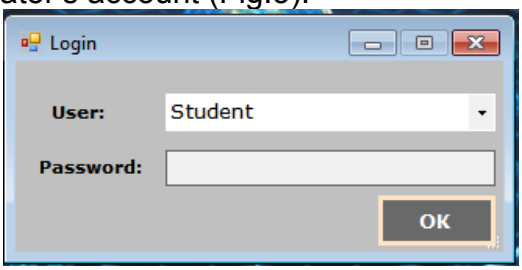

Fig.3. Choosing the logon rights

The student's account provides the possibility to start the course study and further determine the tools for goals accomplishment. The system basic window is displayed as follows (Fig.4):

Fig.4. Basic window of options

- Option for choosing the theme of studying (Fig.5).

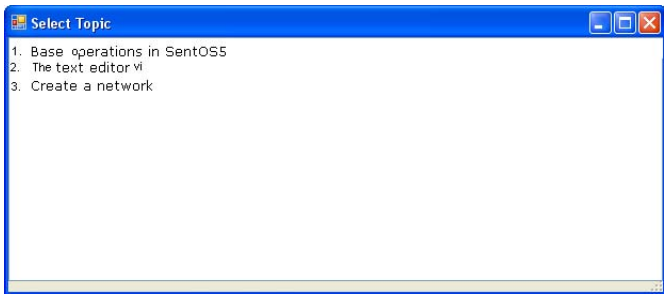

Fig.5. Choosing the theme of studying

- The learner could check his progress by filling a questionnaire on the topic. The answer to every question of the test could be verified by pressing the button “Answer” which displays the correct answer (Fig.6).

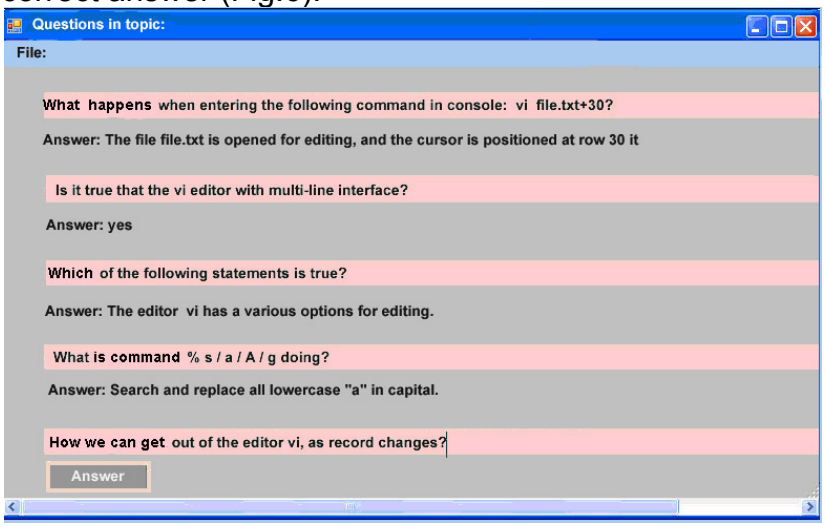

Fig.6. Test result at the end of a topic

- The application is capable in forming the final assessment after student’s completion of the course study.

- Student’s knowledge is evaluated by conducting a test, including all themes of the course (Fig.7).

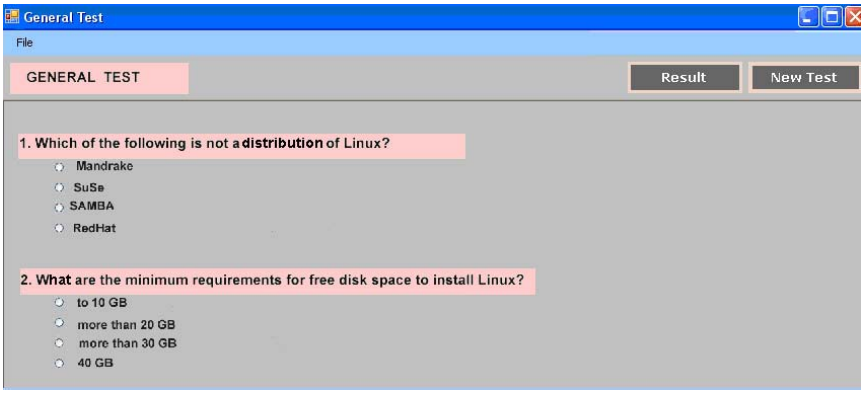

Fig.7. Test control knowledge after studying the whole subject area

- Option for using the incorporated terminological dictionary and abbreviations (Fig.8).

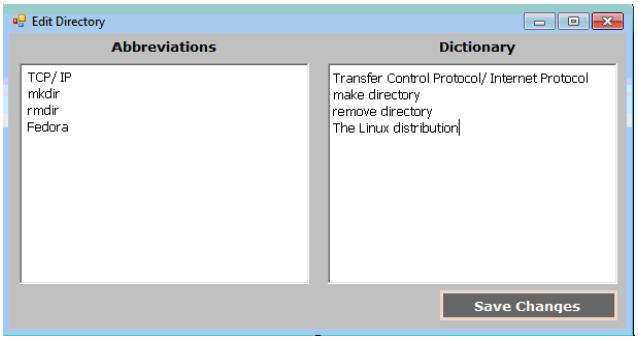

Fig.8. Dictionary of new incorporated terminological and used abbreviations

- Option for video clip addition to any course theme (Fig.9).

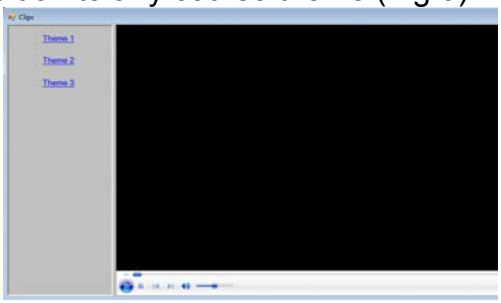

Fig.9. Clip addition

- Opportunity for the advance and inquisitive learners to get further information in the subject domain by following the offered useful links (Fig.10).

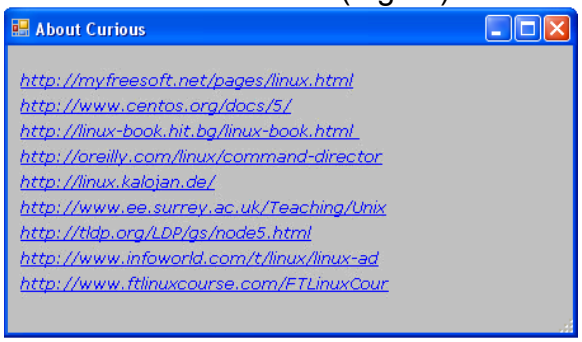

Fig.10. Useful links

The administrator's account is intended only for the tutor. He has the rights to:

- Add or remove a course theme.
- Correct an existing theme.
- Add or remove tests, used for consolidating learners' knowledge after a theme completion. Every test item provides a multiple choice answer (Fig.11).

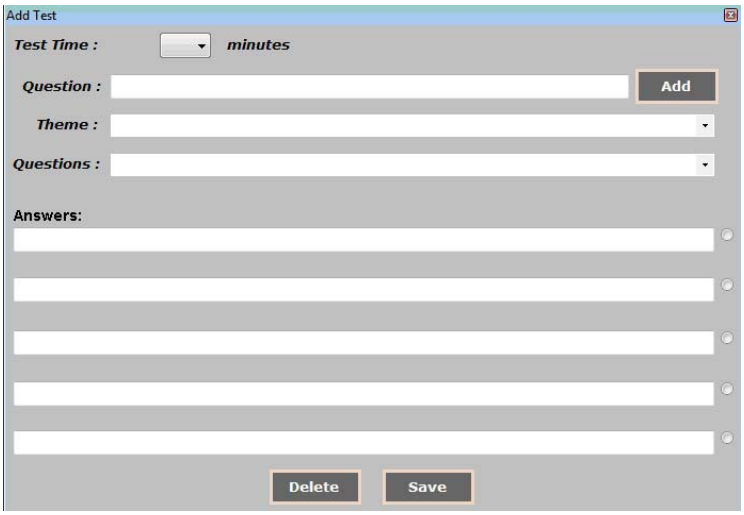

Fig.11. Create a new test

- Add or remove tests intended for consolidating the knowledge gained after the whole course completion.
- Change the set time for conducting the test (Fig.12).

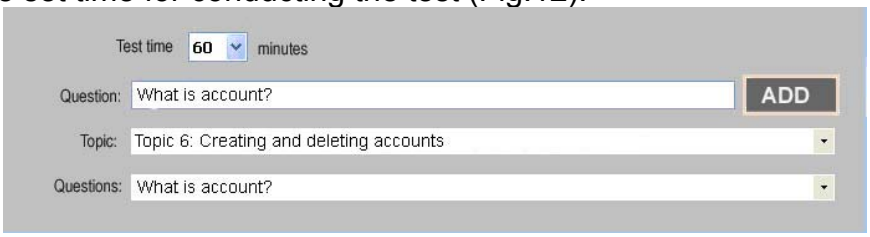

Fig.12. Set the time to finishing the test

- Add new links to websites which could be of interest to the advance learners.
- Add or edit the existing lists of abbreviations and specific terms.
- Add or delete existing video clips attached for a given theme course.

CONCLUSION

The so designed system for E-Textbook could be used by all teachers and students in various disciplines. The system provides a variety of learning opportunities. The main advantage is that the learner himself is to set the pace of adoption of the material. The principal disadvantage is that the system is not yet a web-based.

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ЕЛЕКТРОНЕН УЧЕБНИК ПО ДИСЦИПЛИНАТА ПРАКТИКУМ ПО КОМПЮТЪРНИ МРЕЖИ И КОМУНИКАЦИИ

Виктория Рашкова, Методи Димитров

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Резюме: *С развитието на информационните технологии дистанционното обучение посредством електронни учебници намира все по-широко приложение. Основната причина е, че електронните учебници създават голямо удобство на обучаемия - предоставят му възможност сам да управлява процеса на обучение, съобразно своето темпо и нагласа; предоставят му разнообразни средства за спомагане на процеса на възприемане и обучение, като потребителят сам прави избора си на тема; възможност за увеличаване на изображенията; възможност за проверка на знанията си на всеки етап от обучението; видео уроци; предоставени речници и списъци с използвани съкращения; множество полезни връзки за по- любознателните обучаеми.*

Ключови думи: *дистанционно обучение, електронен учебник, графичен потребителски интерфейс, визуална среда за програмиране, потребителски форми.*

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POSSIBILITIES OF ONLINE FREELANCE PLATFORMS

Metodi Dimitrov, Victoria Rashkova

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Abstract: *In recent years with the development of Internet new online platforms allowing employees and companies to connect and work together are emerging. The current report explores the potential of the online platforms from both points of view - globally and particularly in Bulgaria.*

Key words: *freelance platform, freelancer, online work, online staffing,*

1. INTRODUCTION

Every day people's access to Internet increases rapidly. Many people have daily access to the Internet and communications. This allows many people to live a virtual life and perform various operations such as exchanging ideas and opinions, looking for fun, work, watch TV, follow the news, etc.

In the Internet companies also see the great potential for the development of their business. Many of them use the Internet to offer their goods and services, recruit staff, communicate with business partners, etc.

Internet for those people and companies actually become an integral part of their daily lives. It is therefore not surprising that online platforms, connecting companies and people, began to appear gradually [1]. With the help of these platforms companies have the option to announce their projects, jobs, staff required and people can view companies' announcements and to apply.

The occurrence of these Internet platforms represents a new step in the evolution of society. They established a new form of communication standards and relationships between people and companies. From this point of view these online platforms represent scientific interest. For this reason, current research explores the possibilities and potential of such web platforms. The research covers the leading platforms.

2. POSSIBILITIES OF ONLINE FREELANCE PLATFORMS

In the Internet platforms people and companies see one more tool to make their business more flexible. With the help of the platforms humans and companies are able to: find the right job, find the right specialist for a specific position, share experiences, ideas and more. That is why it is no coincidence that, gradually, multiple web platforms aimed on connecting companies with potential employees appears.

However, despite their great number and variety, there are several platforms that concentrate the attention of companies and people offering and seeking work over the Internet. Proposed jobs are in various fields: software, law, engineering, writing, design, translation, marketing and more. As a rule, most of the proposed jobs do not require the employee to be physically present in the office. In most cases, the only need of an employee is to have a computer and Internet connection.

The wide variety of searched specialists allows people with different abilities to take advantage of such online platforms. Some of them are:

- Flexible working time - each employee defines his own working time.
- Flexible payment - the platforms offer a several ways of payment - hourly, part-time, on finished project, etc. That variety allows everyone to choose the most appropriate option for him.

- Global career – allow specialists to apply for a job anywhere in the world. This highly increases their chance to apply their particular knowledge and skills.
- As for the job apply specialists from all over the world, actually employees compete at global level. This helps the employees to build themselves as professionals.

Table 1
Comparison of some of the leading online freelance platforms

Platform	Jobs offered, in thousands	Visits, in thousands	Earnings of employees, $M	Registered accounts, in thousands	Earnings of BG employees, $k
Freelancer	605,1*	633,8	146,5*	823*	391,4*
Elance	826,0	865,1	200,0	1124	534,3
vWorker	89,4*	93,6	21,6*	122*	57,8*
Guru	300,7*	314,9	72,8*	409*	194,5*
iFreelance	146,3*	153,2	35,4*	199*	94,6*
99designs	927,3*	971,2	224,5*	1262*	599,8*
oDesk	301,6*	315,9	73,0*	410*	195,1*
Total	**3196,4***	**3347,8**	**773,9***	**4350***	**2067,5***

Note: * in Table 1 indicates that the value is obtained by interpolation

Table 1 shows information about the number of visitors and the cost of work, performed by employees for some of the most significant platforms. It is important to be noted that these data are for the period from 01.01.2012 to 31.12.2012. The data about the number of visits are taken from Quantcast [http://quancast.com, 3] and data about the eLance platform are taken from [http://elance.com, 2]. To find values for the remaining fields of the table - an interpolation is being used. Figures 1 and 2 show a graphical interpretation of some of the data in Table 1. Based on the data shown in Table 1, it could be concluded that for the year of 2012 on every 10 seconds a new job was registered on some of the considered platforms.

It is important to be noted, that the average hourly rate for performed work in Bulgaria is $16 [http://elance.com, 2]. On the basis of this number, the man-hours of work carried out in Bulgaria could be calculated, namely 129220 man-hours or 63.8 person-years. In the calculation of the person-years indicator it is assumed that a person works 8 hours a day for 253 days (working days in 2012 for Bulgaria).

In recent years, the jobs proposed in such online platforms are increasing [http://elance.com, 2]. Figures 3 and 4 show the trend of the amendment of registered and hired jobs in the considered platforms in 2012 compared to 2011. According to Figure 4, the employees in 2012 from Bulgaria have risen between 25% and 50%. This means that the number of registered accounts from Bulgaria in the platforms, listed in Table 1 for 2012, were between 4060 and 6764 (calculations are made on the base of the data in table 1 and are obtained by interpolation).

CONCLUSION

Based on the made research, the following conclusions and recommendations could be drawn:

1. For the year 2012, the offered jobs or projects on the considered platforms were approximately 3.2 million. Money, earned by employees worldwide, amounted to nearly 774 million dollars.

2. For the year of 2012, the proportions of completed projects from Bulgarian employees was only 0.3%. This shows the great potential for the Bulgarian freelancers.

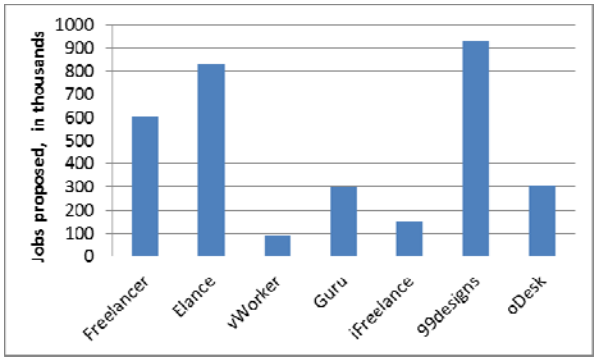

Figure 1. Number of registered employees for 2012

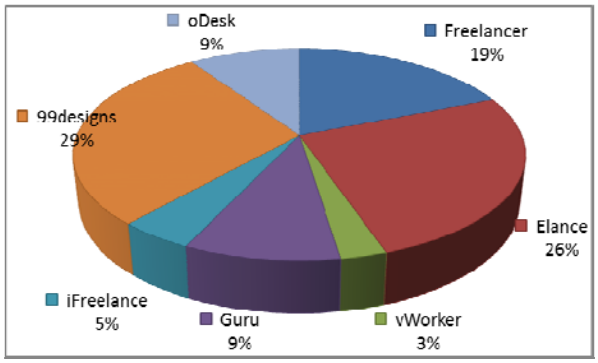

Figure 2. Number of registered employees for 2012, shown as a percentage

Figure 3. Trend of increase in the number of registered jobs in 2012 compared to 2011 [http://elance.com, 2]

Figure 4. Trend of increase in the number of completed jobs in 2012 compared to 2011 [http://elance.com, 2]

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ПОТЕНЦИАЛНИ ВЪЗМОЖНОСТИ НА УЕБ ПЛАТФОРМИТЕ ЗА ЛИЦА РАБОТЕЩИ НА СВОБОДНА ПРАКТИКА

Методи Димитров, Виктория Рашкова

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Резюме: *В последните няколко години с развитието на интернет се появяват онлайн платформи позволяващи на служители и компании да се свържат и да работят заедно. Текущият доклад изследва потенциала на онлайн платформите от две гледни точки – в глобален мащаб и в частност в България.*

Ключови думи*: набиране на персонал, служител, лице, на свободна практика, работа през интернет, уеб платформа*

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DIDACTIC AIMS AND PERSPECTIVES IN COMPUTER SCIENCE TEACHING

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***Abstract**: The article discusses the scope of education in computer science and the didactic objectives on which the introductory courses on algorithms based on those of programming to be built. Attention is paid to the variety aspects of the material and the reasons how to be precisely structured its presentation. The paper proposes an approach of 4 levels for learning and development of skills for writing algorithms. It stresses on the importance students to acquire skills for writing correct and efficient algorithms to achieve success in programming.*

***Keywords:** Computer Science, Algorithm, Algorithm Teaching, Algorithm skills development.*

INTRODUCTION

Computer Science is the study of principles and practices that underpin an understanding and modelling of computation and of their application in the development of computer systems [1]. At its heart the notion of computational thinking lies: a mode of thought that goes well beyond software and hardware and that provides a framework within which to reason about systems and problems. This mode of thinking is supported and complemented by a substantial body of theoretical and practical knowledge and by a set of powerful techniques for analysing, modelling and solving problems.

Computer Science is deeply concerned with how computers and computer systems work and how they are programmed. Students studying computing gain insight into computational systems of all kinds, whether or not they include computers. It allows them to solve problems, design systems and understand the power and limits of human and machine intelligence. It is a skill that empowers and all students should be aware of and have some competence in. Furthermore, students who can think computationally are better able to conceptualise and understand computer-based technology and so are better equipped to function in modern society.

Computer Science is a subject, where invention and resourcefulness are encouraged. Students are expected to apply the academic principles they have learned to understanding real-world systems and creating purposeful artefacts. This combination of principles, practice and invention makes Computer Science an extraordinarily useful and an intensely creative subject, suffused with excitement, both visceral (“it works!”) and intellectual (“that is so beautiful”).

THE BASIC AIMS OF COMPUTER SCIENCE

Education enhances students’ lives as well as their life skills. It prepares young people for a world that doesn’t yet exist; involves technologies that have not yet been invented; presents technical and ethical challenges of which we are not yet aware.

To do this, education aspires primarily to teach disciplines with long-term value, rather than skills with short-term usefulness, although the latter are certainly useful. A “discipline” is characterised by:

- A body of knowledge, including widely-applicable ideas and concepts and a theoretical framework into which these ideas and concepts fit.

- A set of techniques and methods that may be applied in the solution of problems and in the advancement of knowledge.
- A way of thinking and working that provides a perspective on the world that is distinct from other disciplines.
- Longevity: a discipline does not “date” quickly, although the subject advances.
- Independence from specific technologies, especially those that have a short shelf-life.

Computer Science comprises disciplines with all these characteristics. It encompasses foundational principles, such as the theory of computation and widely applicable ideas and concepts, such as the use of relational models to capture structure in data [4]. It incorporates techniques and methods for solving problems and advancing knowledge, such as abstraction and logical reasoning, and a distinct way of thinking and working that sets it apart from other science areas (computational thinking). It has longevity (most of the ideas and concepts that were current 20 or more years ago are still applicable today) and every core principle can be taught or illustrated without relying on the use of a specific technology.

THE KEY DIDACTIC AIM

The key didactic aim is something that a student of Computer Science should be able to do and should know. In Computer Science the key processes focus upon computational thinking. Computational thinking is the process of recognising aspects of computation in the world that surrounds us and applying tools and techniques from computing to understand and reason about both natural and artificial systems and processes.

Computational thinking is something that people rather do than computers and includes the ability to think logically, algorithmically and at higher levels recursively and abstractly [2]. It is, however, a rather broad term. The rest of this paper draws out particular aspects of computational thinking that are particularly accessible to and important for young people at the universities.

A well-grounded student of Computer Science will also be proficient in other generic skills and processes including: thinking critically, reflecting on ones work and that of others, communicating effectively both orally and in writing, being a responsible user of computers and contributing actively to society.

ABSTRACTION: MODELLING, DECOMPOSING AND GENERALISING

A key challenge in computational thinking is the scale and complexity of the systems we study or build [2]. The main technique used to manage this complexity is abstraction. The process of abstraction takes many specific forms, such as modelling, decomposing and generalising. In each case, complexity is dealt with by hiding complicated details behind a simple abstraction or model of the situation. For example, the Ruse Bus Route map is a simple model of a complex reality — but it is a model that contains precisely the information necessary to plan a route from one station to another. A procedure to compute square roots hides a complicated implementation (iterative approximation to the root, handling special cases) behind a simple interface (give me a number and I will return its square root).

Modelling is the process of developing a representation of a real world issue, system or situation, that captures the aspects of the situation that are important for a particular purpose, while omitting everything else. Different purposes need different models. Example: a geographical map of the Bus Route is more appropriate for computing travel times than the well-known topological Bus Route map; a network of nodes and edges can be represented as a picture or as a table of numbers. A particular situation may need more

than one model. For example a web page has a structural model (headings, lists, paragraphs) and a style model (how a heading is displayed, how lists are displayed). A browser combines information from both models as it renders the web page.

A problem can often be solved by decomposing it into sub-problems, solving them and composing the solutions together to solve the original problem. For example “Make breakfast” can be broken down into “Make toast; make tea; boil egg”. Each of these in turn can be decomposed, so the process is naturally recursive. The organisation of data can also be decomposed. For example, the data representing the population of a country can be decomposed into entities such as individuals, occupations, places of residence, etc.

Sometimes this top-down approach is the way in which the solution is developed but it can also be a helpful way of understanding a solution regardless how it was developed in the first place. For example, an architectural block diagram showing the major components of a computer system (e.g. a client, a server, and a network) and how they communicate with each other, can be a very helpful way of understanding that system.

Complexity is often avoided by generalising specific examples, to make explicit what is shared between the examples and what is different about them. For example, having written a procedure to draw a rectangle of sizes 2 and 4 and another to draw a rectangle of sizes 3 and 5, one might generalise to a procedure to draw a rectangle of any sizes N and M, and call that procedure with parameters 6 and 7 respectively. In this way much of the code used in different programs can be written once, debugged once, documented once, and (most important) understood once. A different example is the classification encouraged by object-oriented languages, whereby a parent class expresses the common features of an object, for example, the size or colour of a shape, while the sub-classes express the distinct features (a square and a triangle, perhaps). This process may be called generalisation. It is the process of recognising these common patterns and using them to control complexity by sharing common features.

PROGRAMMING

Computer Science is more than programming, but programming is an absolutely central process for Computer Science. In an educational context, programming encourages creativity, logical thought, precision and problem-solving and helps foster the personal, learning and thinking skills required in the modern university curriculum. Programming gives concrete, tangible form to the idea of “abstraction” and repeatedly shows how useful it is [5].

Every student should have repeated opportunities to design, write, run and debug executable programs [3]. What an “executable program” means can vary widely, depending on the level of the students’ skills and the amount of time dedicated for. In some cases the ability to understand and explain a program is much more important than the ability to produce working but incomprehensible code [4]. Depending on level of their skills, students should be able to:

1. Design and write programs that include
- Sequencing: doing one step after another.
- Selection (if-then-else): doing either one thing or another.
- Repetition (Iterative loops or recursion).
- Language constructs that support abstraction: wrapping up a computation in a named abstraction, so that it can be re-used. (The most common form of abstraction is the notion of a “procedure” or “function” with parameters.).
- Some form of interaction with the program’s environment such as input/output, or event-based programming.

2. Find and correct errors in their code.

3. Reflect thoughtfully on their program, including assessing its correctness and fitness for purpose; understanding its efficiency and describing the system to others.

Effective use of the abstraction mechanisms supported by programming languages (functions, procedures, classes, and so on) is central to managing the complexity of large programs. For example, a procedure supports abstraction by hiding the complex details of an implementation behind a simple interface. These abstractions may be deeply nested, layer upon layer. Example: a procedure to draw a rectangle calls a procedure to draw a line; a procedure to draw a line calls a procedure to paint a pixel; the procedure to paint a pixel calls a procedure to calculate a memory address from an (x, y) pixel coordinate. As well as using procedures and libraries built by others, students should become proficient in creating new abstractions of their own. A typical process is:

- Recognise that one is writing more or less the same code repeatedly. Example: draw a rectangle of sizes 2 and 4; draw a rectangle of sizes 3 and 5.
- Designing a procedure that generalises these instances. Example: draw a rectangle of sizes N and M.
- Replace the instances with calls to the procedure. At a higher level, recognising a standard “design pattern”, and re-using existing solutions, is a key process. For example: simple data structures, such as variables, records, arrays, lists, trees, hash tables.
- Higher level design patterns: divide and conquer, sorting, searching, backtracking, recursion.

Students also must have abilities for debugging, testing, and reasoning about programs. When a programmed system goes wrong, they have to answer the question “How can I fix it?” Computers can appear so opaque that fault-finding degenerates into a demoralising process of trying randomly generated “solutions” until something works. Programming gives students the opportunity to develop a systematic approach to detecting, diagnosing and correcting faults, and to develop debugging skills, including:

- Reading and understanding documentation.
- Explaining how code works or might not work.
- Manually executing code, perhaps using pencil and paper.
- Isolating or localising faults by adding tracing.
- Adding comments to make code more human readable.
- Adding error checking code to check internal consistency and logic.
- Finding the code that causes an error and correcting it.
- Choosing test cases and constructing tests.

A SYSTEMATIC APPROACH IN LEARNING COMPUTER SCIENCE

At the university level of education we need to ensure the process with a systematic approach in knowledge acquiring. Data structures and algorithms are important foundation topics in Computer science education. Students deal with algorithms in many computer science courses and so they must be equipped with solid skills in algorithms [6]. We suggest a progressive building on approach for algorithm teaching, divided in four levels, presenting below. Each level upgrades the knowledge acquired on the previous one.

Level 1 introduces the algorithm concept by following explanations:

- Presenting the algorithms as they are sets of instructions for achieving goals, made up of pre-defined steps. Example: a recipe for making fried eggs.
- Algorithms can be represented in simple formats narrative text.
- They can describe everyday activities and can be followed by humans and by computers.
- Computers need more precise instructions than humans do.

- Steps can be repeated and some steps can be made up of smaller steps.

Level 2 enriches the basic concepts from the previous level 1 with forming the following skills:

- Algorithms representation symbolically (flowcharts) or using instructions in a clearly defined language.
- Building algorithms that include selection (if) and repetition (loops).
- Algorithms decompositions into component parts (procedures), each of which itself contains an algorithm.

At this level 2 students should also be able to make:

- Algorithms should be stated without ambiguity and care and precision are necessary to avoid errors.
- Algorithms are developed according to a plan and then tested. Algorithms are corrected if they fail these tests.
- It can be easier to plan, test and correct parts of an algorithm separately.

The next level 3 comprises the described below concepts:

- An algorithm is a sequence of precise steps to solve a given problem.
- A single problem may be solved by several different algorithms.
- The choice of an algorithm to solve a problem is driven by what is required of the solution (such as code complexity, speed, amount of memory used, amount of data, the data source and the outputs required).
- The need for accuracy of both algorithm and data.

The last final level 4 turns significant milestones for the following students' skills:

- The choice of an algorithm should be influenced by the data structure and data values that need to be manipulated.
- Familiarity with several key algorithms (sorting and searching).
- The design of algorithms includes the ability to easily re-author, validate, test and correct the resulting code.
- Different algorithms may have different performance characteristics for the same task.

CONCLUSIONS AND FUTURE WORK

Future plans are to investigate the suggested four level approach in teaching algorithms for students that learn computer science. Currently, we use the pedagogy based on operational theories approach of learning development [7]. It will be in favor of education quality the two approaces to be compared. Of particular interest are the solving problems principles of suggested approach teaching pedagogy. However, this idea is still in the initial stages of development and much more work is needed. Firstly, further research is needed to refine the ideas regarding this teaching approach and how it can best be used to aid in the delivery of the introductory concepts of programming.

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ДИДАКТИЧЕСКИ ЦЕЛИ И ПЕРСПЕКТИВИ В ОБУЧЕНИЕТО ПО КОМПЮТЪРНИ НАУКИ

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Резюме: *Статията разглежда обхвата на обучението по компютърни науки, дидактическите цели, на базата на които да се изграждат въвеждащите курсове по алгоритми, които са в основата на тези по програмиране. Обърнато е внимание на многоаспектността на материала и причините, поради които трябва прецизно да се структурира поднасянето на материала. Статията предлага подход от 4 нива за усвояване на знания и изграждане на умения за съставяне на алгоритми. Подчертава важността студентите да усвоят умения за съставяне на коректни и ефективни алгоритми, за да постигнат успех в програмирането.*

Ключови думи: *Компютърни науки, Алгоритми, Обучението по алгоритми, Умения за съставяне на алгоритми*

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SOFTWARE SYSTEM FOR PROCESSING MEDICAL DIAGNOSTIC IMAGES

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***Abstract**: This paper proposes an approach to the design and implementation of a software system for processing medical diagnostic images. The system allows acquiring images from x-ray devices, ultrasonic echographs and endoscopic diagnostic devices. It maintains a database of patients' data and medical examination records, containing stored images and the doctor's conclusion. Utilizing this software allows improving the doctors' work in their practice outside the hospital when making diagnosis and tracking the treatment. The system can also be used for remote diagnostics (telemedicine).*

***Keywords**: Medical imaging, Diagnostic devices, Telemedicine, General practitioner, Medical software system.*

INTRODUCTION

The general practitioners' business activity has limited financial resources and the devices used in the medical offices diagnostic do not have the tools for storing the information in a database, assist the diagnostic process and use the results for remote diagnostics. The current paper describes a possible approach for the use of personal computers in the diagnostic activities in the general practitioners' offices. An Overview is given of a software information system for collecting, storing and organizing images from medical devices.

PROPOSED SOLUTION

With the progress of technology, the number of diagnostic devices, which create digital images [1] (radiography, ultrasonography etc.) in the medical science, is increasing. A lot of devices (endoscopes, colposcopes, microscopes etc.) can function with video cameras and the images created during the examinations can then be used for diagnostic purposes. To aid the diagnostic activities of the general practitioners and specialists is developed a software system for personal computer, which allows to:

- Get a live stream from medical devices, which support getting images.
- Give the possibility for recording a single frame or short videos of the conducted examination.
- Support a non complicated database for the patient data.
- Store a chosen set of images and the medical doctor’s conclusion in the database.
- Use the stored data for queries and monitoring the change in the patient’s condition.

The personal computer, used by the system, uses a video capture device, because most of the image diagnostic devices out of hospitals work with analogue video signals. The library DirectShow is used for processing the video signals. The software is developed in Borland Delphi [2] and because of that the free library DSPack is used, which is a set of components and classes for creating multimedia applications by MS Direct Show and DirectX technologies [4,5].

Fig. 1 shows the structure of DSPack as well as the class diagram of the most important class for the application implementation DSUtil.

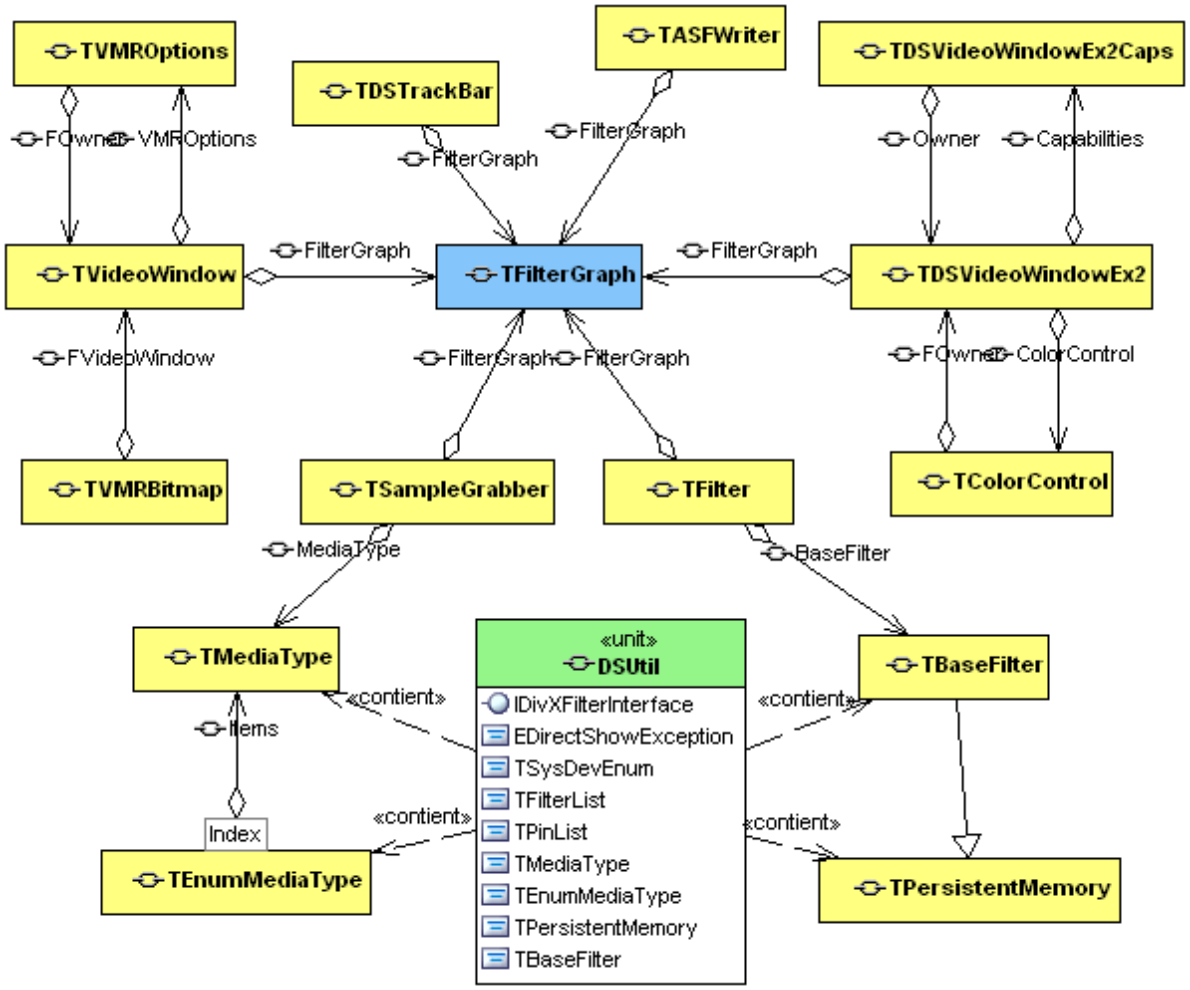

Fig. 1. Structure of DSPack

The software was implemented with an intuitive and user friendly graphical interface. After starting, the system is required to show not only the video stream but also the possibility for storing images and video clips. In fig.2 is the filter-graph, which is used to implement this functionality [4].

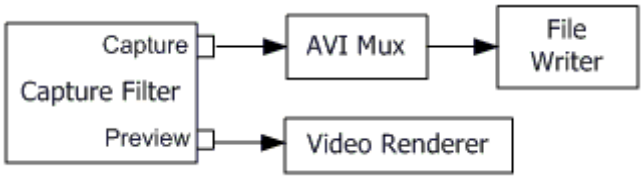

Fig. 2. DirectShow filter-graph used for video stream processing

After choosing the device and the video capture mode, automatically starts a preview of the incoming input stream (shown on fig. 3). After the preview has started and the examination begins the medical doctor has the resources to store a number of images from the diagnostic device. Regardless of the chosen number of pixels of the input image, it is scaled automatically to fit the appropriate window in the main view. The storing of snapshots can be done by pressing a button, keyboard shortcut or with the help of a specially designed for the purpose pedal, which is pressed with a foot by the medical doctor.

Fig. 3. System main windows after starting the video stream

When a single image extraction function is activated, the software system extracts the needed information from the input stream. To achieve this there are two possible approaches: to activate VideoCaptureGraph in the input stream or to extract it from the image preview. In the presented implementation, the second approach is chosen.

The images are stored in a temporary folder, accompanied with a timestamp, which can be used as unique identifier for the moment of extraction. The timestamp can be further used for storing the image into the patient's medical record together with the medical doctor's conclusion (if one was made). This information is also used when the snapshots are exported in the DICOM format, which is done with the purpose of using the functionality "telemedicine", which allows access on a more qualified diagnostic department.

The collection of snapshots during the patient's examination is being saved and at the end the medical doctor can choose to inspect again the records and filter the ones with diagnostic value, which can be further saved and can help for keeping track of the patient's condition (fig. 4). The same figure shows that the system can automatically take snapshots at specific intervals, which can free the medical doctor's attention and let him concentrate more on the examination.

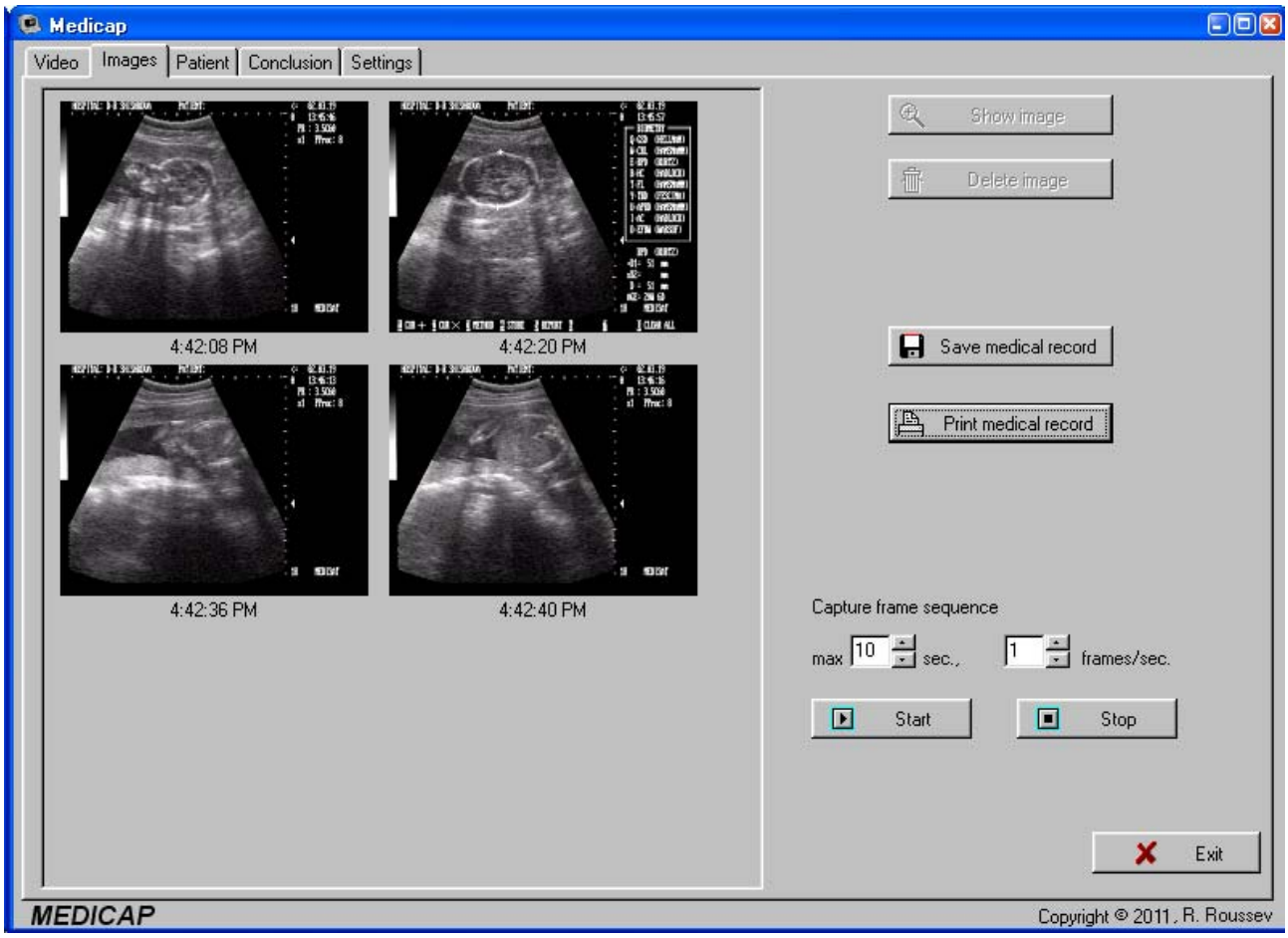

Fig. 4. Visualization of saved images

To achieve the collection of diagnostic information with the option of monitoring the patient's status the system supports a simple database. It stores the most important information about the patient as well as an unique identifier, which can be used for connection with a centralized database. (Fig. 5).

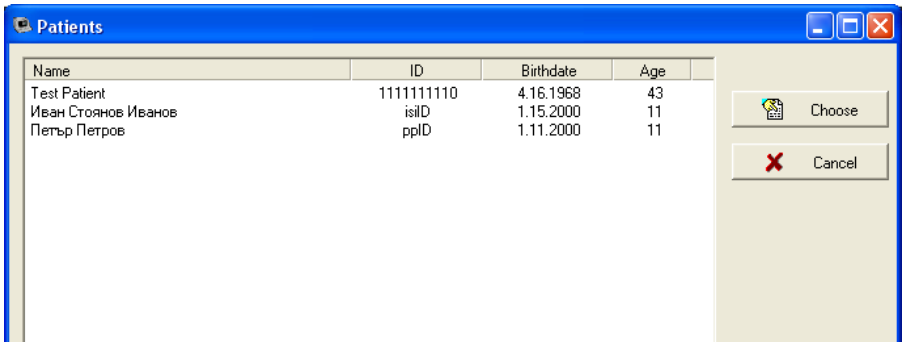

Fig. 5. Patients list in local database

If we want to save the medical record from the current examination or to browse the history of patient's visits, we can choose from the suggested list. The information for the past examination, diagnostic results and the written conclusions are visualized in a tree structure, which is extremely intuitive to work with (Fig. 6).

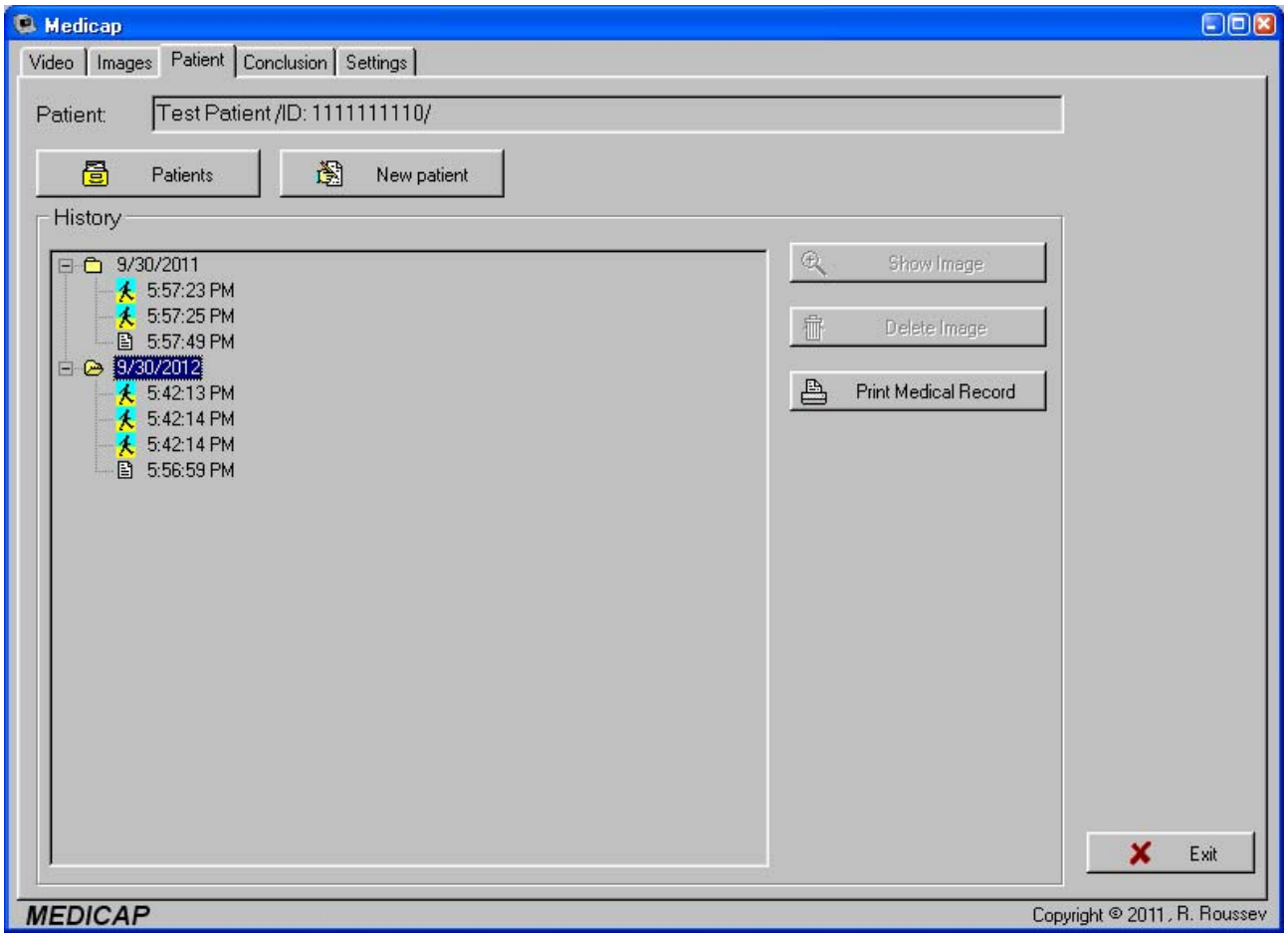

Fig. 6. Tree representation of collected patient's medical data

For simplifying the diagnostics process the medical doctor is offered a graphical editor, which has the option to load templates, on basis of which the diagnostic conclusion can be written. The usage of such templates drastically reduces the amount of paperwork the M.D. has to process, which prevents him from missing important elements in the description of the diagnostics. During any point of the process the medical doctor can choose to save a working copy of the collected data and print medical record of the exam, including any saved images if any and the examination conclusion. To ease the use, those functionalities are accessible during preview of the saved graphical mages, the preview of patient data in the database and during editing the conclusion.

The images are saved either in JPEG format with minimal data loss or in BMP without compression and therefore no data loss. The choice which of the two approaches to be used is a system parameter which is saved and can be edited in a configuration file.

Because some of the cases do not require systematical monitoring of the patient's condition and therefore no medical record is needed, the system can be used as a viewer for the images collected during the examination and/or printing a snapshot of them.

CONCLUSION

The developed software allows storing and managing images from medical devices for diagnostics through a multifunctional personal computer. The supplementary database is used to simplify the medical doctors' paperwork by saving texts, data, and the examination conclusions. The images and the conclusions are saved in a database for patient's medical history. It is stored as medical records and can be used for queries, monitoring the patient's state and for the need of remote diagnostics. The developed

software is used in practice from general practitioner medical doctors and other specialists. Its usage shows good functionality and is simplifying the M.D.'s practice.

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ПРОГРАМНА СИСТЕМА ЗА РАБОТА С ИЗОБРАЖЕНИЯ ОТ МЕДИЦИНСКА АПАРАТУРА ЗА ОБРАЗНА ДИАГНОСТИКА

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Резюме: *Статията разглежда подход при проектиране и реализация на програмна система за работа с изображения, получени от медицинска апаратура за образна диагностика. Системата дава възможност да се получат изображения от ренгенови апарати, ултразвукови ехографи, и ендоскопски уреди за диагностика. Програмната реализация поддържа база от данни за пациентите и медицинските записи от проведените прегледи, съдържащи съхранените изображения и заключението на лекуващия лекар. Използването на софтуера дава възможност да се подобри работата на лекарите в извън болничната практика по поставяне на диагноза, проследяване на лечението, както и да се използва за отдалечена диагностика (телемедицина).*

Ключови думи*: Медицинска образна диагностика, Диагностична апаратура, Общопрактикуващи лекари, Медицински софтуер.*

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AUTOMATIC PROGRAM GENERATION WITHOUT INTERNAL MACHINE REPRESENTATION

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Abstract: *This paper describes some of the problems with automatic program generation, using or not internal machine representation.*

Keywords: *program generation, internal machine representation.*

INTRODUCTION

Internal machine representation [16] has many advantages: high-speed, the developer is confident in his system (it is not open source code, so that is not accessible to manipulate by all), generation is allowed in different programming languages and so on. But there is a significant drawback - presumably if has to be a functionally complete system. That means to be expensive, with a long period of development and testing, because it has finished all necessary modules have to be in a complete form: those for import and export, dialog editors of different nature for task descriptions [12] (text, image, natural-languages, etc.), carefully designed internal machine representation, generators to relevant program languages and so on. It is imperative a service team for system development and maintaining to be kept available.

DETAIL DESCRIPTION

Let consider such a functionally complete system, like as **KATIA 3D** or **IBM Web Sphere Lombardi** [7] (Fig. 1):

An internal machine representation (different structures) of the considered object is created. Usually one already knows the subject areas in which we operate. The end users (engineers, designers and other specialists) put their task using different dialog resources to describe the job: it can be different types of graphical interfaces, dialog interfaces, menu systems and others (depending on the specific needs for each type of interface can be a generator creating it).

After the job has been described, the system converts it to the internal machine representation, appropriate to the corresponding subject area [9]. Usually different internal machine representations are maintained, for different subject areas. If necessary, the support team can create a new internal machine representation or update the existing one. It can also be requested a supplement / specification of the job.

Next the necessary application should be created. This can be a running application, documentation, drawings, description (some of the created applications can be used to create others). To create them various generators are used. Again, if necessary, the support team can create a new generator or update an existing one.

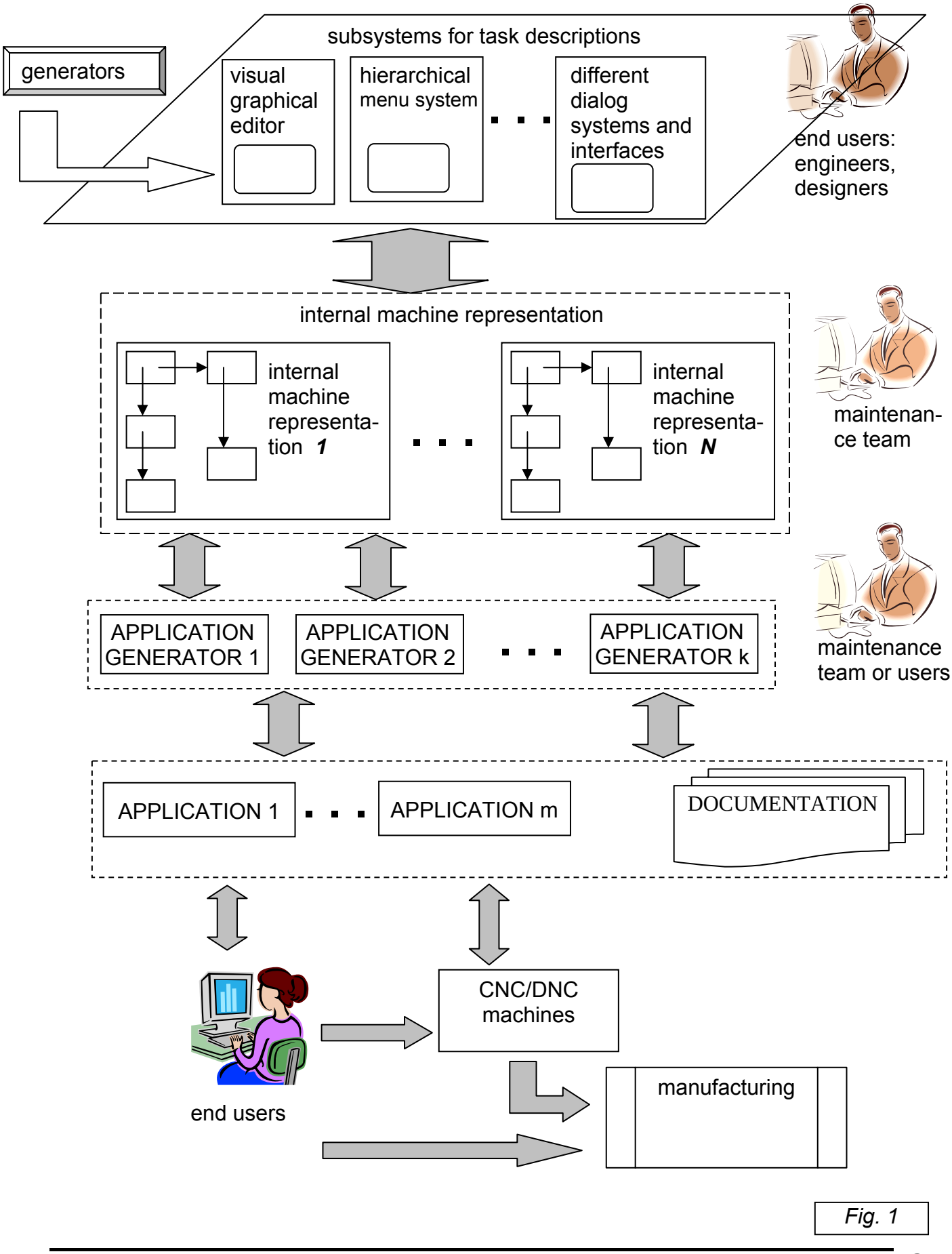
subsystems for task descriptions
generators
visual graphical editor
hierarchical menu system
different dialog systems and interfaces
end users: engineers, designers
internal machine representation
internal machine representa-tion 1
internal machine representa-tion N
maintenan-ce team
APPLICATION GENERATOR 1
APPLICATION GENERATOR 2
APPLICATION GENERATOR k
maintenance team or users
APPLICATION 1
APPLICATION m
DOCUMENTATION
CNC/DNC machines
manufacturing
end users

Fig. 1

It is essential that it is necessary a great support and development team of such a system. Minor modifications of the subject area, adding new ones, creating a new generator - in all cases of manipulation of the system that can not be done by hired for the purpose team without his prior education and training. This is related to time and money again.

It is very important that different generators (for different applications) use the same internal machine representation [10]. Very often one subsystem describes the task, it is fed to the next - to create a project, then it is submitted to the next - to create software and so on. For normal operation of this chain of reciprocity-associated modules it is needed a general (universal) internal machine representation. If an external developer wants to create a new generator and new application or modify an existing one - this is impossible due to the unknown of the internal machine representation.

IS IT POSSIBLE TO SUGGEST SOME OTHER APPROACH?

For a variety of subject areas there are small (relatively simple) systems, making small, well-defined things to automate human activity. Even more - some of them without claim for functional completeness are open source or free. For example: I want to create an information system for library service. It is a need a project to be created with a defined functionality, to describe subject area, to define the database, to create / generate software etc. A system such as ***Enterprise Architect*** [3] or ***IBM Web Sphere Lombardi*** [6], [8] will make a lot of this (or anything) but the price is not low and the maintenance of trained staff will cost as well.

An idea arises: to use multiple unrelated systems [1] (if possible open source or free), each of which makes some of the needful activities. One such a system is the project graphic description (for example: MS Visio for UML-diagrams or [5] - BPM), another - to define the database, a third - to describe data structures (classes [2]) in the program and so on. How different systems to exchange information with each other (i.e. - the result of the work of the first to be used as input for the second)? Different options may be offered, one of which is a common, universal format for information describing a variety of text files, such as XML (eXtensible Markup Language) [19]. Many modern systems support functions for Export and Import of text files and/or variants thereof, including XML-files (Fig. 2).

Thus, developing a problem, business processes can be described in UML diagrams [14], [15] using MS Visio, which comes in handy for drawing but it does not make any logical control over the generated algorithm.

The created (drawing) diagram can be exported as a file in XML-format [4], [17], [18], which is applied as input to the next system (commercial or free) for further processing. And so on, using a lot of internally independent products communicating between by XML-files, we can create a chain of tools that lead us to the final result or bring us closer to it.

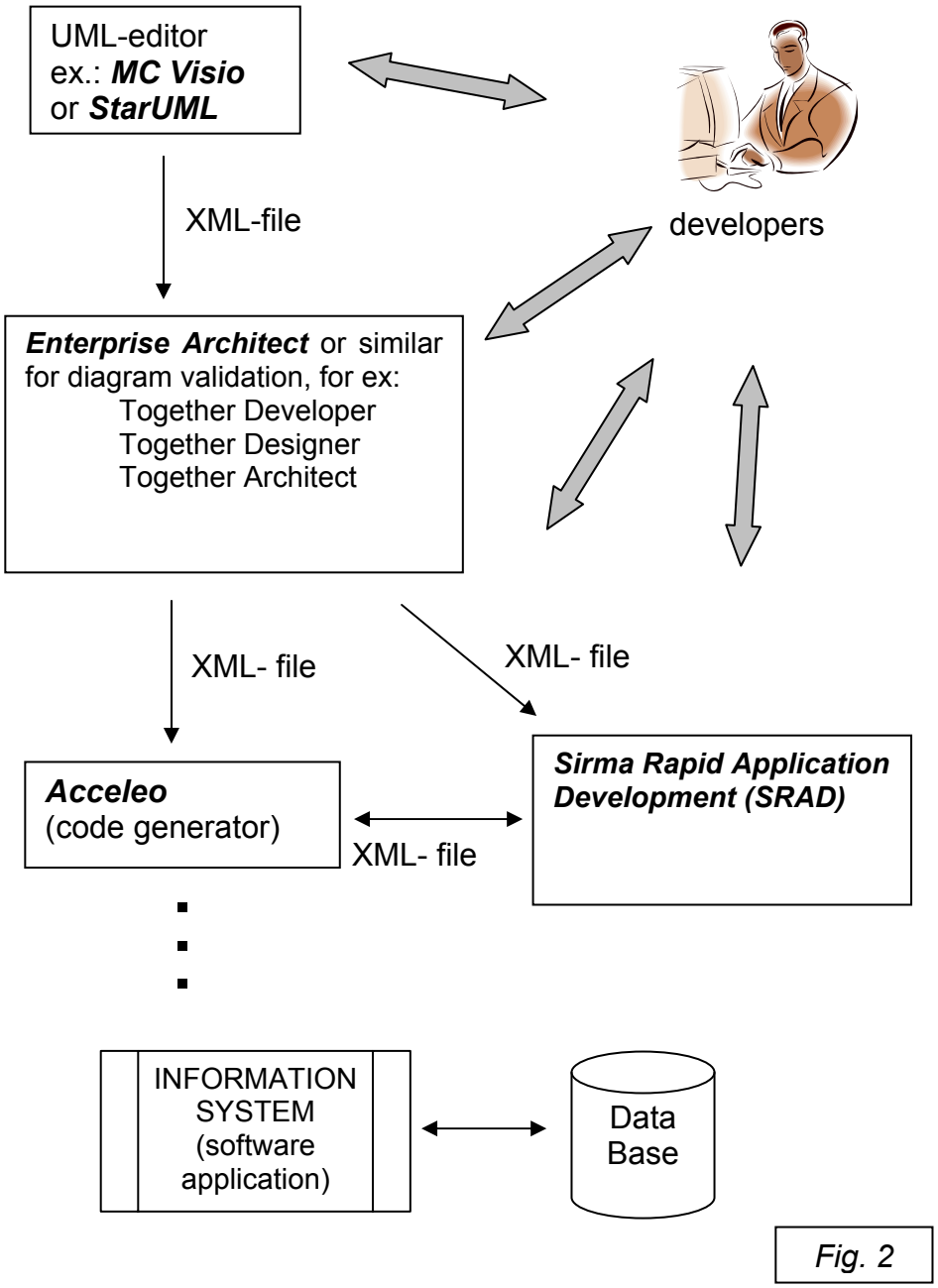

Fig. 2

CONCLUSION

With a good knowledge of the subject area, there are options to select suitable software tools (free or open source [11], [13]) which can automate much of the work in software system creating. Viewed individually, each product works for itself automating "piece" of monotonous work in application development. Passing the results from one system to another (via the corresponding XML-format [4]) we could get the desired end result without using common internal machine representation. Thus, adjusting a chain of open source tools, small companies with insufficient resources can automate some of the activities in the software project implementation.

Moreover: even though we have a commercial functionally complete system of internal machine representation, it is possible to need doing something extra, beyond of the system scope. So in this case we can look for another (external) application and communication between them done in XML-format (Fig. 3).

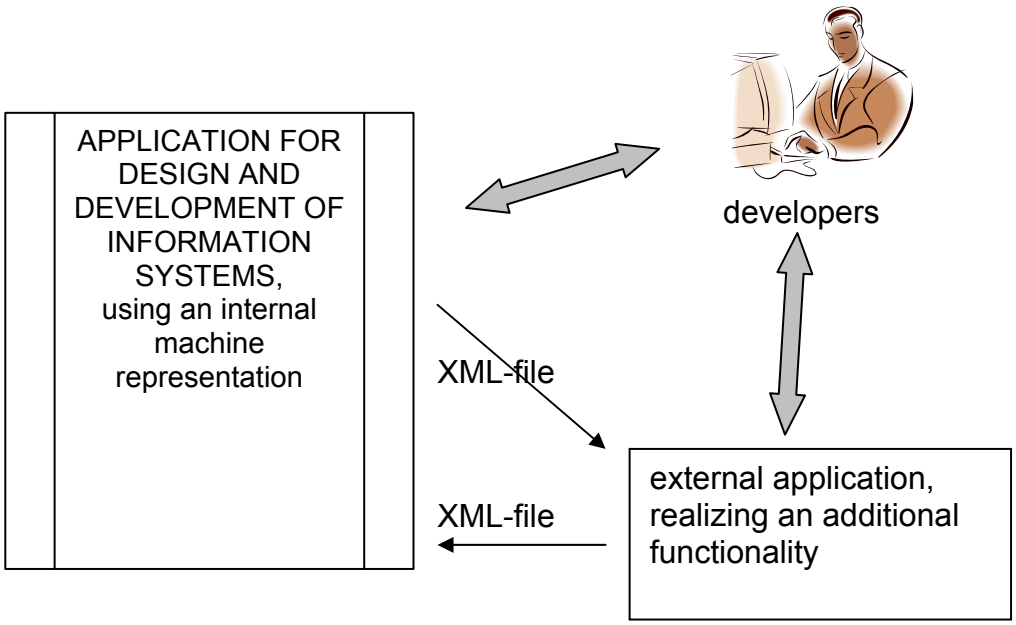

Fig. 3

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АВТОМАТИЗИРАНО ГЕНЕРИРАНЕ НА ПРОГРАМИ БЕЗ ВЪТРЕШНО-МАШИННО ПРЕДСТАВЯНЕ

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Резюме: *Статията дискутира някои от проблемите при автоматизираното генериране на програми при използването на вътрешномашинно представяне и без него.*

Ключови думи*: генерация на програми, вътрешно-машинно представяне.*

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SYSTEMS FOR AUTOMATED SOFTWARE DEVELOPMENT

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Abstract: *This paper presents some of the systems (the most famous ones) for automated software creation/generation and the stages of software development, as well as its documentation.*

Keywords: *Computer Science, program generation, software generation.*

INTRODUCTION

In many places people work in the automatic software generation area. That is a problem, connected not only with human resources and saving time for development. Program generation allows creating syntactically clear and logically correct units – that is connected with time and price for software development, too. There is another reason as well – sometimes it's difficult to find the suitable specialist to create the necessary correction in a software unit.

Another important point is the product or project documentation creation - it takes too much time and resources. This documentation has to respond to different requirements depending on the country or company it was created by. One of the prerequisites for their unification, in addition to the national and international standards, is the IBM's acquisition of Rational Software Corp. (2003) with its iterative software development process framework (RUP).

This article presents some of the leading systems in this area.

DETAILED DESCRIPTION

1. IBM WebSphere Lombardi [3]

This product is one of the leaders in the Automated generation and/or editing the existing three-tier client-server applications for corporate users. It is working in entirely pleasant and comfortable graphical interface. There are subsystems to create and edit SQL-database, to generate custom client interfaces (which will run through the browser on the client), to describe processes through UML, to generate business logic **[6]** (servlets and scriplets, using UML functional description) and others.

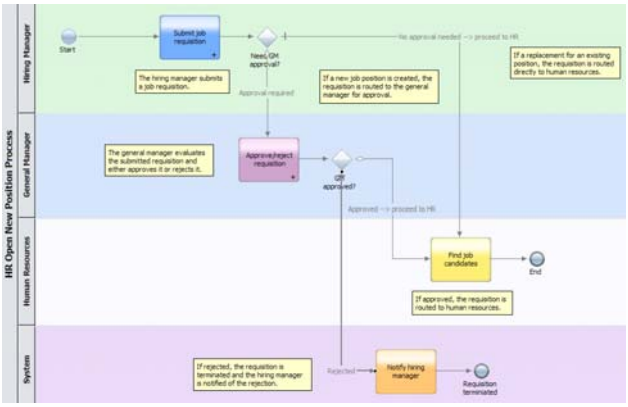

Fig.1 – IBM WebSphere Lombardi - BPML subsystem (editor)

Two types of languages are used mainly for business processes management **[2]** and their automation: BPML (Business Process Modelling Language) and BPEL (Business Process Execution Language). The second group is presented by XML, Java, Java Script, etc. In this group, using some kind of process description (according the certain rules – as data-flow diagram for example) working WEB-pages for database services are generated.

IBM WebSphere Lombardi comprises a number of components (Table 1):

Table 1. IBM WebSphere Lombardi components

Component	Function/Description
Process Center	Provides storage and centralized (single) development environment for many processes creating in ***Process Center Console*** and other interfaces in ***Lombardi Authoring Environment***. Includes a ***Process Center Server*** and ***Performance Data Warehouse*** to enable the processes and applications creation and execution, so and store statistics and performance testing during development.
Process Server	Implements in ***Lombardi Authoring Environment*** the processes and the services in-store Process Center in a position to install them in execution environment.
Performance Data Warehouse	Collects and integrates process data in accordance with the requirements established in в ***Lombardi Authoring Environment***.
Process Center Console	Allows ***Lombardi repository*** management, including process applications management, workspaces, and instant copies. Also allows applications process installation on ***Process Server*** in the execution environment.
Authoring Environment	***Lombardi Authoring Environment*** consists of several interfaces, that allow process creators to model, implement, simulate and verify business processes..
Process Portal	Provides an interface, allowing the actors in process to perform assigned tasks, to view task history, to view statistics of their processes. Using the ***Process Portal***, the participants' process can be connected to the ***Process Center Server*** or a ***Process Server*** in any configured runtime environment, such as test environment and execution environment for example.
Process Admin Console	Provides an interface that allows administrators to set ***Lombardi Process Server*** for arbitrary execution environment, as an execution or testing environment, for example. It also allows administrators to set the ***Process Center Server*** and to manage it
Performance Admin Console	Provides an interface that allows administrators to set ***Lombardi Performance Data Warehouse*** for any run-time environment combination, such as testing or normal operation. Also enables administrators to configure and manage the ***Performance Data Warehouse***, included in the ***Process Center***

The Lombardi Architecture [4]

The diagram in Fig. 2 illustrates a typical ***IBM WebSphere Lombardi Edition*** configuration**[4].**

From ***Lombardi Authoring Environment*** many users can connect to **The *Process Center***

• In ***Lombardi Authoring Environment*** users create process models and supporting applications (process applications) and store those applications and associated items in the ***Process Center*** repository.

The ***Authoring Environment*** users, incorporated into the ***Process Center***, can enjoy together the individual elements.

• ***Process Center*** includes a ***Process Center Server*** and ***Performance DataWarehouse***, allowing users working in ***Lombardi Authoring Environment*** to carry out their application processes and store statistics for testing and reproduction during development.

• From the ***Process Center Console*** administrators install application processes that are ready for establishment, testing and operation in ***the Process Servers*** in the appropriate environment.

• From the ***Process Center Console*** the administrators manage the performance of instances of process applications in all configured environments.

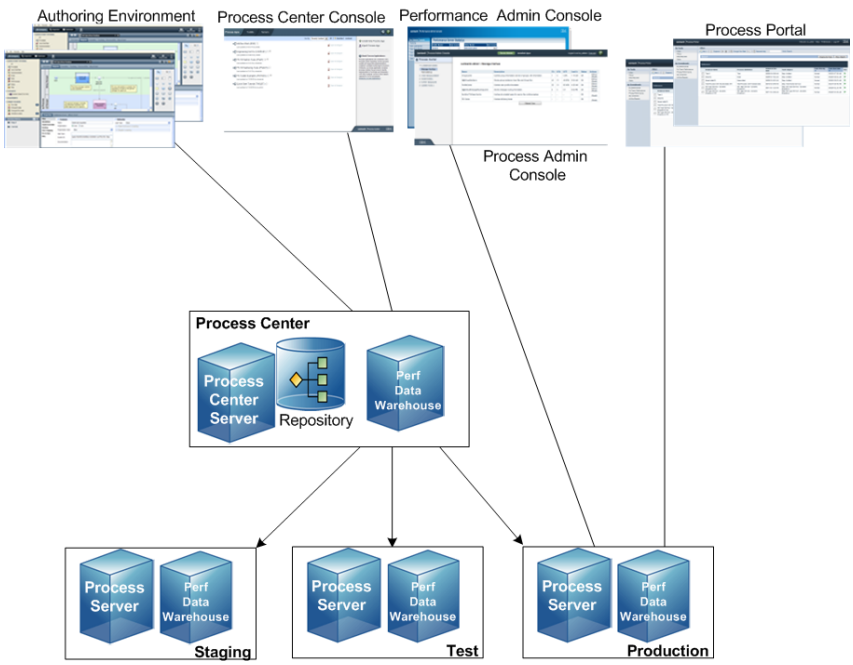

Fig. 2 - Calendar view of events

• From the ***Process Portal*** the end users perform respective tasks. ***Process Center Server*** and ***Process Servers*** in configured execution environments can run application processes that are necessary for the according tasks.

• Using the ***Process Portal*** participants can connect to ***Process Center Server*** or ***Process Server*** in any configured run-time environment depending on whether the corresponding process is in elaboration, testing or is already established in the execution environment.

• ***Lombardi Performance Data Warehouse*** retrieves regularly tracking data from the ***Process Server*** or the ***Process Center Server*** at regular intervals. Users can create and view reports, based on this data, in ***Lombardi Authoring Environment*** and ***Process Portal***.

• From ***Process Admin Console*** and ***Performance Admin Console*** the administrators can manage all ***Lombardi*** servers.

2. Enterprise Architect

Sparx Systems Enterprise Architect [1] is a visual modelling and design tool (Fig. 3) based on the OMG UML. The platform supports: the design and construction of software systems; modelling business processes and modelling industry based domains. It is used by businesses and organizations to not only model the architecture of their systems, but also to process the implementation of these models across the full application development life-cycle.

Overview

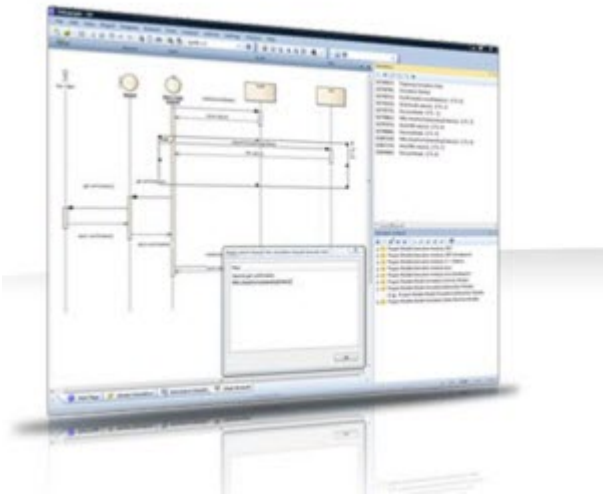

Fig. 3. Sparx Systems Enterprise Architect

Systems modelling using UML provides a basis for modelling all aspects of organizational architecture, along with the ability to provide a foundation for designing and implementing new systems or changing existing systems. The aspects that can be covered by this type of modelling range from laying out organizational or systems architectures, business process re-engineering, business analysis, and service oriented architectures and web modelling, through to application and database design and re-engineering, and development of embedded systems.

Along with system modelling, Enterprise Architect covers the core aspects of the application development life-cycle, from requirements management through to design, construction, testing and maintenance phases, with support for traceability, project management and change control of these processes, as well as, facilities to model driven development of application code using an internal integrated-development platform.

The user base ranges from programmers and business analysts through to enterprise architects, in organizations ranging from small developer companies, multi-national corporations and government organizations through to international industry standards bodies.

Sparx Systems initially released Enterprise Architect in 2000. Originally designed as a UML modelling tool for modelling UML 1.1, the product has evolved to include other OMG UML specifications 1.3, 2.0, 2.1, 2.3 and 2.4.1.

Supported Standards

Enterprise Architect supports a range of open industry standards for designing and modelling software and business systems, as: UML 2.4.1, SysML, BPMN, BPEL, SoaML, SPEM, WSDL, XSD, DDS, ArchiMate, Geography Markup Language (GML), ODM, OWL and RDF.

Enterprise Architect supports industry Frameworks as: Zachman Framework, TOGAF (including FEAF), UPDM framework that supports DoDAF, MODAF and NAF, SOMF.

Enterprise Architect supported Frameworks supplied by industry bodies are TRAK, GeoSciML.

Modelling

Fig. 4 - Simulation in Enterprise Architect

Underlying UML modelling include several key aspects that most modelling tools support. The core aspects supported by Enterprise Architect are: Profiles, Patterns, MOF, OCL, MDA Transforms, Corba IDL. UML Validation can be run against the model.

General features

Requirements management

The common features of Requirements Management supported by Enterprise Architect include customization of how requirements are documented, linking requirements to the design and implementation details, and providing Requirement Traceability through the design and construction phases. These requirements can be subject to change management, workflow processing, baseline comparison and auditing. There is also a model glossary that is interactive with notes for requirements.

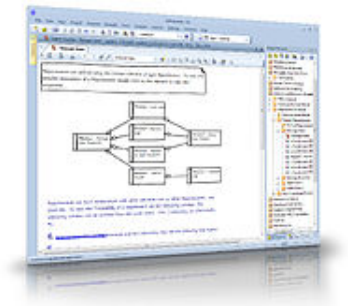

Fig. 5 - Requirements Management in Whiteboard

Business modelling and analysis

Enterprise Architect supports a number of methods of modelling business processes using UML as the foundation modelling language. The core languages for business modelling and analysis include BPMN and BPEL, with various historic profiles such as the Eriksson-Penker profile. Enterprise Architect also supports the definition of Business Rules with the ability to generate executable code from these rules. Business modelling can be combined with GAP analysis to view potential gaps in proposed solutions.

Simulation

Model simulation of Behavioral diagrams is supported for:

- State Machines
- Interaction (Sequence diagrams)
- Activities
- BPMN

Execution flow is defined using Triggers, Guards and Effects. The simulation supports re-runs with alteration to the triggered events and supports viewing variables, the call stack and setting debug markers. There is also support for interaction with emulated User-Interface screens containing common UI fields.

System development

In line with the ***Model Driven*** design principles Enterprise Architect supports ***MDA*** transforms of ***PIM*** Class structures to ***PSM*** Class structures, ***Round-trip engineering*** of code for 10 software languages and several key embedded ***HDL*** systems languages (***Ada***, ***VHDL*** and ***Verilog***). It also supports code generation from Behavioral models.

Supported languages:

ActionScript, ***C***, ***C#*** (for both .NET 1.1 and .NET 2.0), ***C++***(standard, plus .NET managed C++ extensions), ***Delphi***, ***Java*** (including Java 1.5, Aspects and Generics), ***PHP***, ***Python***, ***Visual Basic***, ***Visual Basic .NET***

In accordance with Model Driven Development principles, Enterprise Architect provides an Integrated Development Environment that supports code editing (with Syntax highlighting and **Intellisense**), for Building, Debugging and Code Testing all from within the model.

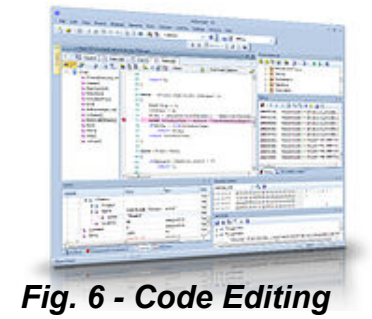

Fig. 6 - Code Editing and Debugging

Supported compilers and interpreters:

- Microsoft Windows Native C
- Microsoft Windows Native C++

- Microsoft Windows Visual Basic
- Microsoft .NET Family (C#, J#, VB)
- Sun Microsystems Java.
- PHP
- GNU Compilers for C++, C and Ada (GCC & GDB)

There are also Add-ins available for integration ***with MS Visual Studio*** and ***Eclipse***.

Test management

For code based testing there is support for:

- xUnit Testing

This involves MDA transformation of Classes to NUnit or Junit Classes with the ability to generate unit tests from the model and automatically record the results against the tested Classes.

- Testpoint testing

This is a model based code testing. It is parallel to test contracts defined in 'Design by Contract' and it runs using debug definitions.

In terms of model based testing; both of these methods support the test definitions and test results being logged against related Classes in the model.

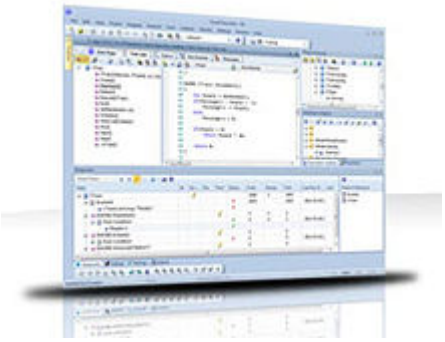

Fig. 7 - Code Editor and Testpoints Manager

Visual execution analysis

Integrated with building and debugging code Enterprise Architect allows the developer to perform abstract analysis of the software using Profiling and Sequence diagram generation:

- Sequence diagram generation provides a means to analyze the general process flow and iron out inconsistencies
- Profiling summarizes, by thread and routine, the code's general efficiency

System engineering

System Engineering is supported with ***SysML 1.2*** modelling which can be coupled with executable code generation. ***SysML*** supports modelling from requirement definition and system composition using ***SysML Blocks*** and ***Parts***, through to parametric model simulation. The executable code generation supports embedded ***HDL*** system languages (Ada, VHDL and Verilog), or it can be coupled with Behavioral code generation of the standard code languages defined above.

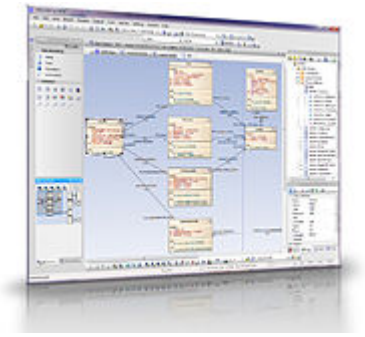

Fig. 8 - Database Modeling using a DDL

Data modelling

Enterprise Architect supports Data Modelling from the Conceptual to Physical levels, Forward and Reverse Engineering of Database Schemas, and MDA transformation of the Logical (platform independent) to Physical DBMS (platform dependant). Diagram types supported include:

- DDL notation
- ERD notation
- IDEF1X notation
- Information Engineering notation

Supported DBMSs:

DB2, Firebird/InterBase, Informix, Ingres, MS Access 97, 2000, 2003, Access 2007, MS SQL Server 2000/ 2005/ 2008, MySQL, SQLite, Oracle 9i, 10g and 11g, PostgreSQL,

Sybase Adaptive Server Anywhere (Sybase ASA), Sybase Adaptive Server Enterprise (Sybase ASE), ArcGIS

Project management

Features supporting project management include:

- Resource Allocation and Tracking using Gantt charts
- Event Logging using model calendars
- Workflow scripting for setting workflow processes
- Security
- Model Metrics
- API Scripting (macros).

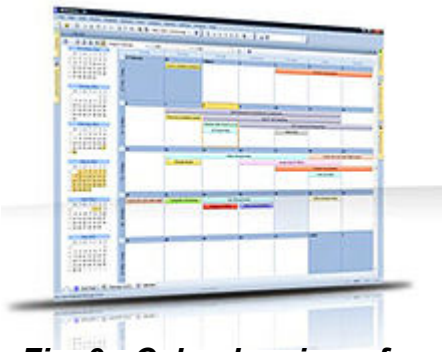

Fig. 9 - Calendar view of events

Change management

The key facilities supporting change management are:

- Auditing
- Baseline Difference and Merge
- Version Control

The auditing feature supports logging changes to the model. The Baseline Management feature allows snapshots of parts of a model to be created periodically. A baseline can be compared and merged with the current model or a branch of that model. This supports Branching model information to another repository, then adding updates and merging them back.

The Version Control interface supports the major version control applications:

- Subversion
- CVS
- Team Foundation Server
- SCC interface to any SCC compatible version control system.

Team-based development and collaboration

The Team Interaction facilities include:

- Model Mail: Internal model based mailing system.
- Team Review: Inter-repository forum for lodging discussion on issues.
- Model Views: User definable Views, prompting users on relevant model updates

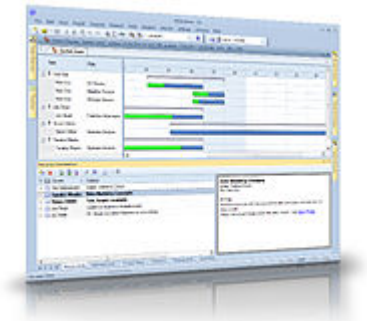

Fig. 10 - Gantt Chart of Project Tasks and Model Mail views

Client Customer Collaboration:

- User definable Word Compatible RTF reporting
- HTML reporting
- EAlite – provides free read-only viewing of models with support for client/customer interaction via the Team Review

Service Oriented Architectures

Supports of the core Service Oriented Architectures: ***SoaML*** and ***SOMF,***
along with Round Trip engineering of WSDL, XSD used to facilitate BPEL generation.

Integration with other tools

The key features that support integration with other tools include:

- XMI Import/Export: Supports the XMI 1.1, 1.2 and 2.1 specifications (and import of .emx files).
- CSV import/Export
- Automation interface – supports a comprehensive API interface for use with any COM based language (and Java).

• MDG Add-ins are available, supporting interfaces to: VS .Net, Eclipse, TcSE, HP Quality Center, Import Visio diagrams, Import DOORS requirements, Atlassion JIRA integration, SAP netweaver Integration

Reporting

Features for creating model documentation include:

• User-definable (MS Word) compatible RTF Reporting
• User-definable HTML generation of the model
• Model slide show presentations
• User definable query based reporting

Deployment

There are many scenarios for deployment. For the multi-user and multi-site development there is support for WAN based connectivity using a WAN Optimizer. Information can be exchanged and merged between repositories for off-site analysis and development or for exchanging models between diverse development groups. The core logistics are:

• Repository Types:

Supports .eap files as well as DBMS repositories (the 10 more commonly used DBMS).

• Tool Deployment:

Supports a simple workstation installation and, if required, a simple DBMS installation.

• Scalability:

The tool is very scalable with options for large scale cross-corporate or WAN based interconnections.

Supported platforms

• Windows XP, Windows Vista, Windows 7 (32 bit and 64 bit)
• Linux & Mac OS installations are supported using CrossOver

3. Sirma Rapid Application Development (SRAD)

The developers' idea is creating a system which facilitates the three-tier client-server applications creation.

It is developed on a modular basis, providing synchronization between business analysts, programmers and customers.

One of the modules provides functionality resembling work with graphs, which create models and describe business data and the relationships between them. Another group of modules implement forms creation for user interface. Third group are modules for creating business logic validation.

Friendly graphical editor allows the user interface formation in the very process of system designing in a dialogue with the potential customer.

Generators, created to validate business rules, provide code for full or partial validation of business data, with various relationships between them, such as:

- Monitor the completion of the required fields;
- imposing rules to restrict some fields;

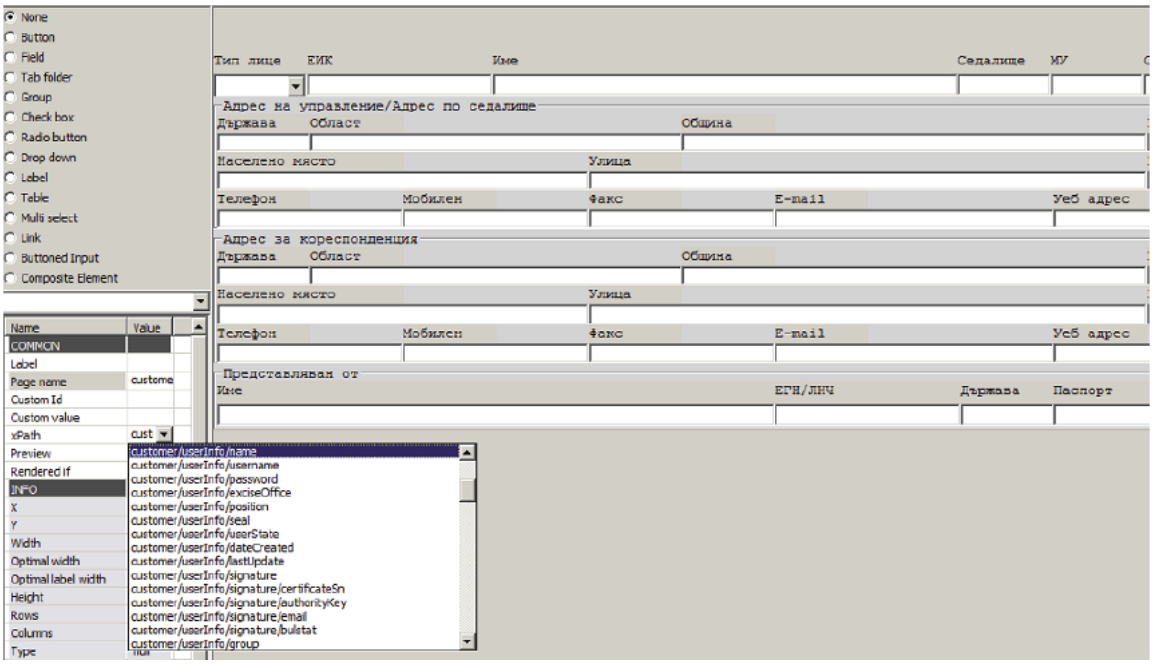

Fig. 11 Graphical editor in SRAD

- rules for initial initialization;
- rules for computing;
- rules for inapplicability.

An interesting possibility is the partial validation - if an event on the client side appears (editing or completing a field from the User Interface) business validation is run on that portion of the processed object only, for which the change is important.

The system development direction is to add more options to describe business rules and automated software generation for work in the interlayer of the client-server application.

4. Rational Rhapsody

Rational Rhapsody **[5]** is an IBM company product (since 2003). It offers opportunities for collaborative design for systems engineers and software developers when creating the real-time systems.

The first version of the product was launched in 1996 by the I-Logix company and was originally developed as a tool for object-oriented modelling. Later, it was purchased from the IBM, which lead to the product improvement, adding the support of a number of established values and separating the product to several editions according to user requirements. There are four editions:

- ***Rational Rhapsody Developer*** provides an environment that allows easy evaluation of the behaviour of different systems to detect defects at an early stage. It supports the possibility of visual debugging. Other features are:
  - o Generate code for C, C + +, Java and Ada applications.
  - o Support for UML, SysML.
  - o Design of object-oriented or functional C applications.
  - o Integrity in the Eclipse environment.
- ***Rational Rhapsody Designer for System Engineers*** is aimed at system engineers. It provides an environment for collaborative design and simulation at an early stage according to specified requirements and architecture.

Its main features are:

- An environment supporting UML and SysML.
- Tracing and analysis of the entire product life cycle.
- Automation of most system engineers tasks.
- Automated documentation creation throughout the product life cycle.
- Configuration and maintenance of parallel development system.

- ***Rational Rhapsody Architect for Software*** provides a UML- and SysML-based environment for developing real-time systems.

Its main features are:

- Architecture visualization through UML.
- Generation of C, C++ or Java code.
- Reverse Engineering of C, C++ or Java code.
- Integration with Eclipse.
- Automatic integrity maintenance of the architecture, code and documentation.
- Application development using AUTOSAR.

- ***Rational Rhapsody Architect for System Engineers*** helps systems engineering teams design complex functionality in less time.

Its main features are:

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The disadvantage of the product is the relatively high complexity of individual modules, which requires a better users' knowledge of the particular area and makes most of the features unusable by end users.

CONCLUSION

1. There is a great number of various software systems, accomplish documents generation at different levels (XML and other formats), diagrams, software.
2. The majority of these systems are not open-source, i.e. - Internal machine representation of the structures is not available (Know-How). Even be found it, in question is the possible manipulation of these structures (copyright, etc.).
3. Best products are commercial. Their price is such, that even big Bulgarian developers and potential customers would think.
4. The last opens up a market niche for new developments in the field.

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СИСТЕМИ ЗА АВТОМАТИЗИРАНО РАЗРАБОТВАНЕ НА СОФТУЕР

Валентин Великов

Русенски университет "Ангел Кънчев"

Резюме:: *Статията представя някои (водещи) от системите за автоматизирано разработване/генериране на софтуер или на етапите в неговата разработка, а също и на документация към него.*

Ключови думи*: информатика, генерация на програми, генерация на софтуер.*

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INVESTIGATION OF THE INFLUENCE OF THE TYPE OF SURFACE ON THE QUALITY OF LASER MARKING

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***Abstract**: The possibility of marking tools have been examined with three types of surfaces: brushed, nickel-plated and oxidized. The change of the contrast has also been analyzed according to two technological parameters - power density and speed of marking. The intervals of power density and speed of marking for optimal contrast have been defined. Experiments were made with two types of lasers - Nd: YAG disc laser D16 and CuBr laser, in purpose to assess the influence of the wavelength on the marking process. Moreover, the physical mechanism for marking on the three surfaces is discussed further.*

***Keywords:** laser marking, surface, optimal contrast, power density, speed.*

INTRODUCTION

Under modern requirements of production standards on cutting tools, offered on the market, there should be marking such as: company logos, serial numbers or lately matrix and barcode containing important technical information etc [1, 8, 9]. In the final stage of the manufacturing process on the instruments, coatings must be applied with purpose to protect from both weathering and aggressive environments during their operation. Without damage significantly of these coatings, the affixed marking should also be durable and should have good readability both visually and by automated control systems. In the study of the process of marking is established that these two criteria for quality are significantly influenced by many factors, and also choosing a suitable method of marking [2, 3]. In many instruments such as taps, dies, drills, lathe tools, main background is dark in result of the final operation on their oxidation. In this case, the method of marking should ensure maximum contrast by removing the top layer and revealing a bright surface of the base material from which the instrument has been made. In the nickel-plated tools, the surface is bright and should be selected a method for marking, that ensures respectively darker marked area to ensure required contrast. Compliance with these requirements in conjunction with the complex relationships that exist between the various thermo-physical and optical properties of the materials of which the instruments on the one hand and the technological parameters related to the laser source and system technologies such as wavelength, power, speed of processing and others required to perform preliminary experimental research in each case in order to establish the optimum process parameters.

EXPERIMENT

In the paper are studied the real products (knives, drills, cutters, taps, dies) from rapid tool steel P6M5 with standard coatings [10, 11]. In purpose to monitor the influence of the wavelength of laser radiation on the process of absorption from the surface of the article for laser marking, the experiments were made with two types of lasers - Nd: YAG disc laser D16 [12] and laser CuBr [13] (see Table 1).

Table 1

Laser Parameters	Nd:YAG disc laser D16	CuBr laser
Wavelength λ, nm	1 064	511 & 578
Power P, W	16,0	10,0
Frequency ν, kHz	5÷50	19,0
Duration of pulses τ, ns	1000 –10000	30
Pulse energy E_p, mJ	0,32 – 3,2	0,51
Pulse power P_p, kW	0,032 – 3,2	17,0

Experimental method provides a research on contrast of the marking k^*, the basic criteria for quality, in:

- a function of power density $k^* = k^*(q_S)$;
- a function of the speed of processing $k^* = k^*(v)$.

Series of experiments are conducted onto:

1. Products with nickel-plated surfaces, as compared are made and control measurements in marking at a polished surface of basic material.

(Method of marking by melting on the surface layer)

2. Products with oxidized surfaces.

(Method of marking by evaporation on the surface layer of the background material)

In both series of experiments the power density of laser radiation q_S is amended in the intervals:

- for disc laser D16 $q_S \in [0{,}91.10^{10}; 2{,}27.10^{10}]$ W/m^2 with a step $1{,}13.10^9$ W/m^2;
- for CuBr laser $q_S \in [0{,}72.10^{10}; 1{,}34.10^{10}]$ W/m^2 with a step $0{,}715.10^9$ W/m^2.

In experiments related to the study of the influence of technological parameters speed v of processing changes its interval $v \in [10, 120]$ mm/s with a step 10 mm/s.

The criteria for quality contrast, according to the standard for the perception of visual marking and automated readout of information, are indicated in [4, 7, 14].

In order to define more precisely the contrast by the method of comparison with the surrounding surface [5], onto prepared samples perform a marking of areas (squares of side 5 mm) on the faster method, such as the paths are written with step Δx = 50 μm.

1. Investigation of the marking process on a nickel-plated surfaces

The results from study of $k^* = k^*(q_S)$ with disc laser D16 and CuBr laser are presented in graphic form in Fig. 1 (a, b).

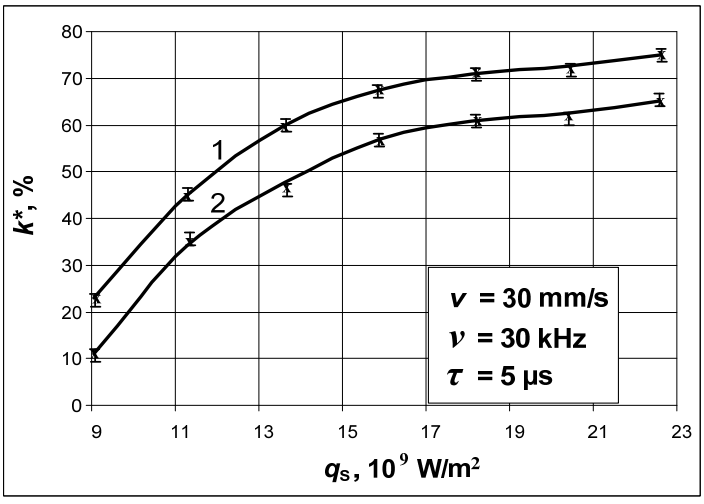


Fig. 1a Amendment of contrast k as a function of power density q_S for marking with disc laser D16 for samples of steel P6M5 with: 1 - brushed surface, 2 - nickel-plated surface.*

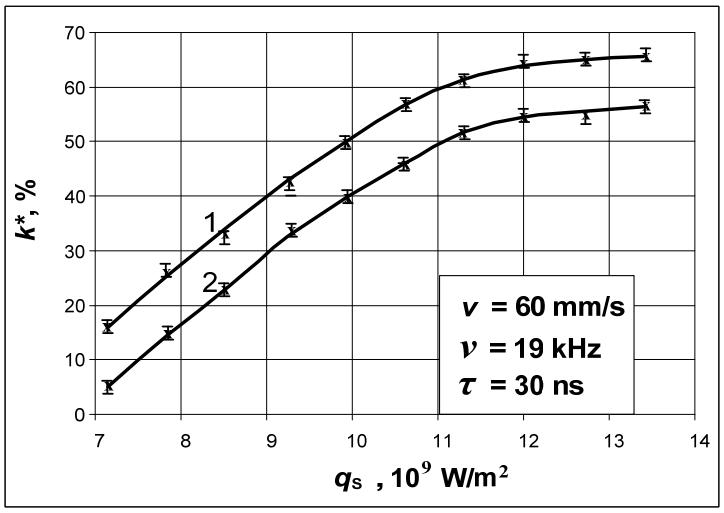


Fig. 1b Amendment of contrast k as a function of power density q_S for marking with CuBr laser for samples of steel P6M5 with: 1 - brushed surface, 2 - nickel-plated surface.*

The following conclusions can be drawn from the analysis of the graphs:

• For polished and nickel-plated surfaces to impact with disc laser D16 and CuBr laser, contrast of marking *k** increases with the increase of surface power density q_S of laser radiation.

• For disc laser the speed of change of contrast is therefore in the interval of power density of laser radiation q_S Є [0,91.10^{10}; 1,36.10^{10}] W/m^2 - 8,2.10^{-9} %/(W/m^2 for both surfaces and in the interval q_S Є [1,36.10^{10}; 2,27.10^{10}] W/m^2 - 1,65.10^{-9} %/(W/m^2).

• For CuBr laser the speed of change of contrast is therefore in the interval of power density of laser radiation q_S Є [0,72.10^{10}; 1,13.10^{10}] W/m^2 - 11,1.10^{-9} %/(W/m^2) for both surfaces and in the interval q_S Є [1,13.10^{10}; 1,34.10^{10}] W/m^2 - 2,4.10^{-9} %/(W/m^2).

• For brushed surface the contrast of marking is with 10-12% higher than that of the nickel-plated surface at impact with disc laser and 10-11% - in impact with CuBr laser. The explanation for this phenomenon can be found in the greater absorbability of the sanded surface than the nickel surface [6].

• Optimal intervals for power density of laser radiation for marking with disc laser D16 at speed v = 30 mm/s.

in the visual perception of the marking	brushed surface nickel-plated surface	q_S Є [1,19.10^{10}; 2,27.10^{10}] W/m^2 q_S Є [1,42.10^{10}; 2,27.10^{10}] W/m^2
in using of special readers	brushed surface nickel-plated surface	q_S Є [0,91.10^{10}; 2,27.10^{10}] W/m^2 q_S Є [1,00.10^{10}; 2,27.10^{10}] W/m^2

• Optimal intervals for power density of laser radiation for marking with CuBr laser at speed *v* = 60 mm/s.

in the visual perception of the marking	brushed surface nickel-plated surface	q_S Є [0,99.10^{10}; 1,34.10^{10}] W/m2 q_S Є [1,11.10^{10}; 1,34.10^{10}] W/m^2
in using of special readers	brushed surface nickel-plated surface	q_S Є [0,75.10^{10}; 1,34.10^{10}] W/m^2 q_S Є [0,83.10^{10}; 1,34.10^{10}] W/m^2

• Due to the higher absorbence of the laser radiation in CuBr (A = 45%) compared with disc laser (*A* = 30%) work with 25-30% less surface power density of laser radiation.

The results of the study of the dependence of the contrast as a function of speed of marking - *k** = *k**(*v*), with both types of lasers are shown in Fig. 2a and Fig. 2b. The remaining technological parameters for this series of experiments are kept constant.

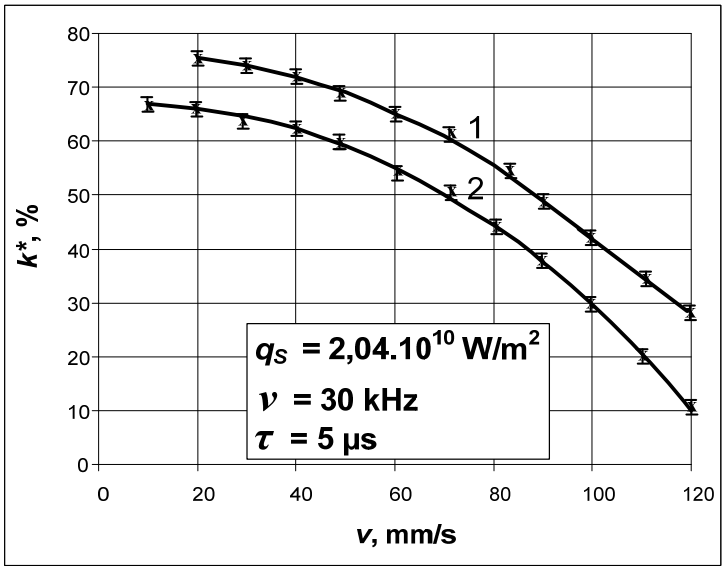


Fig. 2a. Amendment of contrast k as a function of speed v of marking with disc laser D16 for samples of steel P6M5 with: 1 - brushed surface, 2 - nickel-plated surface.*

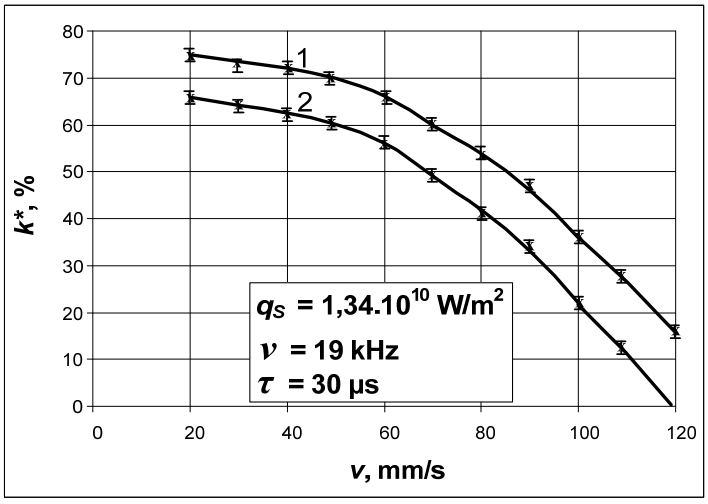


Fig. 2b. Amendment of contrast k as a function of speed v of marking with CuBr laser for samples of steel P6M5 with: 1 - brushed surface, 2 - nickel-plated surface.*

It can be concluded of the constructed graphs of *k** = *k**(*v*) that:

• For brushed and nickel-plated surface speed steel P6M5 with increasing velocity decreases contrast of marking it with the same speed of change (- 0,41%/(mm/s)) in impact with disc laser and in CuBr laser - speed of change (- 0,5%/(mm/s)).

• Optimal intervals of speed of marking *v* for surface power density of laser radiation q_S = 2,04.10^{10} W/m^2 with disc laser.

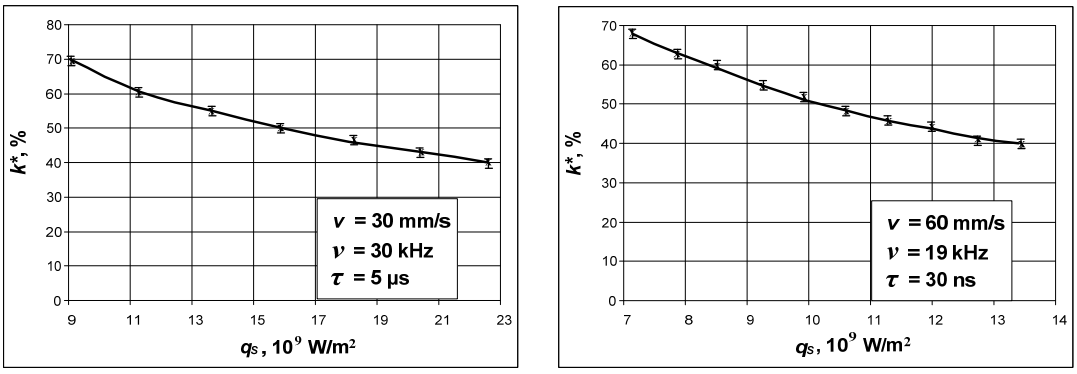
in the visual perception of the marking	brushed surface nickel-plated surface	v Є [20, 88] mm/s v Є [10, 70] mm/s
in using of special readers	brushed surface nickel-plated surface	v Є [20, 120] mm/s v Є [20, 103] mm/s

• Optimal intervals of speed of marking v for surface power density of laser radiation q_S = 1,34.10^{10} W/m^2 with CuBr laser.

in the visual perception of the marking	brushed surface nickel-plated surface	v Є [20, 85] mm/s v Є [20, 68] mm/s
in using of special readers	brushed surface nickel-plated surface	v Є [20, 116] mm/s v Є [20, 110] mm/s

2. Investigation of the laser marking process on the oxidized surfaces

The results of amendment of the contrast k^* as a function of power density q_S with disc laser and CuBr laser are presented in graphic form in Fig. 3 (a, b).



а) *б)*

Fig. 3. Experimental dependence $k^* = k^*(q_S)$ for marking of samples of P6M5 steel with oxidized surfaces for: a) disc laser D16; b) CuBr laser

From the analysis of obtained results the following points can be brought out:

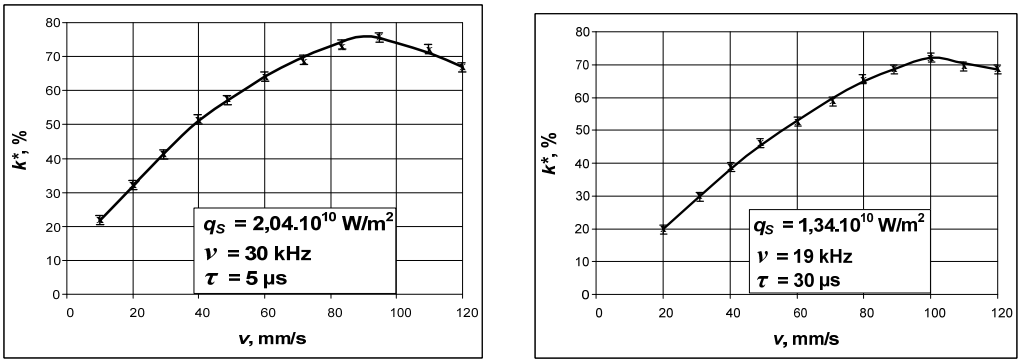
• In the research interval of power density q_S occurs withdrawal of layer and melting of the basic material, which leads to darkening, and thus reduce the contrast.

• There is an almost linear decrease the contrast k^* with increasing power density of laser radiation q_S such as in impact with a disc laser speed of this change is 2,2.10^{-9} %/(W/m^2) and in CuBr laser - 4,4.10^{-9} %/(W/m^2).

• The optimal interval of power density q_S to obtain quality marking.

in the visual perception of the marking	disc laser CuBr laser	v = 30 mm/s v = 60 mm/s	q_S Є [0,91.10^{10}; 1,58.10^{10}] W/m^2 q_S Є [0,72.10^{10}; 1,03.10^{10}] W/m^2
in using of special readers	disc laser CuBr laser	v = 30 mm/s v = 60 mm/s	q_S Є [0,91.10^{10}; 2,27.10^{10}] W/m^2 q_S Є [0,72.10^{10}; 1,34.10^{10}] W/m^2

The results of the study of the dependence of the contrast k^* in a function from velocity v in marking on samples with oxidized surfaces are illustrated in Fig. 4a, b.



а) *б)*

Fig. 4. Experimental dependence k = k* (v) for marking of samples of steel R6M5 with: a) disc laser D16; b) CuBr laser*

From the resulting graphical dependences it can be determined that:

• With increasing of speed v is observed nonlinear increase on the contrast *k** in impact with both laser sources:

- for disc laser the maximum of speed is at *v* = 90 mm/s;
- for CuBr laser the maximum of speed is at *v* = 100 mm/s.

• The optimal working intervals of speed *v* for obtaining of quality marking.

in the visual perception of the marking	disc laser CuBr laser	q_S = 2,04.10^{10} W/m^2 q_S = 1,34.10^{10} W/m^2	*v* Є [38, 120] mm/s *v* Є [56, 120] mm/s
in using of special readers	disc laser CuBr laser	q_S = 2,04.10^{10} W/m^2 q_S = 1,34.10^{10} W/m^2	*v* Є [20, 120] mm/s *v* Є [20, 120] mm/s.

This amendment of contrast is explained as follows. In speed of marking *v* = 20 mm/s obtains the withdrawal of the layer and melting of the basic material in the zone of impact. The increase of speed reduces the depth of the melt. At a speed *v* = 90 mm/s is attained only removal of oxidized layer and so the contrast is maximized. At speeds above *v* = 90 mm/s to obtain the withdrawal of part of the layer where the greater the speed is, the lower part of it is taken. This leads to reduction of the contrast in this range of speeds.

CONCLUSION

Experimental researches on laser marking process of tool steels can continue for other technological parameters - frequency, pulse duration, step in raster selection, number of repetitions, defocus on one hand, but also for other types of lasers and brands instrumental steel. The results obtained in experimental studies can be laid in the technological tables, building a database of optimal process parameters in each case. For given input data in them the operator can quickly reach optimal technological parameters necessary to obtain quality marking of manufactured product in this moment.

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ИЗСЛЕДВАНЕ НА ВЛИЯНИЕТО НА ВИДА НА ПОВЪРХНОСТТА ВЪРХУ КАЧЕСТВОТО НА ЛАЗЕРНАТА МАРКИРОВКА

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Резюме: *Изследвана е възможността за маркировка на инструменти с три вида повърхности: шлифована, никелирана и оксидирана. Анализира се изменението на контраста в зависимост от два технологични параметъра - плътността на мощността и скоростта на маркиране. Определени са интервали на плътността на мощността и скоростта на маркиране за получаване на оптимален контраст. Експериментите са реализирани с два типа лазери – Nd:YAG шайбов лазер D16 и лазер на CuBr с цел да се оцени и влиянието на дължината на вълната върху процеса на маркиране. Дискутира се и физическия механизъм при маркиране върху трите повърхности.*

Ключови думи: *лазерно маркиране,оптимален контраст, повърхност, плътност на мощността, скорост.*

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OPTIMIZATION OF THE PROCESS OF LASER MARKING OF METAL PRODUCTS

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Technical University of Gabrovo

Abstract: *The impact of power density and speed on the process of laser marking of aluminum, copper and nickel is studied. Experiments were performed with a fiber laser and a CuBr laser. The dependencies of the contrast of marking on the power density and speed were obtained and analyzed. The optimum intervals of the power density and the speed of laser marking on these metals were determined.*

Keywords: *laser marking, metals, fiber laser, CuBr laser, optimization.*

INTRODUCTION

Laser marking is a complex process which is influenced by a number of factors [1]. Power density of the laser marking and speed are the most important parameters, which help to achieve high contrast and quality marking products. On the speed depends the time of impact upon the sample and the energy, which is absorbed in the material in the zone of impact. It is decisive in the choosing of the method of marking. The power density of the laser radiation should be sufficient in order to induce structural changes in the product, melting of the material in the area of processing and/or partial evaporation. Another fact, which needs to be considered, is that with its rise increases the absorption of the metals.

Because of the critical importance of power density and speed on the laser marking process, each case should be optimized by conducting experiments.

The objective of the study is to obtain optimal intervals of the power density of laser radiation and the laser marking speed on aluminum, copper and nickel products. Experiments were made with a fiber laser and a CuBr laser.

EXPERIMENT

1. Methodology for determination of the contrast of marking

The determination of the contrast in a gray scale is made in the following way:

A program for raster area marking with measures 3 mm x 3 mm, 5 mm x 5 mm or 10 mm x 10 mm, is applied (the necessity is dictated by the requirements for marking of 2D codes and graphics [4, 5, 6]). A black and white photo of the marked test flight is made using a digital camera.

The contrast k^* is determined by an etalon scale in relative units or percentages. By the gray hue any bitmap image can be represented as a number between 0 (black) and 255 (white). A benchmark number N_f for an image on the surface around the zone of marking is defined. In a particular image of a marking, the corresponding value of N_x is measured by comparing the gray scale (the merger with reference image). The contrast k_x^* is determined by a linear interpolation from the expression:

$$k_x^* = \frac{N_f - N_x}{N_f}.100\%. \quad (1)$$

The described method allows to be studied the influence onto the contrast k* of a number from technological factors, including power density of laser radiation qS and velocity v of marking.

2. Used materials and lasers

For the experiments are used samples of aluminum, copper and nickel. Their characteristics are shown in Table 1 [2, 7].

Table 1

Metal Magnitude	Aℓ	Cu	Ni
Thermal conductivity k, W/(kg.K)	236	401	94
Density ρ, kg/m^3	2,70.10^3	8,92.10^3	8,91.10^3
Specific heat capacity c, J/(kg.K)	830	380	440
Thermal diffusivity a, m^2/s	1,05.10^{-4}	1,18.10^{-4}	2,40.10^{-5}
Temperature of melting T_m, K	933,5	1358	1728
Temperature of evaporation T_v, K	2792	2840	3186

The experiments refer to laser fiber SP-40P and laser CuBr. The main technological parameters of laser marking systems for these lasers are given in Table 2 [3, 8].

Table 2

Laser Parameter	CuBr laser	Fiber laser SP-40P
Wavelength λ, nm	578	1 062
Power P, W	10,0	40,0
Frequency v, kHz	19,0	250
Duration of pulses τ, ns	30	8 ÷ 250
Pulse energy E_p, mJ	0,51	0,16 ÷ 1,33
Pulse power P_p, kW	17,0	5,32 ÷ 17,8
Quantity of beam M^2	< 1,7	< 1,1
Positioning accuracy, μm	2,5	2,5
Efficiency, %	10	40

3. Realized tasks:

3.1. Investigation of the dependence of the contrast of the marking on the power density of laser radiation.

3.1.1. Results for fiber laser SP-40P

Parameters, which are kept constant during the experiments, are shown in Table 3. Power density q_S of laser radiation is amended in the interval $q_S \in [1{,}03.10^{10}; 2{,}15.10^{10}]$ W/m^2 with a step 1,60.10^9 W/m^2.

Table 3

Parameter	Value
Speed v, mm/s	60,0
Diameter d, μm	40,0
Frequency ν, kHz	30
Duration of pulses τ, ns	250
Step Δx, μm	50
Defocusing Δf, mm	0

In Fig. 1 is shown the experimental dependence graphs $k^* = k^*(q_S)$ for samples A ℓ, Cu and Ni. From their analysis can be drawn the following conclusions:

- By increasing the power density of the laser radiation is observed nonlinear increase in the contrast of the three marking materials. The speed of increase of contrast marking v_c is:

Metal	Interval of q_S, W/m^2	v_c, %/(W/m^2)
Nickel	$1{,}03.10^{10} \div 1{,}83.10^{10}$	$3{,}88.10^{-9}$
Aluminum	$1{,}03.10^{10} \div 1{,}83.10^{10}$	$7{,}88.10^{-9}$
Copper	$1{,}35.10^{10} \div 1{,}83.10^{10}$	$10{,}2.10^{-9}$

- In the interval $q_S \in [1{,}83.10^{10}; 2{,}15.10^{10}]$ W/m^2 the contrast of the marking increases very slightly for the three materials.

- Optimum intervals of the power density in marking with a fiber laser with speed v = 60 mm/s are:

$q_S \in [1{,}18.10^{10}; 2{,}15.10^{10}]$ W/m^2 for Ni;
$q_S \in [1{,}58.10^{10}; 2{,}15.10^{10}]$ W/m^2 for Aℓ;
$q_S \in [1{,}76.10^{10}; 2{,}15.10^{10}]$ W/m^2 for Cu.

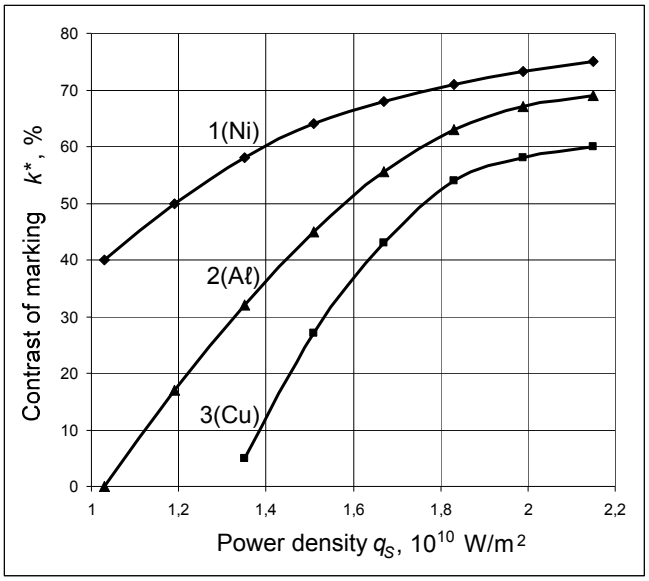

Figure 1. Graphs of the dependence $k^* = k^*(q_S)$ for marking with fiber laser on samples from: Ni - 1; Aℓ - 2; Cu - 3.

3.1.2. Results for CuBr laser

The parameters, which are kept constant during the experiments, are shown in Table. 4. Power density of laser radiation q_S is amended in the interval $q_S \in [7{,}08.10^{9}; 13{,}4.10^{9}]$ W/m^2 with a step $7{,}08.10^{8}$ W/m^2.

Table 4

Parameter	Value
Speed v, mm/s	60,0
Diameter d, μm	30,0
Frequency ν, kHz	20
Duration of pulses τ, ns	30
Step Δx, μm	50
Defocusing Δf, mm	0

In Fig. 2 are presented graphs of the experimental dependence $k^* = k^*(q_S)$ for samples of Al, Cu and Ni. From their analysis can be drawn the following conclusions:

- By increasing the power density of the laser radiation is observed nonlinear increase in the contrast of the three marking materials.

- Comparison of the graphs in Fig. 1 and Fig. 2 shows that in order to be obtained the same contrast of marking for a metal is required 30-35% less power density of CuBr laser compared to a laser fiber. This is explained by the greater absorbability of the radiation with a wavelength λ = 578 nm than that of a fiber laser.

- The optimum intervals of the power density when marking with a fiber laser with speed v = 60 mm /s are:

$q_S \in [8{,}70.10^9; 13{,}4.10^9]$ W/m^2 for Ni;

$q_S \in [10{,}1.10^9; 13{,}4.10^9]$ W/m^2 for Aℓ;

$q_S \in [11{,}3.10^9; 13{,}4.10^9]$ W/m^2 for Cu.

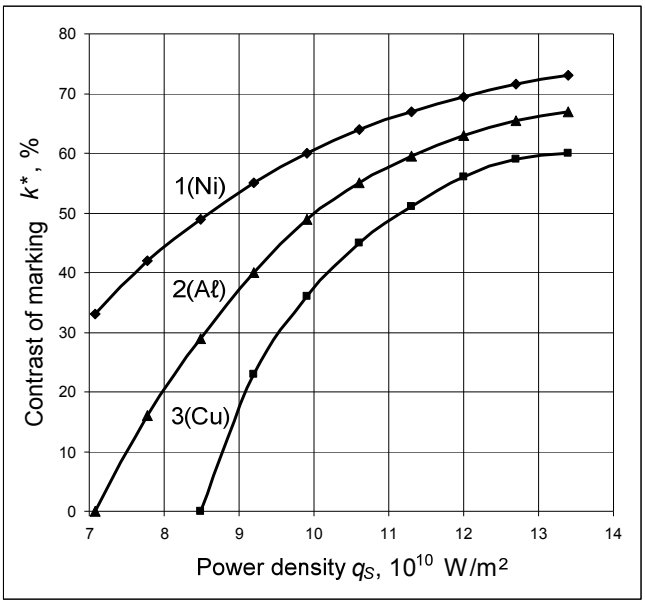

Figure 2. Graphs of the dependence $k^* = k^*(q_S)$ for marking with CuBr laser on samples from: Ni - 1; Aℓ - 2; Cu - 3.

3.2. Investigation of the dependence of the contrast of the marking on the speed.

The parameters of technological system with CuBr laser, which are kept constant during the experiments, are shown in Table 4. Speed marking laser radiation is amended in the interval $v \in [30; 100]$ mm/s with a step of 10 mm/s.

Table 4

Parameter	Value
Power density q_S, W/mm^2	1,20.10^{10}
Diameter d, μm	30,0
Frequency ν, kHz	20
Duration of pulses τ, ns	30
Step Δx, μm	50
Defocusing Δf, mm	0

In Fig. 3 are shown the experimental dependence graphs $k^* = k^*(v)$ for samples of Al, Cu and Ni. From their analysis can be drawn the following conclusions:

- By increasing the speed of the marking is observed a nonlinear decrease in the contrast of the three studied materials.

- The speed of increase in the contrast of marking in the interval $v \in [60, 100]$ mm/s is:

0,79 %/(mm/s) for Ni;
0,82 %/(mm/s) for Aℓ;
0,92 %/(mm/s) for Cu.

- The optimum intervals of speed of marking with CuBr laser for power density $q_S = 1{,}20.10^{10}$ W/m^2 are:

$v \in [30; 90]$ mm/s for Ni;
$v \in [30; 83]$ mm/s for Aℓ;
$v \in [30; 73]$ mm/s for Cu.

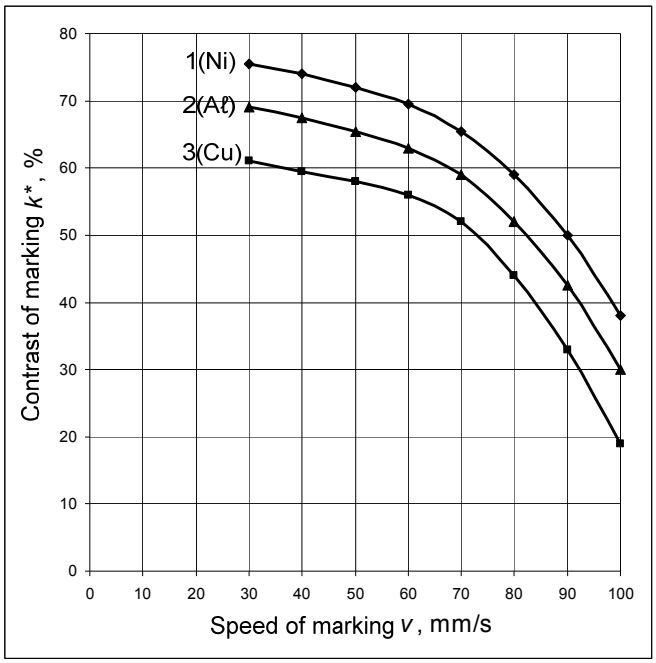

Figure 3. Graphs of the dependence k = k*(v) for marking with CuBr laser on samples from: Ni - 1; Aℓ - 2; Cu - 3.*

CONCLUSION

Results from the experimental studies can be provided in tables with optimum technological process parameters for each case of laser marking. Thus, the work of the laser systems operator is optimized.

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ОПТИМИЗИРАНЕ НА ПРОЦЕСА ЛАЗЕРНО МАРКИРАНЕ НА ИЗДЕЛИЯ ОТ МЕТАЛИ

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Резюме: *Изследвано е влиянието на плътността на мощността и скоростта за лазерно маркиране на алуминий, мед и никел. Експериментите са извършени с файбър лазер и лазер на CuBr. Получени и анализирани са зависимостите на контраста на маркировката от плътността на мощността и от скоростта. Определени са оптимални интервали за плътността на мощността и скоростта за лазерно маркиране тези метали.*

Ключови думи: *лазерно маркиране, метали, файбър лазер, лазер на CuBr, оптимизация.*

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DETERMINATION OF PRELIMINARY INTERVALS OF THE SPEED OF LASER WELDING ON ELECTRICAL STEEL

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Abstract: *Numerical experiments on determination of preliminary intervals on the speed of laser welding of electrical steel M530-50A were conducted. They refer to a diode laser and CO_2-laser. The obtained graphic dependences between the maximum depth of the melt and the speed for two types of lasers were compared and analyzed.*

Keywords: *laser welding, electrical steel, diode laser, CO_2-laser, preliminary intervals.*

INTRODUCTION

On the laser welding process influence a number of factors related to:

- Material properties - optical characteristics (reflectance, absorption, penetration depth) and thermo-physical properties (thermal conductivity, thermal diffusivity, specific heat capacity).
- Laser parameters - power, power density and volume density of absorbed energy.
- Technological parameters - speed, defocusing.

These factors are in particular physical dependencies between them. They are important for the understanding of the physical nature of the process [1, 2]. The speed, in conjunction with the power density, is a basic factor in the realisation of the research process.

The objective of the work is to obtain preliminary speed intervals through numerical experiments on laser welding of products of electrical steel from M530-50A. They are necessary to reduce the timing of real experiments in the study of this process.

EXPERIMENT

1. Used software, material and lasers

In conducting numerical experiments is used a specialized software TEMPERATURFELD3D [3] - the working environment for the calculation of temperature fields with a wide range of input parameters and capabilities for analysis of the calculated results. For the calculations are necessary the following input parameters: program parameters; geometric parameters; parameters of the laser; parameters of the material.

After the calculations, on the output are received the following options: approximation of the results; animation of the entire process; temperature profile of the material at a particular moment of time; profile of the maximum temperature; dependence on the temperature from the time; amendment on the temperature of material in depth.

The experiments are related to electrical steel M530-50A, which is widely used in the manufacturing process of rotor and stator packs for electric motors [4, 5, 6]. The chemical composition of the steel is shown in Table 1.

Two laser systems with diode laser [7] and CO_2-laser [8] are used. Their parameters are shown in Table 2 and are suitable for welding steels.

Table 1

Element	C	Si	Mn	P	S	Cr	Mo	Aℓ
Content, %	0,027	1,45	0,236	0,097	0,020	0,06	0,021	0,358

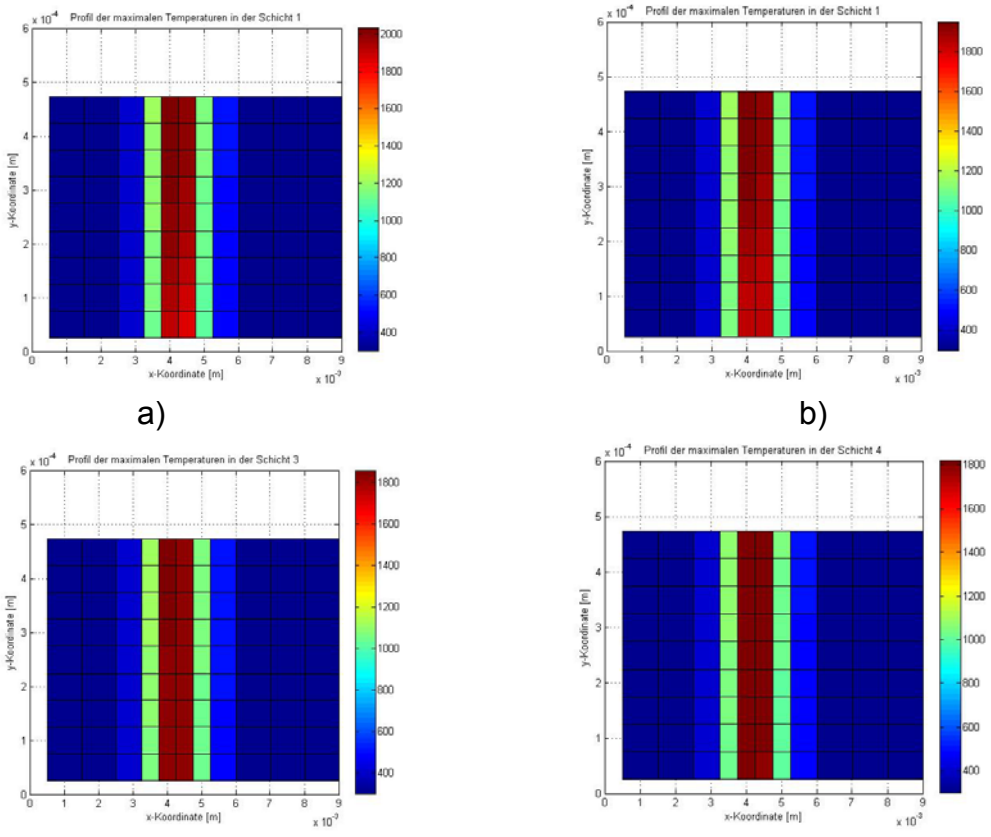
Table 2

Laser Parameter	Diode laser	CO_2-laser
Wavelength λ, nm	1070	10 600
Power P, W	6000	5 000
Frequency ν, kHz	continuous wave (cw)	continuous wave (cw)
Beam quality M^2	< 10	< 10
Positioning accuracy, μm	2,5	2,5
Efficiency, %	40	20

2. Results for Diode laser

In order to determine the intervals of changing of speed at different power densities, the dependence between the depth of the melt and the speed for the two lasers is examined.

With the use of the program Temperaturfeld3D a series of numerical experiments were held; the speed changes in the interval $v \in [20, 120]$ mm/s with a step 10 mm/s. From the obtained temperature fields on the surface of the sample and layer by layer is determined the depth of the melt. In Fig. 1 are shown the temperature fields of the speed v = 70 mm/s and power density of the laser radiation $q_S = 6{,}19.10^9$ W/m^2. On the surface of the sample (h_1 = 0) and in layers 2 (h_2 = 100 μm), 3 (h_3 = 200 μm) and 4 (h_4 = 300 μm), the temperature of the specimen in the impact zone is above the melting point.



a) b)

c) d)

Fig. 1.

The results are summarized in Fig. 2, where graphs of the dependencies of the depth of the melt h to the speed of welding v are presented: 1 - for the power density q_S = 6,19.10^9 W/m^2; 2 - for the power density q_S = 7,43.10^9 W/m^2. From their analysis can be drawn the following conclusions:

- With the increasing of speed is observed a decrease in the depth of the melt. It is explained by the fact that with the increasing of speed the time for impact is reduced. Also, this reduces the energy absorbed in the impact zone, the volume and depth of the melt.
- The average speed of the reduction of the depth of melt is 4,30 μm/(mm/s) for power density q_S = 6,19.10^9 W/m^2 and 5,70 μm/(mm/s) for q_S = 7,43.10^9 W/m^2.
- Intervals for the speed of laser welding are received:

- v Є [20, 57] mm/s for power density q_S = 6,19.10^9 W/m^2;
- v Є [20, 94] mm / s for power density q_S = 7,43.10^9 W/m^2.

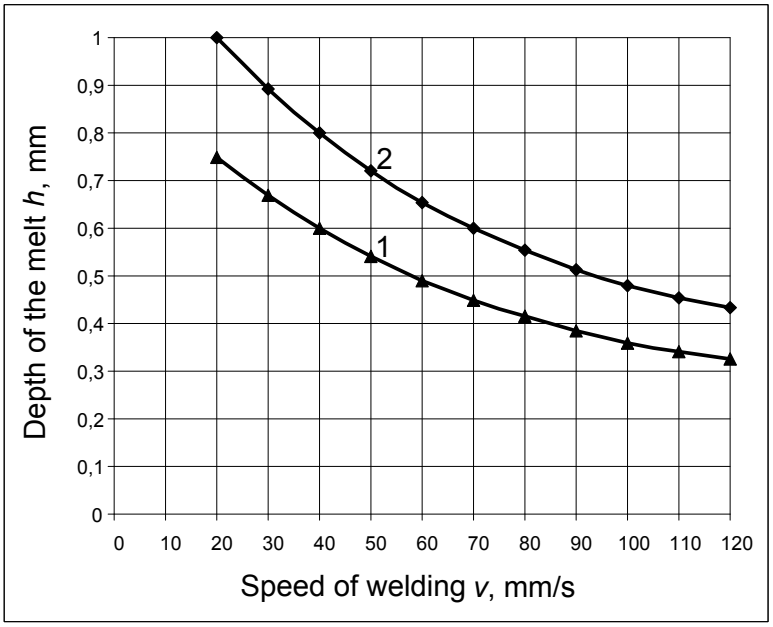


Fig. 2.

3. Results for CO_2-laser

A series of numerical experiments were conducted; the speed changed its interval v Є [20, 120] mm/s with a step of 10 mm/s, and the power density of the laser radiation was kept constant. In Fig. 3 is shown a graph of the dependence between the depth of the melt and the speed of welding: 1 - for the power density q_S = 1,23.10^{10} W/m^2; 2 - for the power density q_S = 1,56.10^{10} W/m^2. From these could be drawn the following conclusions:

- With the increasing of speed is observed a decrease in the depth of the melt.
- The average speed of reduction of the depth of melt is 4,00 μm/(mm/s) for power density q_S = 1,23.10^{10} W/m^2 and 5,20 μm/(mm/s) for q_S = 1,56.10^{10} W/m^2.
- Power density of laser radiation for the CO_2-laser is more than two times larger than that of the diode laser; the melt depth for the studied range of speed is almost the same. This is explained by the larger absorption of the radiation of diode laser compared to that of the CO_2-laser for electrical steel M530-50A.
- Intervals of the speed of laser welding are received:

- v Є [20, 53] mm / s for power density q_S = 1,23.10^{10} W/m^2;
- v Є [20, 80] mm / s for power density q_S = 1,56.10^{10} W/m^2.

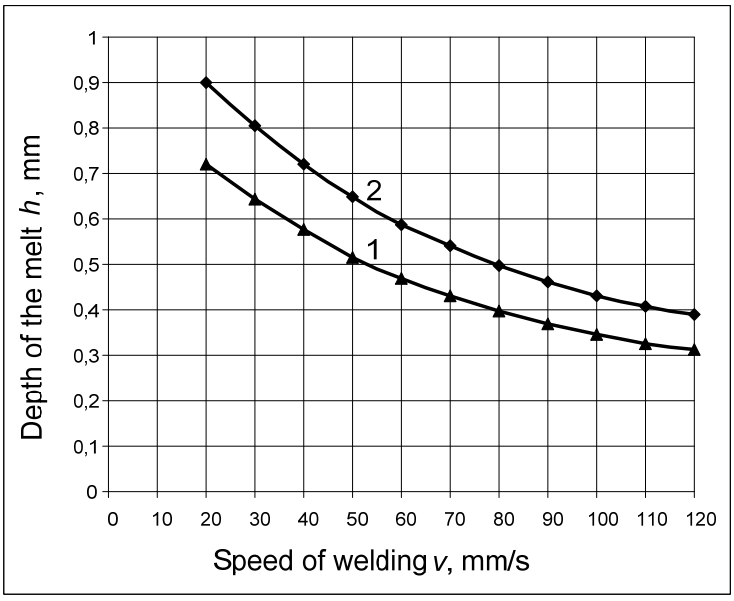


Fig. 3.

CONCLUSION

The received intervals are the preliminary step for the optimization of the process of laser welding of electrical steel products. Numerical experiments can be supported by the study of the influence of the power density of laser radiation on the depth of the melt. These results can serve as a basis for further experiments related to the study of the process.

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ОПРЕДЕЛЯНЕ НА ПРЕДВАРИТЕЛНИ ИНТЕРВАЛИ ЗА СКОРОСТТА НА ЛАЗЕРНО ЗАВАРЯВАНЕ НА ЕЛЕКТРОТЕХНИЧЕСКА СТОМАНА

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Резюме: *Числени експерименти на предварителни интервали за скоростта при лазерно заваряване на електротехническа стомана М530-50А са проведени. Те се отнасят за диоден лазер и CO_2-лазер. Получените графични зависимости на максималната дълбочина на стопилката от скоростта за два типа лазери са сравнени и са анализирани.*

Ключови думи: *лазерно заваряване, електротехническа стомана, диоден лазер, CO_2-лазер, предварителни интервали.*

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USAGE OF THE SYSTEM *MATHEMATICA* IN TEACHING AND LEARNING NUMBER THEORY

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Angel Kanchev University of Ruse

***Abstract**: The aim of this talk is to present some of the possibilities of the system Mathematica® for using it in the BSc degree course in Number Theory taught for the students in Mathematics and Informatics at the Angel Kanchev University of Ruse. The version used is Mathematica 5.1 and most of the problems discussed follow* [3] *used as a textbook for the course.*

***Keywords:** Division algorithms, Fermat's and Euler's theorems, Congruences, Linear Diophantine equations.*

INTRODUCTION

The system for computer algebra *Mathematica*® gives possibilities for good illustrations of many of the problems included in a BSc degree course in Number Theory. This illustration helps not only the teaching process but the learning process as well. It contradicts at some extent the traditional conception that Mathematics is learned only with a pencil at hand. A good lecturer in Mathematics could prepare good practical lessons on different topics and could reach a better students' understanding of mathematical ideas and motivate many of the students for further work both on mathematical problems and the usage of systems for computer algebra.

The development of modern science and technology depends on the development of adequate methods for representing integers and doing integer arithmetic. Starting as long as 5000 years ago different methods using different integers as bases, have been devised to denote integers. The ancient Babylonians used 60 as the base for their number systems and the ancient Mayans used 20. Our method of expressing integers, the decimal system using 10 as its base, was first developed in India approximately six centuries ago. Today, the binary system, which takes 2 as its base, is used extensively by computing machines.

PROBLEMS CONCERNING THE DIVISION ALGORITHM, THE EXPANSION OF INTEGERS AND APPLICATIONS OF FERMAT'S AND EULER'S THEOREMS

- The division algorithm allows finding the quotient and the remainder when dividing two integers m and n. By *Mathematica* this is done using the functions **Quotient**$[m,n]$ and **Mod**$[m,n]$**.**

For example

Quotient[67,3]

22

Mod[67,3]

1.

This means that $67 = 3.22 + 1$.

- Finding the base p expansion of a in decimal notation (i.e. using digits to represent multiples of powers of ten) is done in the system by the function **IntegerDigits**$[a, p]$, which gives this p expansion, namely $a = a_n a_{n-1} ... a_{0(p)}$.

For example

IntegerDigits[235,4]

{3,2,2,3}

IntegerDigits[116,7]

{2,2,4}

IntegerDigits[336,5]

{2,3,2,1}.

This means that $235 = 3223_{(4)}$, $116 = 224_{(7)}$ and $336 = 2321_{(5)}$.

- Obtaining the decimal expansion is done directly from the corresponding p expansion, namely

3*8^2+3*8+7

223

1*7^2+5*7+5

89.

This means that $337_{(8)} = 223$ and $155_{(7)} = 89$.

- Transforming p expansion to q expansion for $p, q \neq 10$ needs an intermediate step, namely the decimal expansion of the integer.

For example

IntegerDigits[3*5^2+3*5+3,6]

{2,3,3}

IntegerDigits[7*9+8,8]

{1,0,7}.

This means that $333_{(5)} = 233_{(6)}$ and $78_{(9)} = 107_{(8)}$.

- Finding the greatest common divisor GCD and the least common multiplier LCM of integers by *Mathematica* is done using the functions **GCD[a,b,…]** and **LCM[a,b,…].**

For example the solution of problem 10 on page 2 i.e. Pr.10/2 of [3] is

GCD[126,238]

14

GCD[123456789,987654321]

9

GCD[646,3003]

1.

The last result shows that the integers 646 and 3003 are relatively prime.

For the solution of problem 11 on page 3 of [3] we get

GCD[270,126,342]

18.

The system *Mathematica* has the advantage to find *LCM* only by one function while working by hand we need the formula

$$GCD\{a,b\}.LCM\{a,b\} = ab \qquad (1),$$

i.e. we have to find GCD at first.

For Pr.12/3 we have

LCM[126,238]

2142

LCM[646,3003]

1939938

LCM[270,54,342]

5130.

The system *Mathematica* could illustrate (1) leading to better knowledge of this relation.

For example

GCD[12,33]

3

LCM[12,33]

132

3*132

396

12*33

396.

- The prime-power factorization of an integer n is done by *Mathematica* using the function **FactorInteger[n].** It gives both the prime divisors and its degrees.

For example

FactorInteger[243]

{{3,5}}.

This means that $243 = 3^5$.

- The integer n could be a very big one, its prime divisors and their degrees as well.

For example

FactorInteger[243!]

{{2,237},{3,121},{5,58},{7,38},{11,24},{13,19},{17,14},{19,12},{23,10},
{29,8},{31,7},{37,6},{41,5},{43,5},{47,5},{53,4},{59,4},{61,3},{67,3},
{71,3},{73,3},{79,3},{83,2},{89,2},{97,2},{101,2},{103,2},{107,2},
{109,2},{113,2},{127,1},{131,1},{137,1},{139,1},{149,1},
{151,1},{157,1},{163,1},{167,1},{173,1},{179,1},{181,1},
{191,1},{193,1},{197,1},{199,1},{211,1},{223,1},
{227,1},{229,1},{233,1},{239,1},{241,1}}.

By *Mathematica* we could check if an integer n is prime or not. For this we need the function **PrimeQ[n].**

For example

PrimeQ[13]

True

PrimeQ[3*3+27-4*2]

False.

The last result shows that the integer $3.3 + 27 - 4.2 = 28$ is not prime.

We could define the n-th prime integer p_n by the function **Prime[n].** Thus

Prime[13]

41

Prime[3*3+27-4*2]

107.

This means that $p_{13} = 41$ and $p_{28} = 107$.

- Some applications of Fermat's and Euler's theorems, namely $a^{p-1} \equiv 1(\mathrm{mod}\, p)$ for p - prime not dividing a and $a^{\varphi(n)} \equiv 1(\mathrm{mod}\, n)$ for a and n relatively prime could be realized by *Mathematica* with the function **PowerMod[m,n,p]** calculating $m^n(\mathrm{mod}\, p)$.

Thus the solution of Pr.30/5 is

PowerMod[137,42,100]

69,

meaning that the last two digits in the decimal expansion of 137^{42} are 6 and 9.

If the first digits are 0 they are not given by *Mathematica*. It means that the correct answer needs more digits to be defined. Thus the solution of Pr.28/5 will be

Mod[783^{15},100]

{7}

PowerMod[783,15,1000]

207,

meaning that the last two digits of 783^{15} are 0 and 7.

- By *Mathematica* one could define the Euler's function $\varphi(n)$, which gives the number of integers less or equal to n and relatively prime to n. For this one uses the function **EulerPhi[n].**

Pr. 45/8 has the following solution

EulerPhi[11088]

2880,

meaning that $\varphi(11088) = 2880$.

The function **EulerPhi[n]** for n prime could illustrate as well the formula $\varphi(n) = n - 1$.

For example

EulerPhi[113]

112.

Such a result shows in another way (except by **PrimeQ[n])** that an integer is a prime number.

The better learning of all stated properties of integers could be helped by the students' self work on Pr.13,14,16/3, Pr.32/6, Pr.38/7, Pr.39/7 and Pr.42/8 from [3] for example.

PROBLEMS CONCERNING CONGRUENCES AND LINEAR DIOPHANTINE EQUATIONS

- First degree congruences could be solved by **Solve[eqns && Modulus=p]**. Thus we have

Solve[2x+5==0 && Modulus==15]

{{Modulus->15,x->5}}.

The same function illustrates both the cases when the congruence does not have a solution and when the solutions are more than one. For example

Solve[21x+4==0 && Modulus==9]

{}

Solve[12x+15==0 && Modulus==21]

{{Modulus->21,x->4},{Modulus->21,x->11},{Modulus->21,x->18}}.

- By *Mathematica* we could solve higher degrees' congruences as well. For example

Solve[25x^3-21x^2+33x-13==0 && Modulus==8]

{{Modulus->8,x->1},{Modulus->8,x->5}}

Solve[x^2+4x-1==0 && Modulus==125]

{}.

The last result means that the congruence $x^2+4x-1=0(\mathrm{mod}125)$ does not have a solution.

- If the congruence is a quadratic one and we don't need its solution but only to know if it exists or not, we work with the function **JacobiSymbol[n,m],** calculating $\left(\frac{n}{m}\right)$.

The example

JacobiSymbol[426,491]

-1

shows that the congruence $x^2 \equiv 426(\mathrm{mod}\,491)$ does not have a solution, while

JacobiSymbol[-1,17]

1

gives that $x^2 \equiv -1(\mathrm{mod}17)$ has a solution. By

Solve[x^2+1==0 && Modulus==17]

{{Modulus->17,x->4},{Modulus->17,x->13}}

we get the two solutions $x \equiv 4(\mathrm{mod}17)$ and $x \equiv 13(\mathrm{mod}17)$ of the congruence $x^2+1 \equiv 0(\mathrm{mod}17)$.

We illustrate one more advantage of the system *Mathematica*: Working analytically to find if a congruence has a solution or not and the modulus is not prime we have to make additional transformations in order to use the Gauss' law of reciprocity. In *Mathematica* there are no restrictions for the parameters of the function **JacobiSymbol[n,m]** (the **Jacobi symbol[*n*, *m*]** reduces to the **Legendre symbol** $\left(\frac{n}{m}\right)$ when *m* is an odd prime).

Thus

JacobiSymbol[860,11021]

1.

- For solving systems of first degree congruences *Mathematica* works with the function **ChineseRemainder[list$_1$, list$_2$],** giving the least nonnegative integer r for which **Mod[r, list$_2$] = list$_1$.**

Let list$_1 = \{5,3\}$ and list$_2 = \{12,14\}$. Thus $r \equiv 5(\mathrm{mod}12)$ and $r \equiv 3(\mathrm{mod}14)$. It means that r is a solution of the system

$$\begin{vmatrix} x \equiv 5(\mathrm{mod}\,12) \\ x \equiv 3(\mathrm{mod}\,14) \end{vmatrix}.$$

The function **ChineseRemainder[list$_1$, list$_2$]** is a part of the standard package **NumberTheory`NumberTheoryFunctions`,** which has to be activated before the action of the function.

The solution of the above system by *Mathematica* is

<<NumberTheory`NumberTheoryFunctions`
ChineseRemainder[{5,3},{12,14}]
17
LCM[12,14]
84.

This means that the solution of the system is $x \equiv 17(\mathrm{mod}\,84)$.

We give two examples more: For the system $\begin{vmatrix} x \equiv 1(\mathrm{mod}\ 3) \\ x \equiv 3(\mathrm{mod}\ 7) \end{vmatrix}$ by *Mathematica* we get

ChineseRemainder[{1,2},{3,7}]
16
LCM[3,7]
21

and the solution is $x \equiv 16(\mathrm{mod}\,21)$. For the system $\begin{vmatrix} x \equiv 6(\mathrm{mod}\ 7) \\ x \equiv 1(\mathrm{mod}\ 49) \end{vmatrix}$ we get

ChineseRemainder[{6,1},{7,49}]
{},

meaning that the last system does not have a solution.

We point that the considered congruences have to be of the form $x \equiv a(\mathrm{mod}\,m)$. Thus the system $\begin{vmatrix} 19x = 17(\mathrm{mod}\ 5) \\ 19x = 17(\mathrm{mod}\ 7) \end{vmatrix}$ has the following solution by *Mathematica* in activated **NumberTheory`NumberTheoryFunctions`**package):

Solve[19x-17==0 && Modulus==5]
{{Modulus->5,x->3}}
Solve[19x-17==0 && Modulus==7]
{{Modulus->7,x->2}}
ChineseRemainder[{3,2},{5,7}]
23
LCM[5,7]
35.

When the congruences in the system differ only by the modules being relatively prime we could solve the system in another way as well. For example the above system could be solved as

Solve[19x-17==0 && Modulus==35]
{{Modulus->35,x->23}}.

For higher degree congruences *Mathematica* gives all logical possibilities. For example

Solve[7x^{11}-4==0 && Modulus==221]
{{Modulus->221,x->44}}
Solve[x^4-37==0 && Modulus==41]
{{Modulus->41,x->8},{Modulus->41,x->10}
{Modulus->41,x->31},{Modulus->41,x->33}}
Solve[9x^{12}-11==0 && Modulus==29]
{}.

In the last case the congruence $9x^{12}-11\equiv 0(\mathrm{mod}\,29)$ does not have a solution.

- For solving linear Diophantine equations with two unknowns there is no function in *Mathematica*. The equation has to be written as two congruences but the general solution has to fulfill the conditions for its existence, namely: If (x_0,y_0) is a solution of the equation $ax+by=c$, its general solution is given by the formula

$$\begin{vmatrix} x=x_0+\dfrac{b}{(a,b)}t \\ y=y_0-\dfrac{a}{(a,b)}t \end{vmatrix}.$$

Let us consider the equation $4x+19y=5$. As $GCD\{4,19\}=(4,19)=1$, then the general solution is $\begin{vmatrix} x=x_0+\dfrac{19}{(4,19)}t=x_0+19t \\ y=y_0-\dfrac{4}{(4,19)}t=y_0-4t \end{vmatrix}$ for $4x_0+19y_0=5$.

In *Mathematica* by

ExtendedGCD[4,19]

{1,{5,-1}}

we get $4.5+19.(-1)=1$ being equivalent to $4.25+19(-5)=5$. Then we find x_0 as a solution of $25\equiv x(\mathrm{mod}19)$, namely

Solve[25==x && Modulus==19]

{{Modulus->19,x->6}}.

Now we find y_0 by

Solve[4*6+19y==5]

{{y->-1}}.

Substituting in the above formula we get that the equation $4x+19y=5$ has the solution $\begin{vmatrix} x=6+19t \\ y=-1-4t \end{vmatrix}$.

If the equation $ax+by=c$ has no solution (for (a,b) not dividing c) then the congruence $ax\equiv c(\mathrm{mod}\,b)$ has no solution as well.

As an example we use Pr.78(g)/19 i.e. the equation $60x+18y=97$. From

ExtendedGCD[60,18]

{6,{1,-3}}

we get $6=1.60+(-3).18$ but 6 does not divide 97. Solving the congruence $60x\equiv 97(\mathrm{mod}18)$, namely

Solve[60x==97 && Modulus==18]

{}

we get the same answer.

Now we consider the equation $6x+9y=15$. As $(6,9)=3$ and 3 divides 15 the equation has a solution. For it by *Mathematica* we have:

ExtendedGCD[2,3]
{1,{-1,1}}
Solve[-5==x && Modulus==3]
{{Modulus->3,x->1}}
Solve[6*1+9y==15]
{{y->1}}.

Thus the general solution will be

$$\begin{cases} x = x_0 + 9t = 1 + 9t \\ y = y_0 - 6t = 1 - 6t \end{cases}.$$

The students' self work could include Pr.51-52/10, Pr.56/11, Pr.58/12, Pr.62-63/14, Pr.72/17, Pr.78/19 and Pr.81/20 from [3].

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ИЗПОЛЗВАНЕ НА СИСТЕМАТА *MATHEMATICA* ПРИ ПРЕПОДАВАНЕ И ОБУЧЕНИЕ ПО ТЕОРИЯ НА ЧИСЛАТА

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Резюме: *Докладът представя някои възможности на системата Mathematica при използването ѝ в бакалавърски курс по Теория на числата за студенти от специалност Математика и информатика в Русенския университет. Използваната версия е Mathematica 5.1 и много от разглежданите примери са по [3], използван като учебник за такъв курс.*

Ключови думи*: Алгоритъм за деление, Теореми на Ферма и Ойлер, Сравнения, Линейни Диофантови уравнения.*

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USING THE MAPLE SOFTWARE PRODUCT IN STUDYING FUNCTIONS

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Abstract: *The paper discusses the use of the MAPLE software product in teaching the subject Higher Mathematics Part 1, and more specifically in working with functions. A sequence of operations is presented for collecting the necessary information about the behavior of the function and for drawing its graph. One fractional-rational and one exponential function are studied. By drawing the graph of each function, the results obtained at earlier stages are confirmed. The visual representation of how a given problem is solved helps students to more easily comprehend and better analyze the results.*

Keywords: *Calculus, MAPLE, Study of a function.*

The use of software products in the training of university students is becoming increasingly widespread in the educational process. Higher Mathematics Part 1 is taught to students of the *Computer Systems and* Technologies and *Telecommunication Systems* courses by applying the MAPLE software program.

The present paper examines the use of the product to study a function of one variable. One fractional-rational and one exponential function are considered.

The study of these functions is accomplished by applying the following sequence of actions intended to gather the necessary information about the behavior of the function and to plot its graph:

1. Determining the domain of the function.
2. Checking if the function is even, odd, or periodic.
3. Studying the behavior of the function at the ends of the domain intervals. Finding the horizontal and vertical asymptotes.
4. Finding the oblique asymptotes.
5. Finding intersections with the *Ox* and *Oy* axes and the intervals in which the function keeps a constant sign.
6. Finding the intervals of monotonicity of the function and its extrema.
7. Finding the intervals of convexity and concavity of the function and its inflection points.
8. Summarizing the results in a table and plotting a graph of the function.

Function 1

The first function chosen to be investigated is $y = \frac{x^3}{2*(x+1)^2}$.

1. The function is entered in the working area of the MAPLE application.

```
> y1:=x**3/2/((x+1)**2);
```

$$y1 := \frac{1}{2}\frac{x^3}{(x+1)^2}$$

In determining its domain it is necessary to exclude those values of x, which lead to division by zero. Then the domain will be $x \in (-\infty,-1) \cup (-1,+\infty)$.

2. The function is neither even nor odd as $f(-x)=\frac{(-x)^3}{2*(-x+1)^2}\neq\pm f(x)$. Therefore, no symmetry can be expected to exist in the graph of the function with respect either to an axis, or to a point.

The studied function is not periodic, either, because when solving the equation $f(x+T)=f(x)$, i.e. $\frac{(x+T)^3}{2*(x+T+1)^2}=\frac{x^3}{2*(x+1)^2}$ no value for *T* is obtained that is a positive number not depending on *x.*

```
> solve((x+T)**3/2/(x+T+1)**2-x**3/2/(x+1)**2=0,T);
```

$$0,\ \frac{1}{2}\frac{(-2x^2-6x-3+\sqrt{-4x-3})x}{x^2+2x+1},\ \frac{1}{2}\frac{(-2x^2-6x-3-\sqrt{-4x-3})x}{x^2+2x+1}$$

3. Now we find the corresponding limits:

```
> a1:=limit(y1, x=-infinity);
```

$$a1 := -\infty$$

```
> a2:=limit(y1, x=-1, left);
```

$$a2 := -\infty$$

```
> a3:=limit(y1, x=-1, right);
```

$$a3 := -\infty$$

```
> a4:=limit(y1, x=infinity);
```

$$a4 := \infty$$

The obtained values of the limits *a1* and *a4* give reason to conclude that there are no horizontal asymptotes on the function graph and limits *a2* and *a3* lead to the conclusion that the line $x=-1$ is a vertical asymptote on the graph of the function being examined.

4. Finding the other asympotes is done in the following way:

```
> k1:=limit(y1/x, x=-infinity);
```

$$k1 := \frac{1}{2}$$

```
> n1:=limit(y1-k1*x, x=-infinity);
```

$$n1 := -1$$

```
> Y1:=k1*x+n1;
```

$$Y1 := \frac{1}{2}x-1$$

```
> k2:=limit(y1/x, x=infinity);
```

$$k2 := \frac{1}{2}$$

```
> n2:=limit(y1-k1*x, x=infinity);
```

$$n2 := -1$$

```
> Y2:=k2*x+n2;
```

$$Y2 := \frac{1}{2}x-1$$

Since the results of the calculation of the limits *κ1* and *κ2* and those of *n1* and *n2* coincide, it follows that the equations of the lines $Y1 := \frac{1}{2}x - 1$ and *Y2*, which are oblique asymptotes on the graph of *y1* with $x \to \pm\infty$ will coincide, too.

5. The next two commands find the intersections of the function with the axes *Ox* and *Oy*:

> **solve(y1=0,x);**

$$0,\ 0,\ 0$$

> **y11:=1/2*0**3/((0+1)**2);**

$$y11 := 0$$

There is only one point of intersection and it is the origin of the coordinates, point *O(0,0).*

6. Finding the intervals of monotonicity of the function and its extrema is connected with finding the first derivative which is accomplished with the next command. The second command is used to simplify the first derivative expression.

> **y1prim:=diff(y1,x);**

$$y1prim := \frac{3}{2}\frac{x^2}{(x+1)^2} - \frac{x^3}{(x+1)^3}$$

> **simplify(%);**

$$\frac{1}{2}\frac{x^2(x+3)}{(x+1)^3}$$

To find the points of a hypothesised extremum, the first derivative is equated to zero and points are searched for in the area where it does not exist. These must be points belonging to the function domain.

> **solve(y1prim=0,x);**

$$-3,\ 0,\ 0$$

The result obtained after completing this command shows that the points of a hypothesised extremum of the examined function are $x = -3$ and $x = 0$. The first derivative does not exist for $x = -1$, but this is not a point in the function domain. Therefore, it does not appear to be a hypothesised extremum point.

The sign of y' is studied with the help of the next two commands:

> **solve(y1prim>0,x);**

$$\mathrm{RealRange}(-\infty,\ \mathrm{Open}(-3)),\ \mathrm{RealRange}(\mathrm{Open}(-1),\ \mathrm{Open}(0)),\ \mathrm{RealRange}(\mathrm{Open}(0),\ \infty)$$

> **solve(y1prim<0,x);**

$$\mathrm{RealRange}(\mathrm{Open}(-3),\ \mathrm{Open}(-1))$$

> **y1max:=1/2*(-3)**3/((-3+1)**2);**

$$y1max := \frac{-27}{8}$$

The results show that *y1* is increasing for $x \in (-\infty,-3) \cup (-1,0) \cup (0,+\infty)$ and decreasing for $x \in (-3,-1)$. At point $x = -3$ *y1* reaches a maximum $y1_{\max} = \frac{-27}{8}$. In the neighbourhood of *x=0* the sign of y' doesn't change. Therefore there is no extremum at this point of the function.

7. The expression $y'' = (y')'$ is derived and simplified with the following two commands:

> **y1sec:=diff(y1prim,x);**

$$y1sec := 3\frac{x}{(x+1)^2} - \frac{6x^2}{(x+1)^3} + \frac{3x^3}{(x+1)^4}$$

> **simplify(%);**

$$3\frac{x}{(x+1)^4}$$

The following three commands are used to look for the points of a hypothesized inflexion.

> **solve(y1sec=0,x);**

$$0$$

> **solve(y1sec>0,x);**

$$\text{RealRange}(\text{Open}(0), \infty)$$

> **solve(y1sec<0,x);**

$$\text{RealRange}(-\infty, \text{Open}(-1)), \text{RealRange}(\text{Open}(-1), \text{Open}(0))$$

The results show that $x = 0$ is a points of the hypothesized inflexion. The function is convex downward for $x \in (0,\infty)$ and convex upward for $x \in (-\infty,-1) \cup (-1,0)$. In the neighbourhood of $x = 0$ y'' changes its sign. Therefore, point *O(0,0)* is an inflexion point for the function under study.

8. The graph of the function *y1* and its oblique asymptote *Y1* are plotted (Fig.1) and confirmation is found for the results obtained above.

> **plot({y1,Y1},x=-10..20,y=-20..30);**

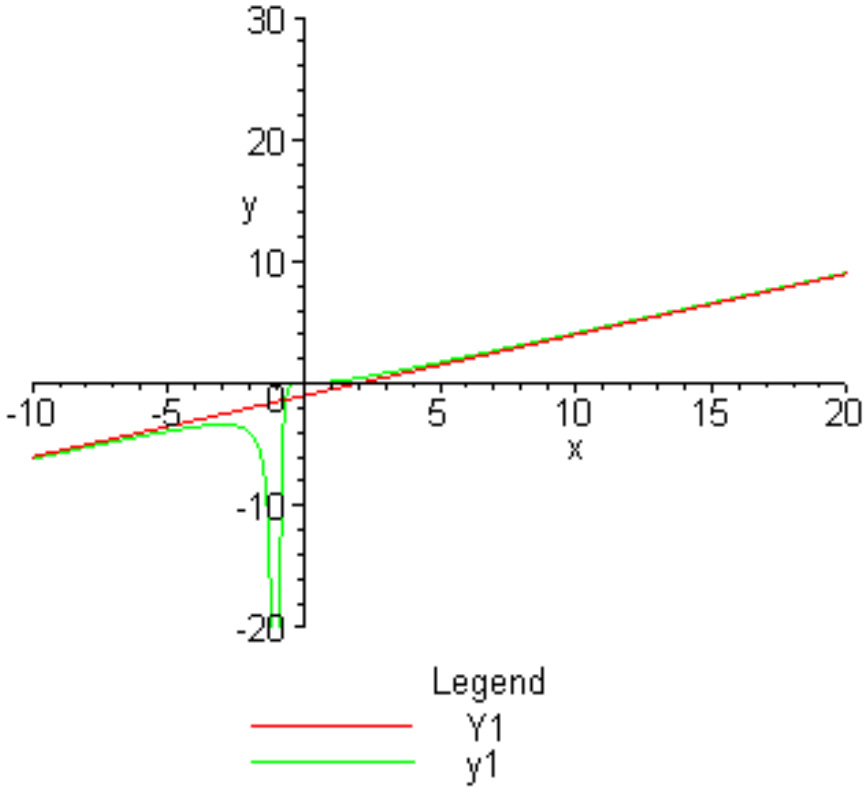

Figure 1. Function 1

Function 2

1. The function is entered in the working area of the MAPLE application.

> **y2:=(x+2)*exp(1/x);**

$$y2 := (x+2)\,e^{\left(\frac{1}{x}\right)}$$

In determining its domain it is necessary to exclude those values of x, which lead to division by zero. Then the domain will be $x \in (-\infty, 0) \cup (0, +\infty)$.

2. The function is neither even nor odd because $f(-x) = (-x+2)*e^{\frac{1}{(-x)}} \neq \pm f(x)$. Therefore, no symmetry can be expected to exist in the graph of the function with respect either to an axis, or to a point.

The studied function is not periodic, either, because when solving the equation $f(x+T) = f(x)$, i.e. $(x+T+2)*e^{\frac{1}{x+T}} = (x+2)*e^{\frac{1}{x}}$ no value for *T* is obtained that is a positive number not depending on *x.*

3. Now we find the corresponding limits:

> **b1:=limit(y2,x=-infinity);**

$$b1 := -\infty$$

> **b2:=limit(y2,x=0, left);**

$$b2 := 0$$

> **b3:=limit(y2, x=0, right);**

$$b3 := \infty$$

> **b4:=limit(y2, x=-infinity);**

$$b4 := -\infty$$

The obtained values of the limits *b1* and *b4* give reason to conclude that there are no horizontal asymptotes on the function graph and limits *b2* and *b3* lead to the conclusion that the line $x = 0$ is a vertical asymptote on the graph of the function being examined.

4. Finding the other asympotes is done in the following way:

> **k3:=limit(y2/x,x=-infinity);**

$$k3 := 1$$

> **n3:=limit(y2-k3*x, x=-infinity);**

$$n3 := 3$$

> **Y3:=k3*x+n3;**

$$Y3 := x + 3$$

> **k4:=limit(y2/x,x=infinity);**

$$k4 := 1$$

> **n4:=limit(y2-k3*x, x=infinity);**

$$n4 := 3$$

> **Y4:=k4*x+n4;**

$$Y4 := x + 3$$

Since the results of the calculation of the limits *κ3* and *κ4* and those of *n3* and *n4* coincide, it follows that the equations of the lines $Y3 := x + 3$ and *Y4*, which are oblique asymptotes on the graph of *y2* with $x \to \pm\infty$ will coincide, too.

5. With the next command, the intersection is found of the function with the *Ox* axes $x = -2$.

The function under study has no point of intersection with the *Oy* axis since point $x = 0$ does not belong to the function domain.

> **solve(y2=0,x);**

$$-2$$

6. Finding the intervals of monotonicity of the function and its extrema is connected with finding the first derivative which is accomplished with the next command. The second command is used to simplify the first derivative expression.

> **y2prim:=diff(y2,x);**

$$y2prim := \mathbf{e}^{\left(\frac{1}{x}\right)} - \frac{(x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^2}$$

> **simplify(%);**

$$\frac{\mathbf{e}^{\left(\frac{1}{x}\right)}(x^2 - x - 2)}{x^2}$$

To find the points of a hypothesised extremum, the first derivative is equated to zero and points are searched for in the area where it does not exist. These must be points belonging to the function domain.

> **solve(y2prim=0,x);**

$$2, -1$$

The result obtained after completing this command shows that the points of a hypothesised extremum of the examined function are $x=-1$ and $x=2$. The first derivative does not exist for $x=0$, but this is not a point in the function domain.

The sign of y' is studied with the help of the next two commands.

> **solve(y2prim>0,x);**

$$\mathrm{RealRange}(-\infty, \mathrm{Open}(-1)), \mathrm{RealRange}(\mathrm{Open}(2), \infty)$$

> **solve(y2prim<0,x);**

$$\mathrm{RealRange}(\mathrm{Open}(-1), \mathrm{Open}(0)), \mathrm{RealRange}(\mathrm{Open}(0), \mathrm{Open}(2))$$

> **y2max:=(-1+2)*exp(1/(-1));**

$$y2max := \mathbf{e}^{(-1)}$$

> **y2min:=(2+2)*exp(1/2);**

$$y2min := 4\,\mathbf{e}^{(1/2)}$$

The results show that *y2* is increasing for $x \in (-\infty,-1) \cup (2,\infty)$ and decreasing for $x \in (-1,0) \cup (0,2)$. At point $x=-1$ *y2* reaches a maximum $y2_{\max} = e^{(-1)}$. The minimum is at point $x=2$ and $y2_{\min} = 4*e^{\frac{1}{2}}$.

7. The expression $y'' = (y')'$ is derived and simplified with the following two commands:

> **y2sec:=diff(y2prim,x);**

$$y2sec := -2\,\frac{\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^2} + \frac{2\,(x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^3} + \frac{(x+2)\,\mathbf{e}^{\left(\frac{1}{x}\right)}}{x^4}$$

> **simplify(%);**

$$\frac{\mathbf{e}^{\left(\frac{1}{x}\right)}\,(5\,x+2)}{x^4}$$

The following three commands are used to look for the points of a hypothesized inflexion.

> **solve(y2sec=0,x);**

$$\frac{-2}{5}$$

> **solve(y2sec>0,x);**

$$\text{RealRange}\left(\text{Open}\left(\frac{-2}{5}\right), \text{Open}(0)\right), \text{RealRange}(\text{Open}(0), \infty)$$

> **solve(y2sec<0,x);**

$$\text{RealRange}\left(-\infty, \text{Open}\left(\frac{-2}{5}\right)\right)$$

The results show that $x = -\frac{2}{5}$ is a points of the hypothesized inflexion. The function is convex downward for $x \in \left(-\frac{2}{5}, 0\right) \cup (0, \infty)$ and convex upward for $x \in \left(-\infty, -\frac{2}{5}\right)$. In the neighbourhood of $x = -\frac{2}{5}$ y'' changes its sign. Therefore, point $A\left(-\frac{2}{5}, \frac{8}{5} * e^{-\frac{5}{2}}\right)$ is an inflexion point for the function under study.

8. The graph of the function *y2* and its oblique asymptote *Y3* are plotted (Fig.2) and confirmation is found for the results obtained above.

> **plot({y2,Y3},x=-20..20, y=-10..20);**

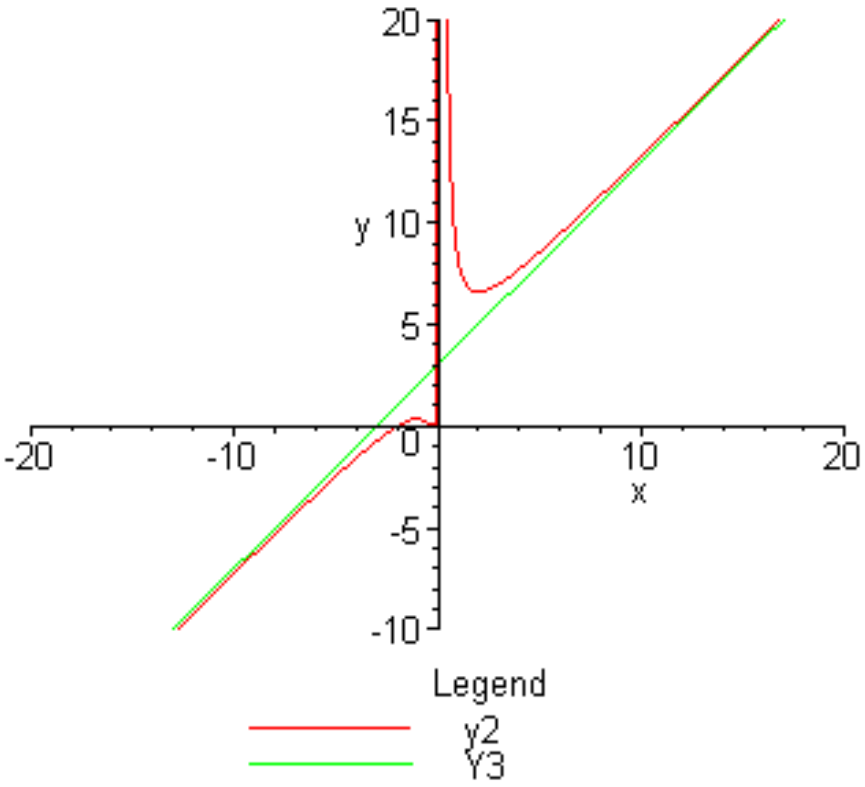

Figure 2. Function 2

Through the use of the MAPLE software product, visualization is achieved of how the given problems are solved which helps the students to comprehend and analyze the results better.

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ИЗПОЛЗВАНЕ НА ПРОГРАМНИЯ ПРОДУКТ MAPLE ПРИ ИЗСЛЕДВАНЕ НА ФУНКЦИЯ

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***Резюме**: В настоящата работа е разгледано използването на програмния продукт MAPLE в обучението по Висша математика 1 в частност при изследване на функция. Показана е последователността от действия за събирането на необходимата информация за поведението на функцията и изчертаване на нейната графика. Направено е изследване на една дробно-рационална и една експоненциална функция. При изчертаването на графиката на функцията е установено потвърждение на получените в предишните етапи резултати. Постигнато е онагледяване на решението на разглежданите задачи, което помага на студентите да осмислят и анализират по-добре резултатите.*

***Ключови думи**: Висша математика, MAPLE, Изследване на функция.*

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PLANE IN SPACE WITH MATHEMATICAL SOFTWARE

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Abstract: *The article presents some mathematical problems suitable for training students on the topic "Plane in Space" included in the discipline Analytical Geometry. Analytical solutions are visualized by using mathematical software Maple 13. This helps students in better understanding the material taught in spatial patterns. Thus, the rate of success increases significantly.*

Keywords: *plane in the space, teaching and learning mathematics, mathematical software, Maple 13.*

INTRODUCTION

The rapid development of information technology and the large number of mathematical software (*Maple, Mathematica, MATLAB, GeoGebra* etc.), that are readily available to students, helps the interesting teaching of mathematics. Using mathematical software is activated and increases the cognitive activity of students. It is very important for the students to have a thorough mathematical knowledge and to be able to verify the results given by the computers [1,7,8,10].

The topic discussed is included in the curriculum of the subject **Linear Algebra and Geometry** for students studying Computer Science, Mathematics & Informatics and Information Technologies in Business. The aim of the study is to show the author's opinion how geometry to be learned easier and to become more interesting by using Maple 13 (Fig. 1).

PRESENTATION

1. Necessary theoretical knowledge ([4, p. 170])

General equation of a plane:

$$\alpha \to Ax+By+Cz+D=0,\ A^2+B^2+C^2\neq 0, \tag{1}$$

$\overrightarrow{n_\alpha}(A,B,C)$ – the normal vector to the plane.

Equation of a plane through point $A(x_0,y_0,z_0)\in\alpha$ and a normal vector:

$$\alpha \to A(x-x_0)+B(y-y_0)+C(z-z_0)=0 \tag{2}$$

Equation of a plane through three points, $A_1(x_1,y_1,z_1)\in\alpha$, $A_2(x_2,y_2,z_2)\in\alpha$ and $A_3(x_3,y_3,z_3)\in\alpha$:

$$\alpha \to \begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0. \tag{3}$$

Equation of a plane through a point $A(x_0,y_0,z_0)\in\alpha$ and two non-collinear vectors $\overrightarrow{m_1}(a_1,b_1,c_1),\overrightarrow{m_2}(a_2,b_2,c_2)$:

$$\alpha\rightarrow\begin{vmatrix} x-x_0 & y-y_0 & z-z_0 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0. \quad (4)$$

Oriented distance from a point to a plane:

$$\delta(M,\alpha)=\frac{Ax_0+By_0+Cz_0+D}{-\operatorname{sgn} D\sqrt{A^2+B^2+C^2}},M(x_0,y_0,z_0). \quad (5)$$

Distance from a point to a plane:

$$d=|\delta| \quad (6)$$

Table1: Commands in Maple 13 ([6], [9])

Command in MAPLE	Action
plots[implicitplot3d]	calculate the area of a surface in 3D / three dimensional space
matrix(A)	input matrix
det(A)	calculate the determinant of a matrix
eval	most often used to calculate the expressions
solve	solving equations
display	visualization
sqrt(a)	$\sqrt{a}$
abs(a)	absolute value
with(linalg)	Linear algebra package
with(plots)	Plots package

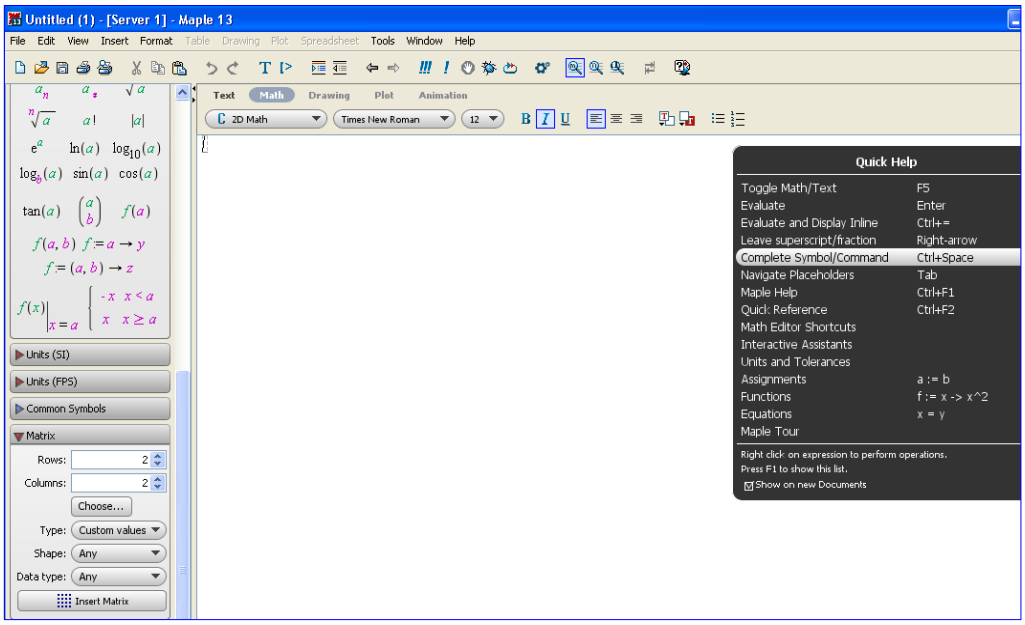

Figure 1: *Maple workspace*

2. Solved problems

Problem1: Find the general equation of a plane, passing through points $M_1(3,0,4)$, $M_2(5,2,6)$ and $M_3(2,3,-3)$ ([4, p. 172]).

Solution:

We are working with Maple 13 and use formula (3)

```
> with(linalg);
```

```
> A:=Matrix([[x,y,z,1],[3,0,4,1],[5,2,6,1],[2,3,-3,1]]);
```

$$A := \begin{bmatrix} x & y & z & 1 \\ 3 & 0 & 4 & 1 \\ 5 & 2 & 6 & 1 \\ 2 & 3 & -3 & 1 \end{bmatrix}$$

```
> det(A)=0;
```

$$-20x + 12y + 8z + 28 = 0$$

```
> det(A)/(-4)=0;
```

$$5x - 3y - 2z - 7 = 0$$

So, we obtain the equation of a plane $\alpha \to 5x - 3y - 2z - 7 = 0$. If we were solving the problem without mathematical software, finding the value of the determinant would have been more difficult.

Problem 2: Find the distance from p. $M(3,-1,0)$ to the plane $\alpha \to 3x - 4y + 12z = 0$ ([4, p. 174]).

Solution:

We are working with Maple 13. After that we use formula (5) to find the oriented distance from a point to a plane:

```
> with(linalg);
```

```
> x:=3;y:=-1;z:=0;
```

$$x := 3$$

$$y := -1$$

$$z := 0$$

```
> A:=3;B:=-4;C:=12;G:=-26;
```

$$A := 3$$

$$B := -4$$

$$C := 12$$

$$G := -26$$

```
> d:=abs(A*x+B*y+C*z+G)/sqrt(A^2+B^2+C^2);
```

$$d := 1$$

Problem 3. It's given the equation of a plane $\alpha \to 23x - 25y + 11z + 13 = 0$. Find the intercept of X, Y, Z with axes and calculate the volume of the parallelepiped constructed using vectors $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ [3].

Solution:

To find the intersection with the axis Ox, we have to make the substitutions in the equation the plane $y = 0, z = 0$ and hence to find x.

```
> solve({eval(23*x-25*y+11*z+13,[y=0,z=0])},{x});
```

$$\left\{x = -\frac{13}{23}\right\}$$

```
> solve({eval(23*x-25*y+11*z+13,[x=0,z=0])},{y});
```

$$\left\{y = \frac{13}{25}\right\}$$

```
> solve({eval(23*x-25*y+11*z+13,[x=0,y=0])},{z});
```

$$\left\{z = -\frac{13}{11}\right\}$$

Consequently $X\left(-\frac{13}{23},0,0\right), Y\left(0,\frac{13}{25},0\right), Z\left(0,0,-\frac{13}{11}\right)$.

To find the volume of the parallelepiped, using the application of the mixed product of three vectors, we have $\overrightarrow{OX}\left(-\frac{13}{25},0,0\right), \overrightarrow{OY}\left(0,\frac{13}{25},0\right), \overrightarrow{OZ}\left(0,0,-\frac{13}{11}\right)$.

```
> M:=matrix([[-13/23,0,0 ],[ 0,13/25,0 ],[ 0,0,-13/11 ]]);
```

$$V := \begin{bmatrix} -\frac{13}{23} & 0 & 0 \\ 0 & \frac{13}{25} & 0 \\ 0 & 0 & -\frac{13}{11} \end{bmatrix}$$

```
> V:=abs((-13/23)*(13/25)*(-13/11));
```

$$V := \frac{2197}{6325}$$

Problem 4: It's given the sphere $x^2 + y^2 + z^2 = 9$ and the plane $x + 2y + z = 1$. Determine what their intersection using Maple 13 [2].

Solution:

```
> with(plots);

> p1:=implicitplot3d (x^2+y^2+z^2=9,x=-3..3,y=-3..3, z= 3..3,color=pink):

> p2:=implicitplot3d(x+2*y+z=1,x=-3..3,y=-3..3,z=-3..3,color=brown):

> display([p1,p2]);
```

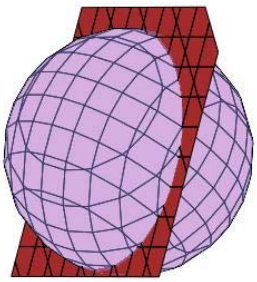

Figure 2: *The intersection of a sphere and a plane*

It can be seen that the plane intersects the sphere, and thus their intersection is a circle (Fig. 2).

Problem 5. Determine the relative positions of the sphere $x^2+y^2+z^2=1$ and the plane $x+2y+z+\sqrt{6}=0$. ([5])

Solution:

After visualization, we can see that the plane does not intersect the sphere (Fig. 3).

```
> with(plots);

> p1:=implicitplot3d(x^2+y^2+z^2=1,x=-1..1,y=-1..1,z=-1..1):

> p2:=implicitplot3d(x+2*y+z+sqrt(6)=0,x=-5..5, y=-5..5,z=5..5, color=red):

> display([p1,p2]);
```

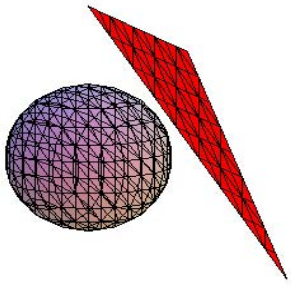

Figure 3: *The intersection of sphere and plane*

CONCLUSION

The usage of mathematical software makes learning mathematics easier and interesting. Also it creates conditions for visualization; motivates and helps students for

successful completion of the course **Linear Algebra and Geometry**. The theme is attractive to students, and would be useful for the work of each teacher in mathematics or anyone with an interest in mathematics. In addition, one can develop other themes used in the learning process. Through the use of mathematical software the goals and objectives set in the curriculum are properly achieved.

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„РАВНИНА В ПРОСТРАНСТВОТО" С МАТЕМАТИЧЕСКИ СОФТУЕР

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Резюме: *Статията представя някои подходящи задачи за обучение на студентите по темата "Равнина в пространството", включени в дисциплината Аналитична геометрия. Аналитичните решения са визуализирани с помощта на математически софтуер Maple13. Този метод помага на студентите за по-добро разбиране на преподавания материал в пространствени модели. По този начин успеваемостта на студентите се увеличава значително.*

Ключови думи: *равнина в пространството, преподаване и обучение по математика, математически софтуер, Maple13.*

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USE OF DYNAMIC SOFTWARE FOR SKETCHES IN GEOMETRY LESSONS

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Abstract: *The article presents some dynamical geometrical software. A teacher really has to include very often personal computers (PC) in his work. The mentioned software allows creation and manipulation with geometric constructions and export in HTML format.*

Keywords**:** sketch, dynamical geometric software, geometric construction.

INTRODUCTION

The application of information and communication technologies is the challenge ahead of education in modern days. The rapid progress of the technologies ranges over areas, associated with the development and the use of new methods and instruments of teaching and lays open new prospects for improvement in the educational process [3]. In an attempt to correspond to the current requirements the teacher certainly has to include very often in his work PC with their large variety of applications in the area of education [1].

EXPOSITION

The use of new technologies in the education is caused by the desire for automation or simulation of different actions, that are performed in classes. When teaching geometry the use of dynamical geometrical software is possible. It allows creation and manipulation with geometric constructions. Similar to the geometric sketches in the notebooks, every sketch starts with making some points. They are being used when defining new objects (line, circumference etc.). The sketches created are dynamical, which means that it is possible to move points (vertex of a triangle, heel of an altitude, center of a circumference) without breaking the geometric connections, that are building the basis of the sketch.

Such software is applied in many geometric situations such as retaining the distance, parallel transition, geometric proportion, etc. The idea is to observe what happens when some elements of one construction are being moved and how other elements are changing their position. The use of a dynamic software not only makes it easier to produce the sketch, when it will take much more time to make a correct one on a paper, but it adds as well to the knowledge of the basic constructions with some built in ones. For example, if it is needed to construct parallel lines by ruler and compass, then it is necessary to sketch a few lines and angles, while the dynamic geometric software allows this to happen just by the selection of the objects, that the sketch is dependent to – line and point for this example. In other words the use of dynamic constructions helps to learn geometry. Wide range of dynamical geometric software is available, but most of it practically has at its disposal instruments for sketching of the basic 2D objects: point, line, segment, circumference, arc, curved line, sector, regular polygon.

Let's consider the special feature of some of the most widespread dynamical geometric software 4]. Each programme includes built in constructions that are the basic characteristics that distinguishes it from the other programs. Universal characteristics for almost all of the programs are the possibilities of constructing perpendicular and parallel lines, angle bisectors, midpoint of a segment, circumference with a given centre, etc.

Important characteristic of some of the programs considered is the possibility of creating macros.

C.a.R. (Compas and Ruler)

Size: 4,41 Mb
Platform: PC/Mac
OS: Windows/Linux/Mac OS
Licence: free
Official site: http://mathsrv.ku-eichstaett.de/MGF/homes/grothmann/java/zirkel/.

The program simulates constructions by line ruler and compass. The creation of the basic geometric figures and constructions is possible. The possibility of measuring distance, area and angles is included. It is allowed to hide/view objects depending of their color. It gives the opportunity of creating macros and generating HTML file in which it is possible to make changes over the sketch. C.a.R. is valuable because of great number of examples included. It is suitable not only when teaching geometry, but for demonstrations and verification of the skills.

Cabri Geometry

Size: 3,5 Mb
Platform: PC/Mac
OS: Windows/Mac OS
Licence: paid; free demo version available
Official site: http://www.cabri.com/.

This is one of the most spread programs that are being used by the mathematicians in the university as a valuable instrument to achieve their science aims. It is developed by the company Cabrilog. It can be used to create sketches and to ask questions – for example if two lines are parallel (perpendicular). It is allowed to compare measures and also the program displays the equations of basic mathematical objects: lines, circumferences, ellipsis. The opportunity of publishing the sketches in web sites is offered.

Cinderella

Size: 15,1 Mb
Platform: PC/Mac
OS: Windows/Linux/Mac OS
Additional requirements: JRE
Licence: paid; free demo version available
Official site: http://cinderella.de.

The program offers wide range of geometric objects. The user friendly interface allows the creation of curves, circumference with given radii, circumference by 3 points. Reflection is a specific build in construction. The main difference from the other programs is that Cinderella offers representation in Euclidean, spheric or hyperbolic geometry at the same time. Animation of geometric places is included. Export in HTML is optional.

EUKLID DynaGeo

Size: 2,4 Mb
Platform: PC
Os: Windows
Licence: paid, free demo version available, but the use of the demo in schools is forbidden
Official site: http://www.dynageo.de/.

All the standard geometric objects can be created by the use of this program. The general geometric transformations are build in – translation, rotation, reflection. Angles and distances can be measured with EUKLID DynaGeo, the geometric objects are available for

filling with some colours. The creation of macros is available for the advanced users. Export in HTML is optional.

GeoGebra

Size: 2,5 Mb

Platform: PC/Mac

OS: Windows/Mac OS/Linux

Additional requirements: JRE

Licence: free

Official site: http://www.geogebra.at/.

This is a dynamical geometric software that combines geometry, algebra and analysis (differentiation and integration processes). It is developed by the University of Salzburg. The program won some awards like the German and European awards for educational software. It allows creation and dynamical change of constructions with points, vectors, segments, lines and functions. The user has an option to insert equations and coordinates by hand. The program has some additional characteristics as the possibility of working with variables, finding some derivatives and integrals, also commands like "root" and "extremum" are available for use. The worksheet is divided into two parts: "algebraic" and "geometric" window. The expression in the "algebraic" window corresponds to the object in the "geometric" window.

Geometer's Sketchpad

Size: 15 Mb

Platform: PC/Mac

OS: Windows/Mac OS

Additional requirements: JRE

Licence: paid

Official site:

http://www.keypress.com/catalog/products/software/Prod_GSP.html.

This is one of the most powerful programs for dynamic geometry. The possibilities of drawing the graph of a given function, of inserting buttons to control animations and of development of macros are the main characteristics, that are making this program so remarkable. Another advantage of this software is that the instructions are visible for the user and in this way the separate steps for the constructions are known for the user.

Geometrix

Size: 10 Mb

Platform: PC

OS: Windows

Licence: free

Official site: http://geometrix.free.fr/.

This program has built in constructions of the basic geometric objects. Some additional constructions are available such as in-circle, tangent line, perpendicular projection etc. The trace that is made by the figure, when it is moved, can be studied. Rotation and translation are also built-in constructions. Diagrams can be created and printed. Some of the general calculations can be performed – sine, cosine, square root, division, multiplication, perimeter and area of a given figure.

Geonext

Size: 5 Mb

Platform: PC/Mac

OS: Windows/Linux/Mac OS

Additional requirements: JRE

Licence: free

Official site: http://geonext.uni-bayreuth.de/.

Geonext is the only one of the programs for dynamic geometry, that is available in bulgarian language. Vectors, right polygon, arc, sector, translation, rotation, reflection, circumference through 3 points etc. are some of the available constructions. The main disadvantage here is that when it is needed to make calculations with length, areas etc. the user has to know how to write commands in XML syntax. Export in HTML is optional. In this process a band with instruments for manipulation of the construction is generated. This is an important characteristic of the program as in this way it is possible changes to be made on the sketch directly from the browser used.

Kig

Size: 11,2 Mb
Platform: PC/Mac
OS: Linux/Unix
Additional requirements: JRE
Licence: free
Official site: http://edu.kde.org/kig/.

This program has a very wide range of constructions and is very friendly user. Most of the options for the constructions and the way the objects are visualized are available through contextual menu. Kig offers built-in constructions that are not available in the other dynamic geometric software. These constructions are related to circumference, curves, axes and directrix, also sum and subtraction of vectors. The main disadvantage is that if calculations are needed, then the user must be good in programming. Geometric transformations such as rotation, translation, inversion, reflection are available.

Example for reflection on some of the programs mentioned above follows.

Euclid DynaGeo:

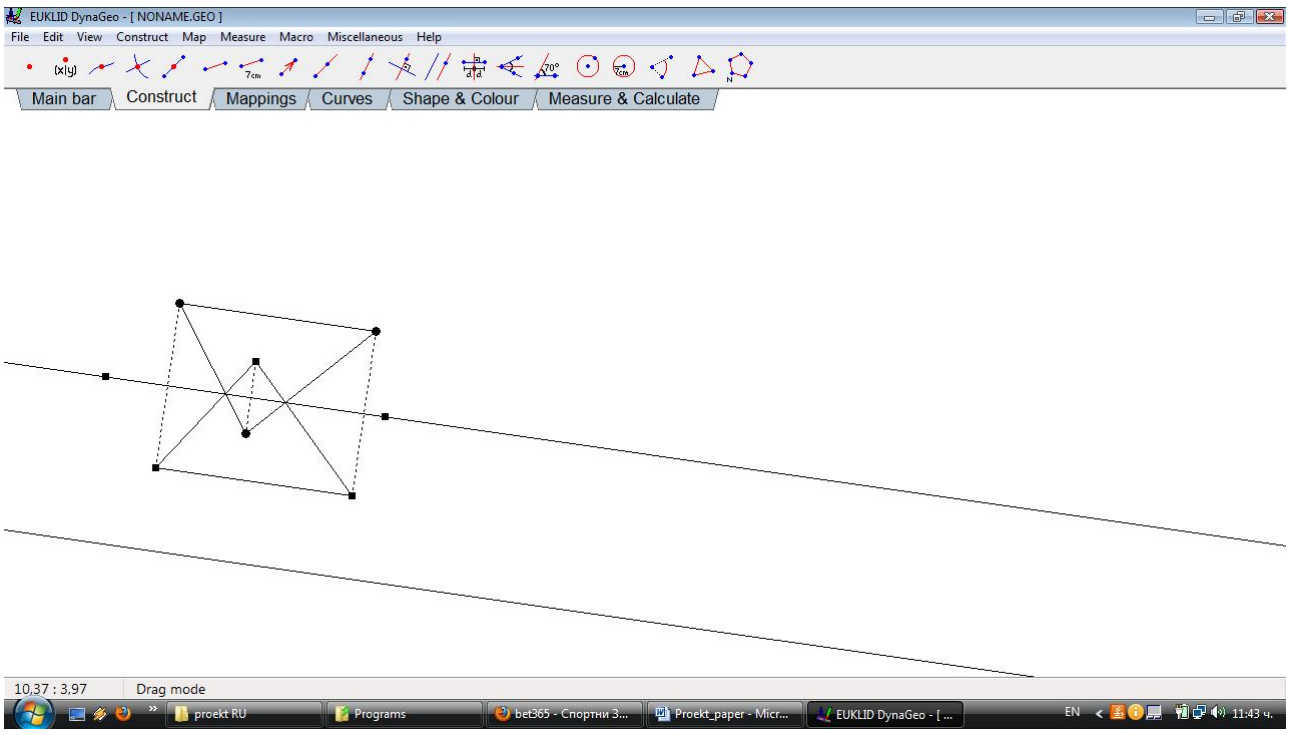

Figure 1.

The same is made in Geogebra and it looks this way:

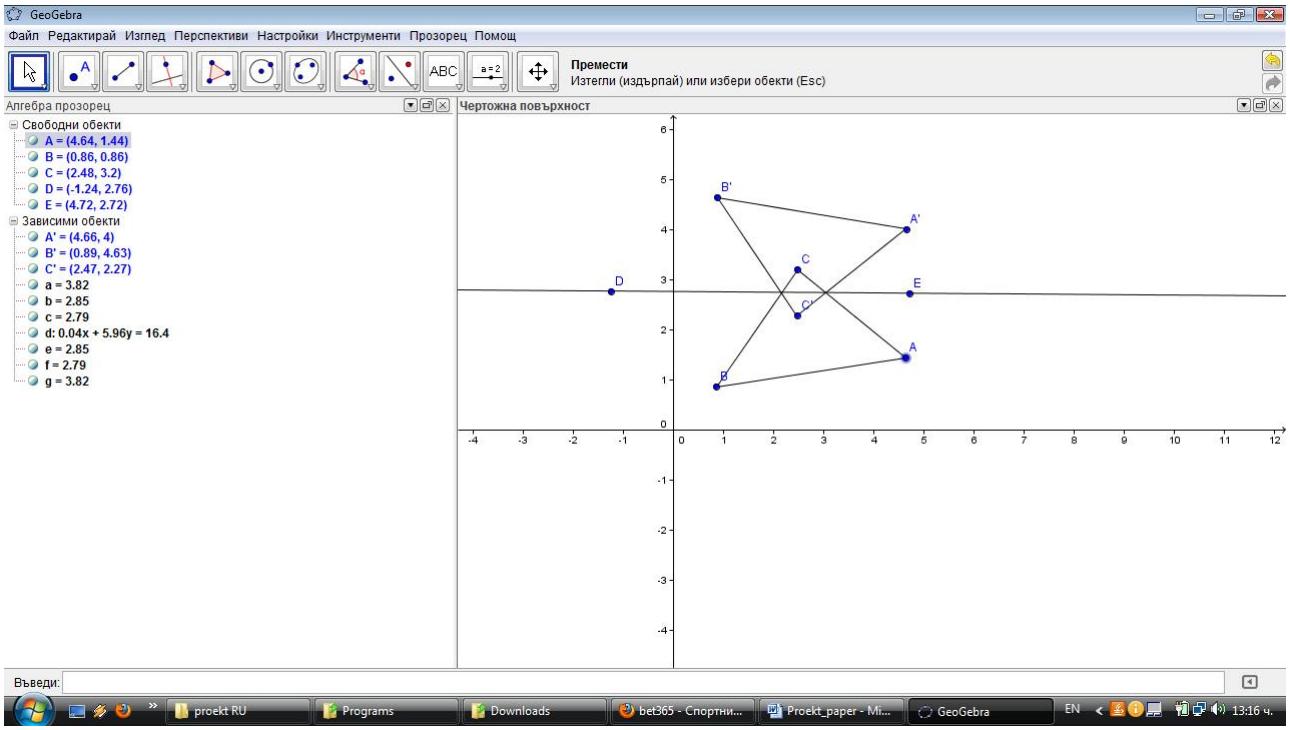

Figure 2.

And at last it is made on Cinderella:

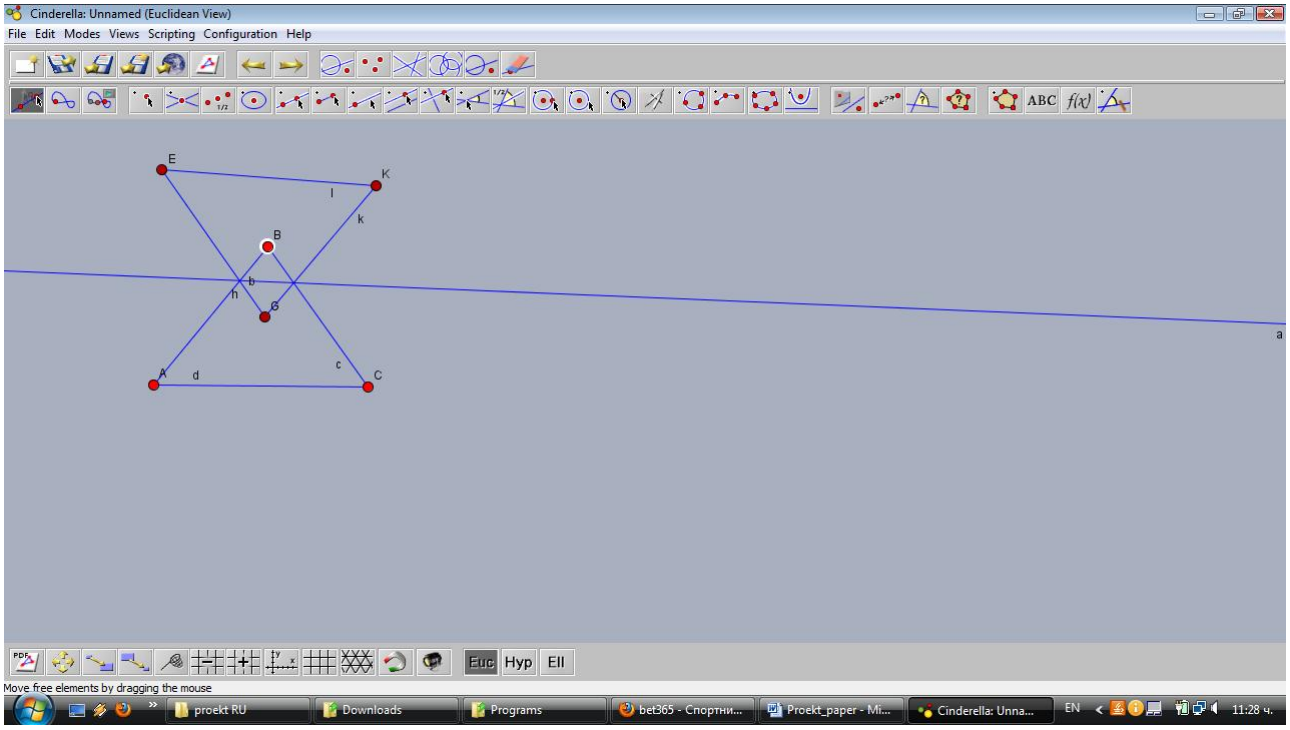

Figure 3.

CONCLUSION

The great interest of students and teachers to the PC determines its use in the process of learning (teaching) as a strong and motivating argument for the implementation

of the purposes that are set. The work of the students in surrounding with PC provokes them to investigate purposefully, it develops their imagination. The advantages of the PC as an assistant of the teacher are indisputable. It expands, increases and enriches the presented subject, but the teacher has the leading role of an instructor, organizer and inspireror of the learning process.

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ИЗПОЛЗВАНЕ НА ДИНАМИЧЕН СОФТУЕР ЗА ЧЕРТАНЕ В ЧАСОВЕТЕ ПО ГЕОМЕТРИЯ

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Резюме: *Статията представя някои софтуерни продукти за динамична геометрия. Съвременният учител е принуден все по-често да използва в своята дейност и персонален компютър. Тези продукти позволяват създаването и променянето на чертежите, както и експортиране в HTML формат.*

Ключови думи: *чертеж, динамичен геометричен софтуер, геометрични построения.*

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GEOGEBRA IN SCHOOL COURSE IN GEOMETRY

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Abstract: *This article provides an incremental approach to integration of mathematical software in teaching future teachers in Mathematics. For this purpose the subject School course in Geometry is selected. The emphasis is on developing and presenting some specific tasks in GeoGebra environment through static and dynamic text, animation, LaTeX formulas and tools - sliders and checkboxes.*

Keywords: GeoGebra, school course in geometry, approach and tasks

INTRODUCTION

The School Course in Geometry (SCG) is a discipline that is studied by students majoring in Mathematics and Informatics in the ***fifth semester*** of their preparation for teachers of mathematics.

The goal of the course is deepening the knowledge of students on a wide range of methods for solving tasks in geometry. The SCG is the basis for teaching disciplines such as ***Methods of Instruction in Mathematics (MIM)***, ***Extracurricular Work in Mathematics (EWM)***, etc.

The curriculum of the course covers the following topics: Logical construction course in geometry; Movements and equality; Figures in the plane; Vectors on the plane and in space; Similarity and homothety; Trigonometric functions of angles; Metrical dependencies in a triangle, polygons and circumferences; Area of figures; Geometrical constructions; Geometrical transformations; Geometry of complex numbers; Parallelism and perpendicularity in the space area; Coordinate method; Bodies in the space area; Volumes; Surface area; Sections [5].

The development includes exemplary elaborated tasks in GeoGebra environment, related to the educational content and in accordance with the curriculum in secondary school.

GeoGebra is a software tool that is used all over the world for creating mathematical constructions and dynamic drawings, which are suitable for application in teaching mathematics to students of different age groups. The positive aspects of the use of GeoGebra, which can be listed, are:

- Expanding and enriching learning environment through the use of interactive methods and strategies of teaching and learning.
- Improvement of the existing static forms of teaching materials and implementation of dynamic (multimedia) developments by integrating image, audio, animation and text.
- Electronic and multimedia teaching materials in the educational process [1, p. 146].

The proposed four basic stages of mastering the knowledge and skills to implement GeoGebra in teaching mathematics are:

- **First (Preparatory) stage.** Formation of interest to software by presenting its characteristics (basic menus, submenus and toolbars) and tasks that require simplified

geometric shapes and structures (in the course ***Mathematical software (MS)*** that is studied by students in ***third semester***).

➢ **Second stage.** Study on application of basic commands by developing a system of examples linked to educational content. Development of simple sample assignments and/or structures with the participation of learners. Presentation of tools ***for advanced learners*.**

➢ **Third stage.** Self-study process and preparation on learning resources by using GeoGebra. Presentation of results to appropriate audiences with an ***Interactive Whiteboard (IWB)*.**

➢ **Fourth stage.** Training on the methods of mathematics teaching and learning, based on GeoGebra software. Application of the knowledge and the skills in real learning process and evaluating the results of specialist [1, p. 147].

The second and third stages shall be presented to the students during their training at SCG and the fourth stage – during the ***Current Pedagogical Practice in Mathematics (CPPM)*** or ***MIM***.

IMPORTANT TERMS (FIGURES 1÷3)

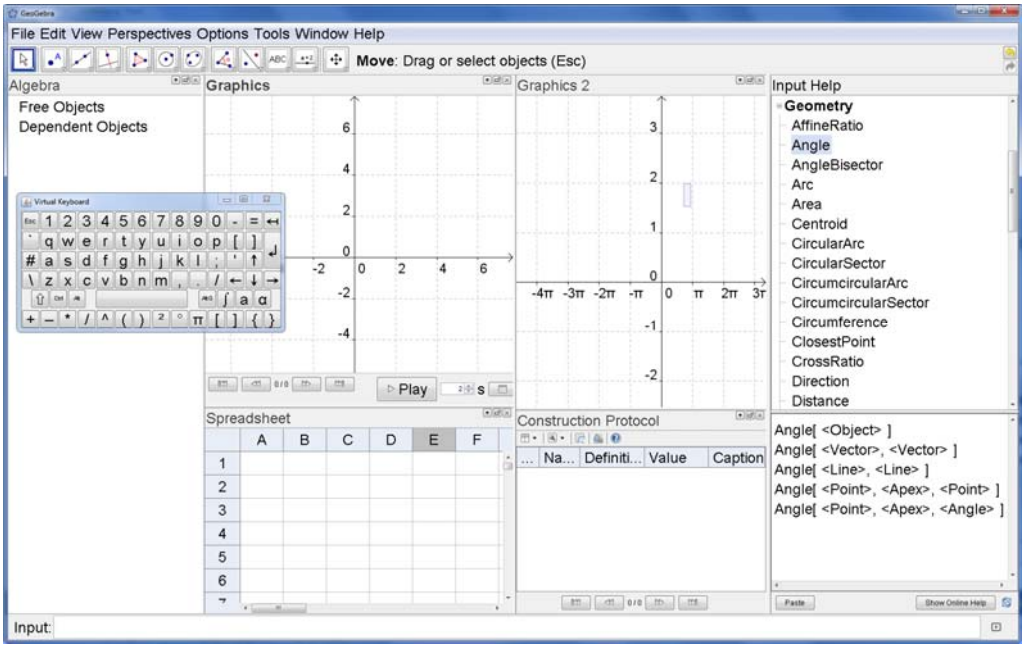

Figure 1: Interface

Figure 2: Toolbars

	***Slider***	Parameter that is changing in a predefined interval
	***Checkbox***	Checkbox for hiding and displaying objects

Figure 3: The tools Slider and Checkbox

THEORETICAL KNOWLEDGE. DEVELOPMENTS

All selected thematic developments are represented through Internet Browser. They have been elaborated in such a way as to provide multi-functional application - in a computer lab, during a presentation, IWB, in an individual work process.

The introduction of mathematical formulas in GeoGebra is performed through **LaTEX.**

What is LaTEX?

LaTEX is a word processing system, widespread in the scientific and academic communities. It is designed for making scientific documentation such as books, articles, dissertations, electronic textbooks in the areas of mathematics, physics, chemistry, engineering and others. LaTEX is using the program **TEX** (established in 1977 by the mathematician **Donald Knuth** for writing texts containing mathematical formulas) as a mechanism for word processing [2, p. iii], [4, p. 1 - 5].

How mathematical formulas in **LaTEX** appear without compiling text visualization?

This command

\frac{a}{b}=\frac{6}{7}

in **LaTEX** is equivalent to the command

$$\frac{a}{b}=\frac{6}{7},$$

or this command

\int{ \frac{4 (cosx)^{2}+3 (sinx)^{2}}{cosx}dx

have the equivalent of

$$\int \frac{4(\cos x)^2+3(\sin x)^2}{\cos x}\,dx.$$

The possibility of using **LaTEX** in the development of mathematical constructions in GeoGebra by students studying mathematics and informatics could contribute to the preparation of high-quality footage for secondary education.

The specifications of each structure are provided in advance.

1. Linear function of an independent variable

The following three examples involve two variables *a* and *b* which change at predefined interval [-50, 50]. This interval can be changed depending on the condition of the task (Figures 4÷6).

The structure includes a color graph, title, two sliders, two input fields of value, dynamic and static text.

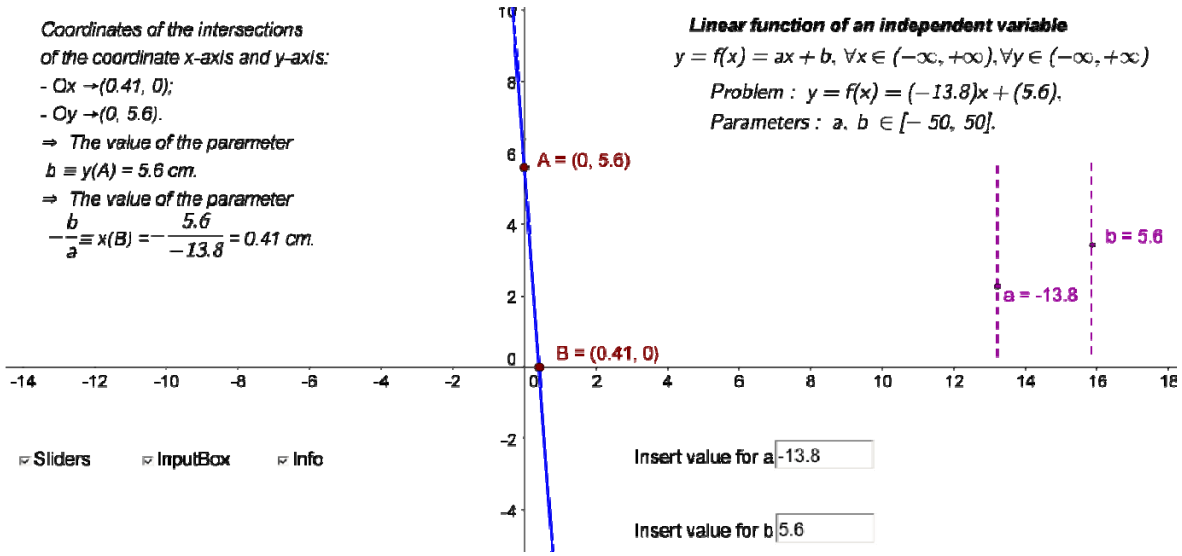

Figure 4: Problem 1

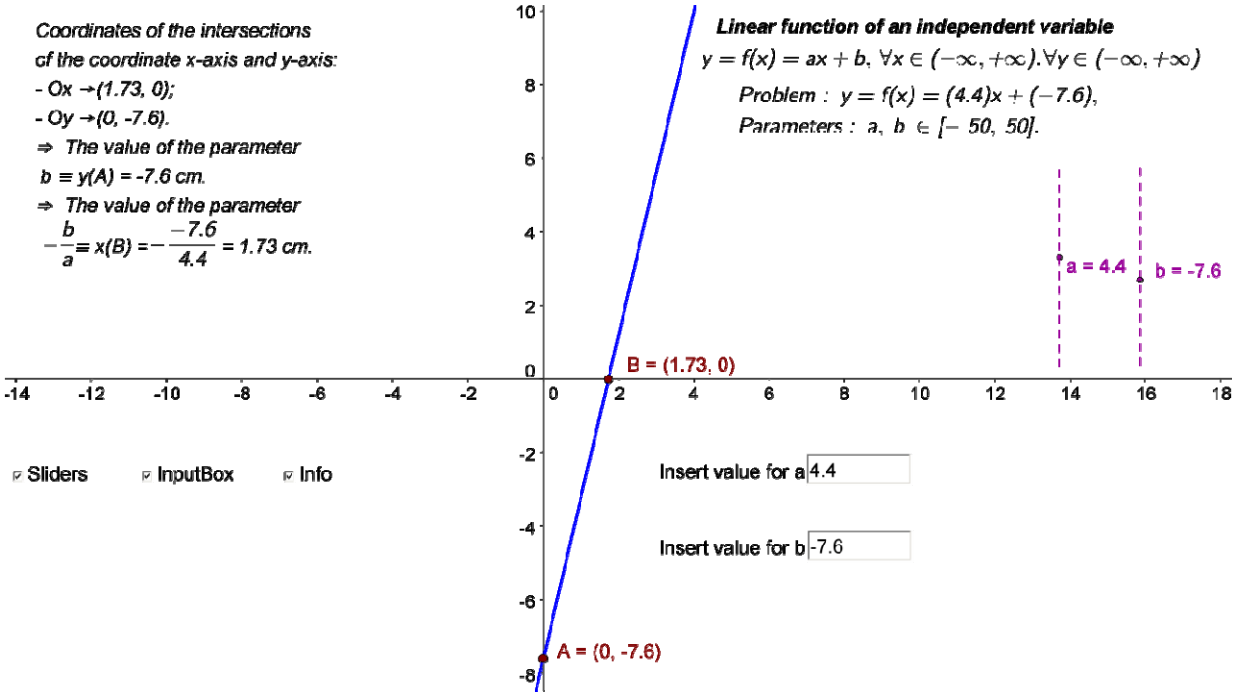

Figure 5: Problem 2

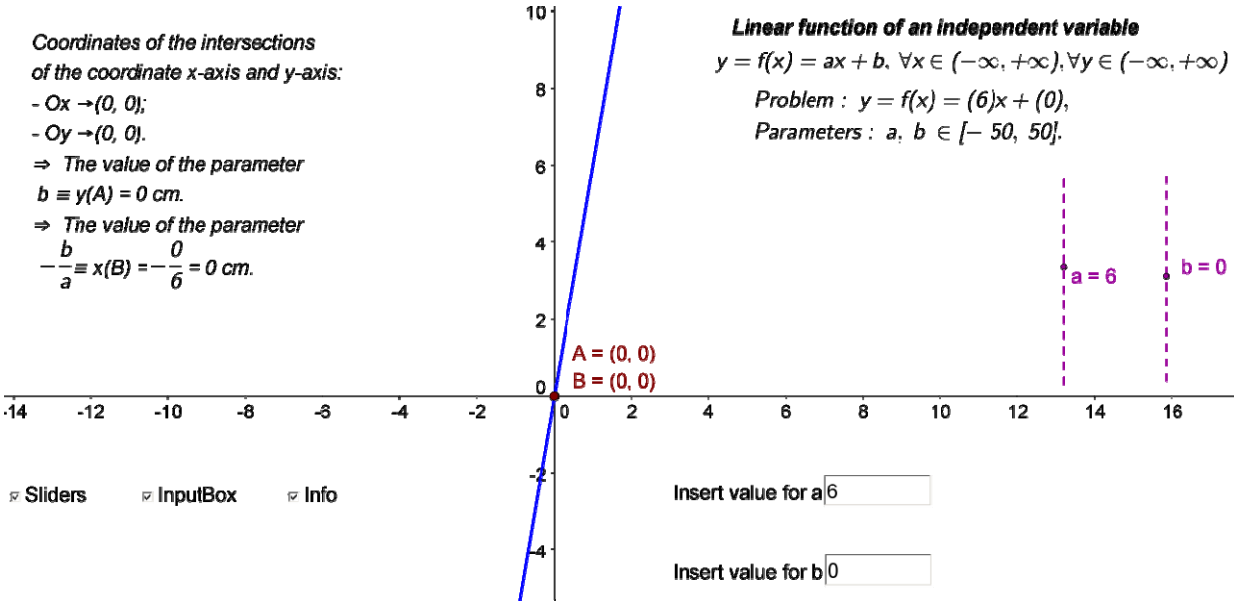

Figure 6: Problem 3

2. *Excircles of a triangle*

The structure includes a basic drawing, dynamic and static text and four sliders.

Construction (Figure 7):

- Construct a random triangle ΔABC.
- Prolong all the sides.
- Construct the three internal bisectors l_a, l_b, l_c.
- Construct the three external bisectors l_a^*, l_b^*, l_c^*.
- The excenters of the excircles are the points O_a, O_b, O_c, obtained by the intersection of the respective internal and external angle bisectors:

$$O_a = l_a \cap l_b^* \cap l_c^*; \quad O_b = l_b \cap l_a^* \cap l_c^*; \quad O_c = l_c \cap l_b^* \cap l_a^*.$$

- Draw the three circumferences, making perpendicular lines from the points O_a, O_b, O_c to the sides of the triangle. These would be the three radii of the three excircles [3, c.94].

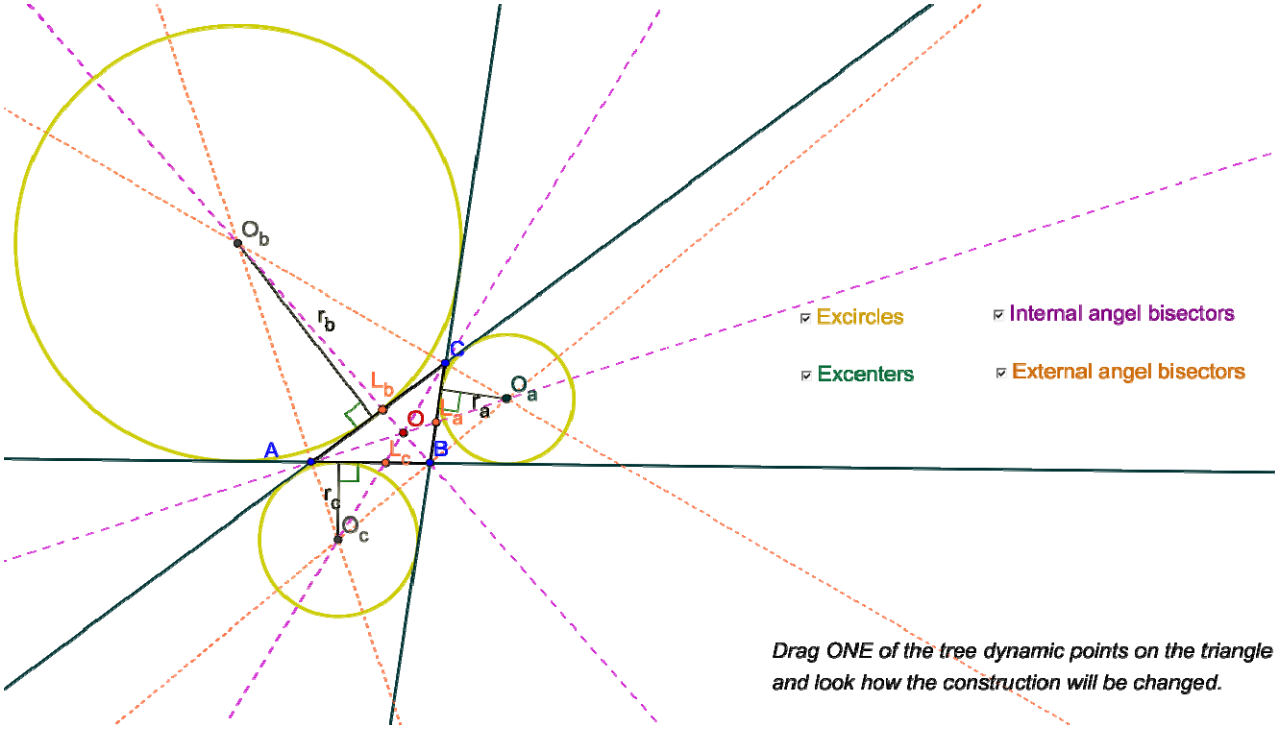

Figure 7: Excircles of a triangle with internal and external angle bisectors

3. Euler line.

The orthocenter, the centroid and the circumcenter of any triangle are aligned (Figure 8÷11). They belong to the same straight line, called ***line of Euler*** [6].

The construction includes drawing, static and dynamic text, four checkboxes.

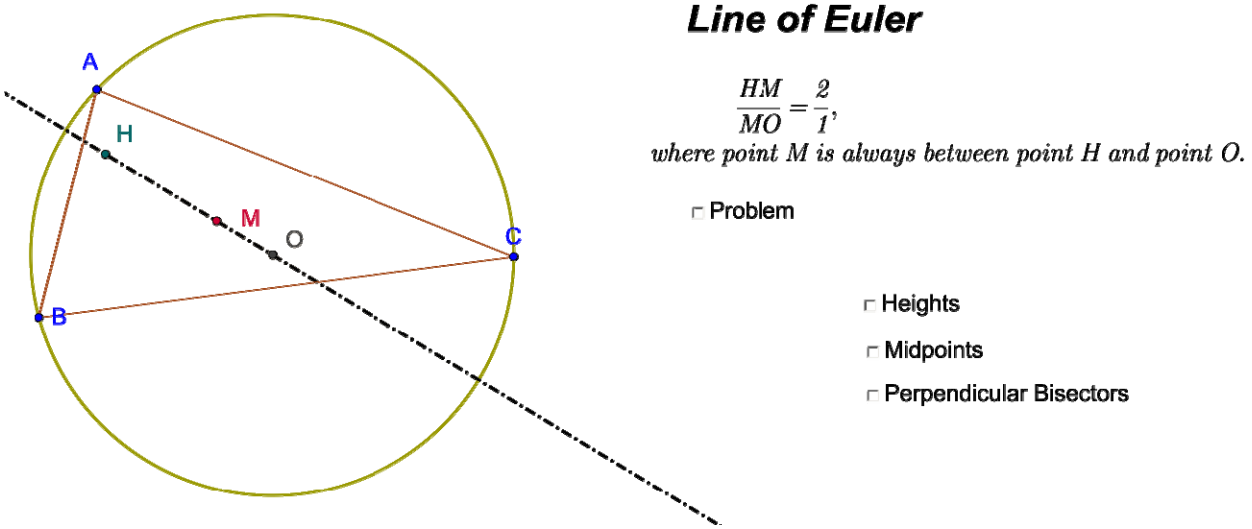

Figure 8: Line of Euler

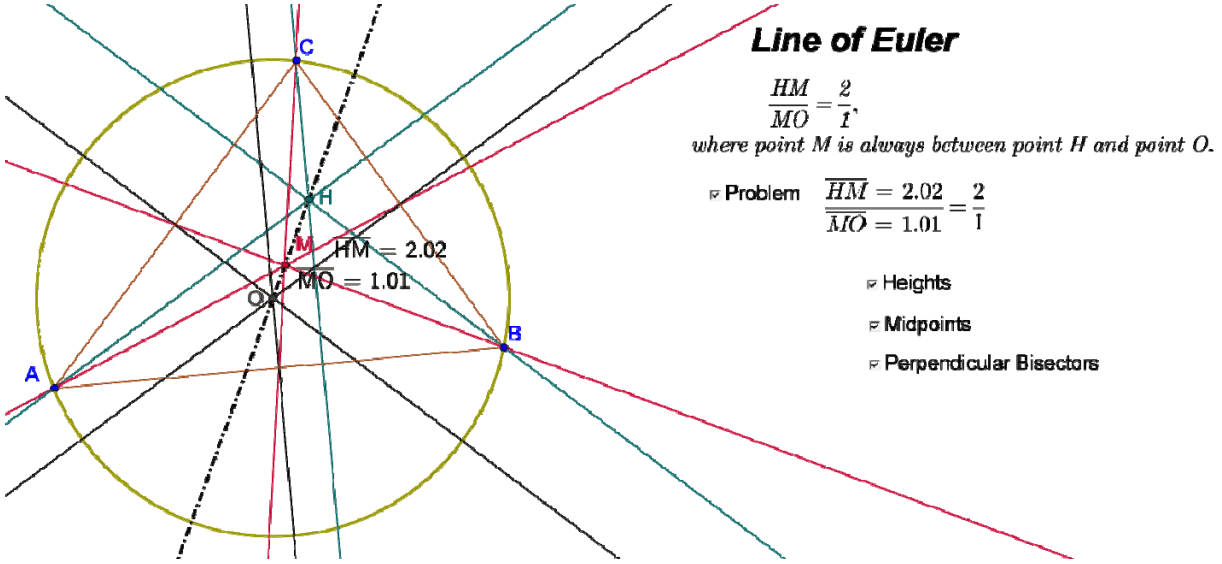

Figure 9: Problem 1 (acute-angled triangle)

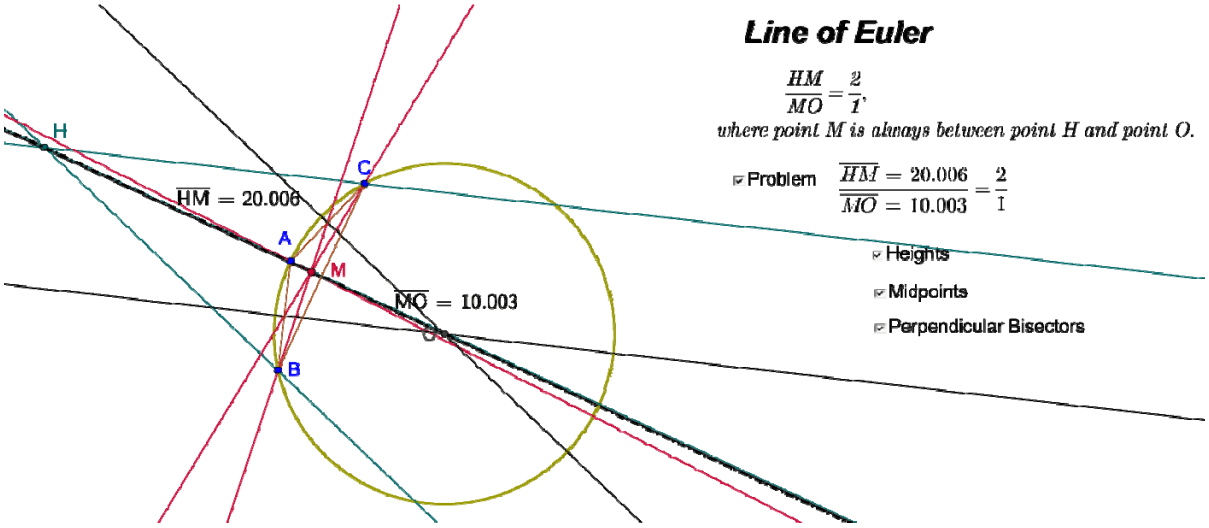

Figure 10: Problem 2 (obtuse-angled triangle)

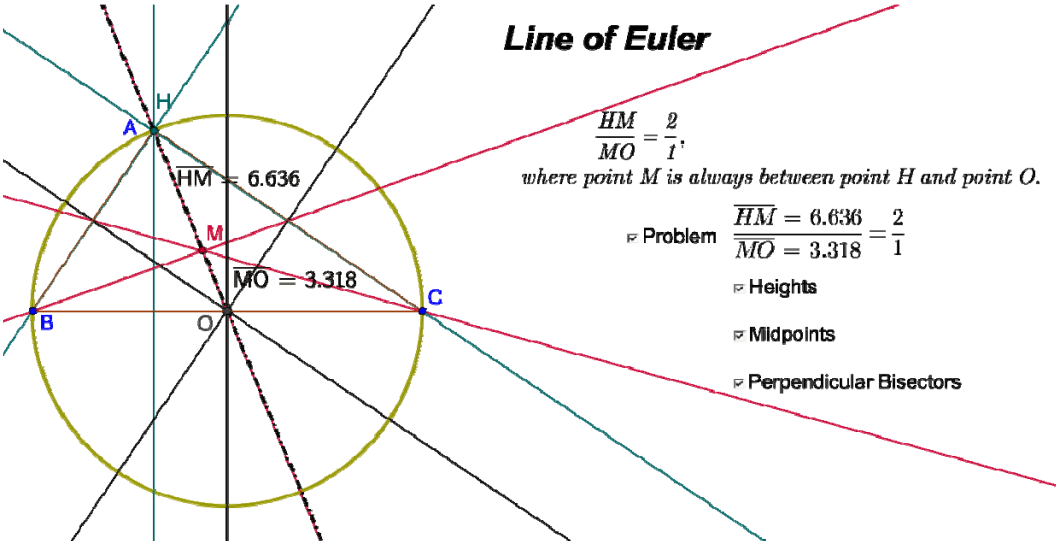

Figure 11: Problem 3 (right triangle)

4. Regular hexagon.

The construction contains a drawing, a slider (parameter), settings to start and stop the animation, static text, and tracking.

Construction (Figure 12 and 13):

- A parameter $r \in [1,25]$ is developed, where r is the length of the radii of all seven circumferences in the figure.
- The first circumference is the center is drawn, which appears central for the construction with a center and a point on it (point B).
- The point B will be the center of the second circumference with radius r.
- The intersections of the first to the second circumference are exactly those points, which are the centers of the next two circumferences with radii equal to r.
- Continue until all new sections are used as centers of circumferences, i.e. there must be six circumferences without central circumference.
- The following settings are additional: add color, insert animation, place trail static text, export to **html** format (suitable for training and implementation in presentations).

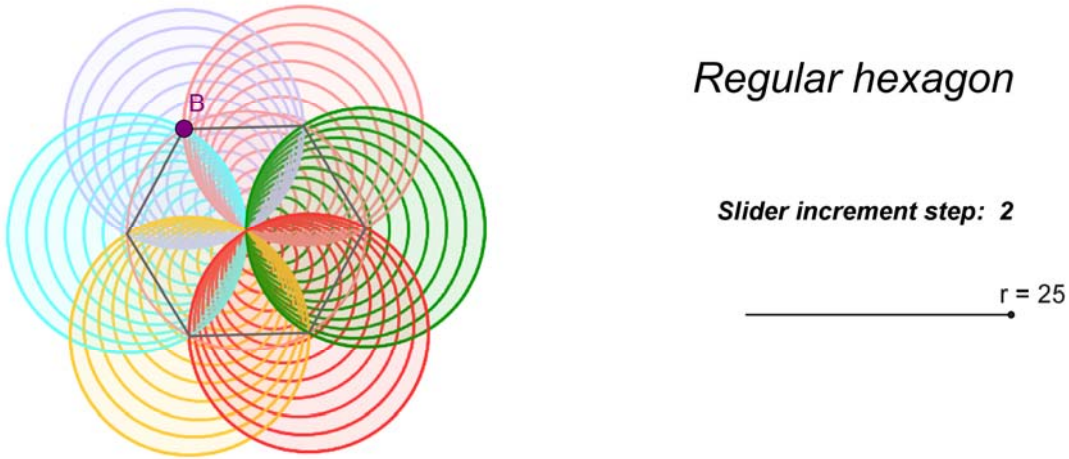

Figure 12: Regular hexagon (slider increment 2.00)

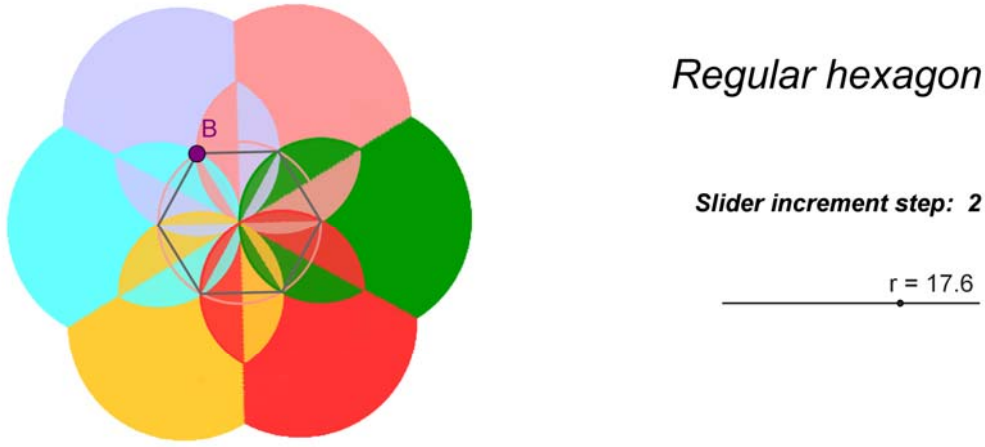

Figure 13: Regular hexagon (slider increment 0.02)

CONCLUSION

Working with software GeoGebra helps to raise the level of teaching mathematics. This software can be modified to the needs of the users (students, teachers, researchers). It is an easy tool for organizing examples of current teaching/ presentation and for complex structures, requiring extensive research in a given area.

The study of GeoGebra in ***SCG*** contributes to the creative development and improvement of future math teachers during their training.

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GEOGEBRA В УЧИЛИЩНИЯ КУРС ПО ГЕОМЕТРИЯ

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Резюме: *В тази статия е представен примерен постъпков подход за интегриране на математически софтуерен продукт по време на обучаването на бъдещи учители по математика. За тази цел е избрана дисциплината Училищен курс по геометрия. Акцентирано е върху разработването и представянето на няколко конкретни теоретични постановки в GeoGebra среда чрез статичен и динамичен текст, анимация, LaTeX формули и инструменти - плъзгач, чекбокс (поле за отметка).*

Ключови думи: *GeoGebra, Училищен курс по геометрия, подход и примери*

web: suruse.uni-ruse.bg

APPLICATION OF MATLAB SOFTWARE FOR DIGITAL IMAGE PROCESSING

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Angel Kanchev University of Ruse

***Abstract**: In this article an application of software product MATLAB in digital image processing using recursive discrete Kalman filter is presented. The potentiality of the product is analyzed examining the influence of the number of iterations in different disperse of the adaptive Gaussian noise upon the quality of the filtered image.*

***Key words**: digital image, discrete recursive Kalman filter, Gaussian noise, iterations, MATLAB.*

INTRODUCTION

The system MATLAB („MATrix LABoratory") is a software environment for numerical analysis and independent fourth generation programming language. Created by the multinational software corporation for technical applications "The MathWorks", MATLAB allows operations using matrices, functions' drawing, data presenting, algorithm program implementation, development of human-machine interfaces and interfaces with different software products written in different programming languages [6]. One of the most important advantages of the system MATLAB is its ability to expand in purpose to be used as a tool for resolving new scientific tasks. This can be realized by creating a sequence of packages which can serve for expanding the common mathematical abilities of the system (symbolic calculations, optimization, statistics) as well as realization of the directions in computer mathematics like mathematical modeling, building an architecture of neural networks and fuzzy conclusions. MATLAB is used by more than 1 million academic and business users [6]. One possibility for applying the program environment MATLAB in digital image processing using discrete Kalman filter is presented.

DIGITAL IMAGE PROCESSING USING DISCRETE KALMAN FILTER

No matter the ways of obtaining, a given image is always accompanied with existence of noise and embarrassment of different kind. A main problem in its processing is removing the noise when keeping the important data for further details [7].

In most of the cases the most important goal of the image processing is to obtain a contour. This requires a sequence of procedures (filtration of Gaussian and impulse noise, segmentation aiming to receive black and white image, extraction of the contour line) which should be picked correctly of certain type and in determined sequence. An application of discrete Kalman filter in filtration of Gaussian at radiolocational images of dynamic objects (aircrafts) will be the case under review.

The Kalman filter is consistent recursive algorithm, using a model of dynamic system for receiving marks, which can be edited after every new measurement in temporal sequence [2,9]. This algorithm finds appliance in the process of managing complex dynamic processes, in which it is important to know the stage in each moment of time. The Kalman filter has a possibility to adjust in case of not precisely chosen parameters of the mathematical models. The advantage of the approach is that it does not require huge computational resources and can evaluate the condition vector in real time [5].

▪ ***Mathematical model of Discrete Kalman filter***

The equations of the condition and observation are given:

$$x_k = Ax_{k-1} + \omega_{k-1} \quad (1)$$

$$z_k = Hx_k + \nu_k, \quad (2)$$

where $x_k \in R^n$ is the state variable in *k*-th moment, and $z_k \in R^m$ is the observation variable. Here A is $n \times n$ matrix, connecting the variables of the conditions of the current step *k* and the previous step *k-1*, and H is $m \times n$-matrix connecting the observation and condition in the *k*-th step. The vectors w_k and v_k are normally distributed random variables with zero mathematical expectation and covariance matrixes:

$$Q_k = E\left[w_k w_k^T\right], \qquad R_k = E\left[v_k v_k^T\right], \quad (3)$$

where E is the mathematical expectation [8].

With $\hat{x}_k^- \in R^n$ is marked the priori mark of the condition variable in the *k*-th step, and to $\hat{x}_k \in R^n$ corresponds a posteriori evaluation in given observation z_k. Priori and posteriori errors are defined in the following way:

$$e_k^- \equiv x_k - \hat{x}_k^-, e_k \equiv x_k - \hat{x}_k \quad (4)$$

Then the priori covariance matrix of the error will look like:

$$P_k^- = E\left[e_k^- e_k^{-T}\right], \quad (5)$$

and the posterior covariance matrix of the error will be respectively:

$$P_k = E\left[e_k e_k^T\right]. \quad (6)$$

Posterior condition mark is a nonlinear combination of the priori assessment $\hat{x}_k^-$ and the difference between provided measurement $H\hat{x}_k^-$ and the real measurement z_k, multiplied with $n \times m$ weight matrix K_k [4].

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - H\hat{x}_k^-\right). \quad (7)$$

The Kalman matrix K_k in (7) has the look:

$$K_k = P_k^- H^T\left(HP_k^- H^T + R_k\right)^{-1}. \quad (8)$$

▪ ***Software realization of the model in MATLAB environment***

The function which realizes the Kalman algorithm is the function s=kalmanf(s) [9]. Typical for this function is that it is executed for every dot of the image. It uses the primary data from function start_kal (Nt). In the beginning a validation check is executed to ensure the main parameters and corresponding assignments. Based upon the choice made concerning the number of filtering, Nt times filtering are executed. This is set as a parameter in function start_kal and is chosen from a menu. Appliance of Kalman filter for every row of the image starts, as it starts from the second column. The typical special feature here is that the input data initialized in the beginning are edited for each pixel from the image based on the previous filtered pixel and if the previous pixel is the first it is based on it. The filtration algorithm includes procedure for brightness segmentation (Otsu method) [3] and contour receiving (Roberts operator).

```
function varargout = kfig(varargin)
% Visualization Function
gui_Singleton = 1;
gui_State = struct('gui_Name', mfilename,
if nargin & isstr(varargin{1})
  gui_State.gui_Callback = str2func(varargin{1});
end
if nargout
  [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
  gui_mainfcn(gui_State, varargin{:});
end
function kfig_OpeningFcn(hObject, eventdata, handles, varargin)
% Choose default command line output for kfig
handles.output = hObject;
% Update handles structure
guidata(hObject, handles);
% Executing actions when visualizing the object
if strcmp(get(hObject,'Visible'),'off')
  I = imread('I1gaus.bmp','bmp');    % Load file (with matrix size 75x100) in I
  mshow(I);
end

% --- Outputs from this function are returned to the command line
function varargout = kfig_OutputFcn(hObject, eventdata, handles)
varargout{1} = handles.output;

% Start (pushbutton1) button Kalman filtering
function pushbutton1_Callback(hObject, eventdata, handles)

  global I;                  % We create I global matrix
bw = get(handles.slider1,'Value') % Load from Slider in bw (value for black/white)
popup_sel_index = get(handles.popupmenu1, 'Value'); % From the drop down menu
(n) times repetitions in k filter.
switch popup_sel_index
  case 1 % 1 път
          start_kal(1); % Function Start Kalman
          fig_bw(bw);   % Function towards black/white
  case 2 % 3 times
          start_kal(3);
          fig_bw(bw);
  case 3 % 5 times
          start_kal(5);
          fig_bw(bw);
  case 4
          start_kal(10);
          fig_bw(bw);
  case 5
          start_kal(20);
          fig_bw(bw);
```

```
  case 6
          start_kal(30);
          fig_bw(bw);
end
function start_kal(Nt)
figure(1);
global I
Im=I
clear s
s.x = 1;
s.A = 1;
s.Q = 2^2;              % Measurement
s.H = 1;
s.R = 2^2;
s.B = 0;
s.u = 0;
s.x = nan;
s.P = nan;

for t2=1:Nt
for t1=1:346
for t=2:572
  tr(end+1)=I(t1,t);
  s(end).z=tr(end) ;
  s(end+1)=kalmanf(s(end));        % Filtration
  Im(t1,t)=s(end).x;
end
  clf;
  J = imresize(Im,4);
  imshow(J);
end
I=Im;
end
global Im;

function fig_bw(bw)
figure(2);
global Im;
bw_210 = Im > bw;
bw_210 = imresize(bw_210,4);
imshow(bw_210);title(['Prag B/W # ',int2str(bw)]);
global bw_210;
end
return
```

AN APPROACH FOR DETERMINING THE OPTIMAL NUMBER OF ITERATIONS FOR DIFFERENT DISPERSE OF GAUSSIAN NOISE IN FILTRATION WITH DISCRETE KALMAN FILTER

The optimal number of iterations in different disperse of Gaussian noise is determined by basing on quantity characteristics of two geometrical characteristics of the object (aircraft) – fuselage axis length and wingspan of the aircraft. The fuselage axis length and the wingspan are measured in number of pixels, respectively between the "nose" and "tail" of the aircraft and the end points of the two wings. The optimal number of iterations is determined by using the following criteria: the values of the chosen geometrical characteristics, received after image filtering to be most similar to the corresponding etalon aircraft before filtration. The change of the geometrical characteristics is examined in different number of iterations with discrete Kalman filter. Digital images from 10 types of aircrafts, excepted as etalon, have been used. They have been chosen by random pick of data bases [1]. They are: F16, An 124, Mc Donnell, B 52, Buccaneer, F117, Jaguar, Mig 29, Miraj 2000, Su34. A random image is chosen out of the choice. It is noised with adaptive Gaussian noise with normal distribution at zero mathematical expectation and disperse relatively 0,03; 0,05; 0,07. The results of the filtration are examined after 5, 10, 20 and 30 iterations. The results after filtration of etalon object are given respectively in Table 1, Table 2 and Table 3.

Table 1. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,03

Geometrical characteristics	**Number of iterations**			
	5	10	20	30
Fuselage length	60	62	63	58
Wingspan	39	40	41	37

Table. 2. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,05

Geometrical characteristics.	**Number of iterations**			
	5	10	20	30
Fuselage length	55	57	60	56
Wingspan	35	36	38	33

Table.3. Geometrical characteristics of an object, received after filtration with Kalman filter-disprse 0,07

Geometrical characteristics	**Number of iterations**			
	5	**10**	**20**	**30**
Fuselage length	40	42	42	38
Wingspan	28	29	30	27

The results are given graphically on *Fig. 1.* They show that the filter gives best results at 20 iterations and Gaussian noise disperse 0,03

For etalon object, presented in black color, the wingspan is 69 pixels and the fuselage axis length – 45 pixels.

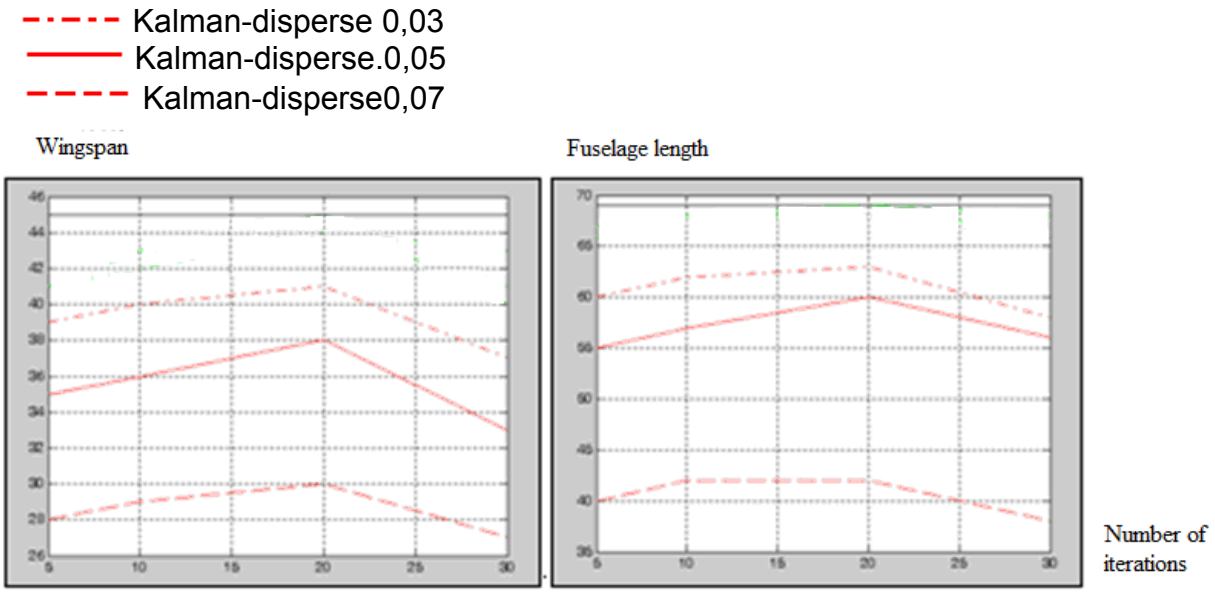

Fig. 1. Dependence of the values of the geometrical characteristics upon the number of iterations in Discrete Kalman filter

Stages of the image processing wish Discrete Kalman filter at 0,03 and 20 iterations are visualized on *Fig. 2*. In Table 4 are given the geometrical characteristics of all the aircrafts, noised with adaptive Gaussian noise (disperse 0, 03) after filtrating with Kalman filter.

Table. 4 Aircrafts geometrical characteristics from the sample noised with adaptive Gaussian noise with adaptive Gaussian noise (disperse 0, 03) after filtration with Kalman filter (20 iterations)

Type of aircraft		Etalon object		After filtration of Kalman Filter	
		Fuselage length	Wingspan	Fuselage length	Wingspan
1	**F16**	50	33	47	29
2	**AN 124**	91	100	89	90
3	**Mc Donnell**	69	59	66	57
4	**Buccaneer**	50	36	47	33
5	**F117**	51	38	48	35
6	**Jaguar**	80	45	75	41
7	**Mig29**	43	38	39	32
8	**Miraj 2000**	50	31	47	26
9	**Su34**	87	61	84	55
10	**B52**	69	45	68	44

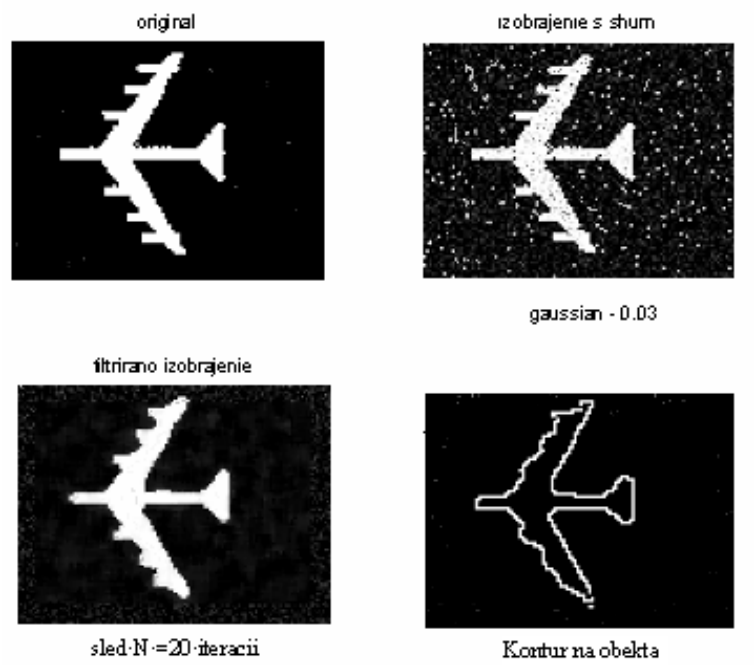

Fig. 2. Stages of image processing with Discrete Kalman filter in disperse 0,03 and 20 iterations

As a criteria for accuracy is used the relative error [8]:

$$\hat{\delta}_x = \frac{\delta_x}{x_{изм}}.100, \quad \%, \tag{9}$$

where $\delta_x = |x_{em} - x_{изм}|$ - absolute error;

x_{em} is the value of the parameter of the etalon object:

$x_{изм}$ - value of the parameter's object after Kalman filter applying

Values of the relative error for the examined types of aircrafts, calculated on (9), are showen in Table 5.

Table.5 Values of the relative error for the studied types of aircraft

Type of aircraft	$\hat{\delta}_x$, %	
	Length of the fuselage axis	Wingspan
F16	6.38	13.79
AN 124	1.12	11.11
Mc Donnell	4.55	3.5
Buccaneer	6.38	9.09
F117	6.25	8.57
Jaguar	6.67	9.75
Mig29	10.25	18.75
Miraj 2000	6.66	19.23
Su34	3.57	10.9
B52	3.7	6.25

For image filter using the Kalman method is created graphical user interface (GUI). The overall appearance is presented on *Fig. 3.*

Fig. 3. Overall appearance of GUI after filtration for digital image using the Kalman method

The interface gives the following possibility for choice:

- Number of repetitions for filtration;
- Grey threshold – for receiving the binary image;
- Object detecting in the image.

The input and the filtered image are visualized.

CONCLUSION

The presented development shows one application of program product MATLAB in software realization for created mathematical model, which gives possibility for visualizing the stages of image processing as well as examining the influence of iteration numbers of Discrete Kalman filter upon the quality of the filtered image.

For reasonable error usually is accepted the value $\hat{\delta}_x$=5 %. From the results we can see that the relative error when using Kalman filter reaches up to 10% for length of the fuselage axis and up to 19% for wingspan. The result gives rise for looking to another filter (eventually based on neural network), which would give better results (minimization the error), connected with the inputted geometrical characteristics.

This work is appropriate for classes, related to Digital image processing. It could be an interesting subject for graduate students studying Computer systems, Mathematics, Informatics, Radiolocation and navigations and so on.

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ПРИЛОЖЕНИЕ НА ПРОГРАМЕН ПРОДУКТ MATLAB ПРИ ОБРАБОТКА НА ЦИФРОВА ИНФОРМАЦИЯ

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***Абстракт**: В статията е представено приложение на програмен продукт MATLAB при обработка на цифрово изображение чрез дискретен рекурсивен Калманов филтър. Анализирани са възможностите на продукта при изследване влиянието на броя итерации, при различна дисперсията на адитивния Гаусов шум, върху качеството на отфилтрираното изображение.*

***Ключови думи**: цифрово изображение, дискретен рекурсивен Калманов филтър, Гаусов шум, итерации, MATLAB.*

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